

# Optimizing Railroad Tank Car Safety Design to Reduce Dangerous Goods Transportation Risk

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## Abstract

This paper describes an optimization model developed to determine the most efficient way to enhance the safety design of tank cars used to transport toxic inhalation hazard (TIH) materials in North America and a quantitative risk analysis conducted to assess the risk and potential benefit of implementing enhanced safety-design tank cars. The conditional probability of release and the car's weight were used as the proxy for safety and cost, respectively. The concept of Pareto optimality is introduced to evaluate possible alternative tank car safety designs. A spatial analysis of the rail routes for TIH transport was conducted using geographic information system (GIS) data and software. The results were combined with probabilistic estimates of a release accident and the likelihood of exposure of the population along the routes. Risk profiles or "F-N curves" for both the baseline and alternate design tank cars are presented along with the corresponding risk reduction estimates. The results of this research were used by the Association of American Railroads in their development of new specifications for TIH tank cars, which are estimated to reduce the risk of rail transportation of these products by approximately 60 to 70 percent.

## Introduction

Railroads are the leading transporter of dangerous goods in North America. In 2006, there were approximately 1.5 million rail shipments of these materials in the U.S. and Canada, and approximately 87% of them were transported in tank cars [4]. While rail safety has improved considerably over the past two decades [7], several recent high-profile train accidents [11, 12], that resulted in fatalities due to releases of toxic inhalation hazard (TIH) chemicals have put rail safety under increased scrutiny. In addition to ongoing rail infrastructure and train control improvement efforts to reduce the likelihood of accidents, the railroad industry via its standard setting organization, the Association of American Railroads (AAR), initiated development of new, safer tank car design specifications for transportation of TIH materials [2]. The objective of this effort was to reduce the likelihood of a release if a tank car is involved in an accident. A key question facing the industry was what enhancements to tank car design would most effectively improve their safety?

This paper presents an optimization model that was used to identify the most efficient combination of enhancements to improve tank car safety design in the AAR effort. A risk analysis was also conducted to estimate the potential reduction in risk through use of safer tank cars. The results of this analysis are illustrated by comparing the risk associated with the baseline tank car designs currently used for transportation of Chlorine and Anhydrous Ammonia, with the new, enhanced specifications adopted by the AAR.

Risk in general can be defined as the product of the probability and the consequences of an event [5]. In the context of railroad hazardous materials transportation, a simplified definition of risk is as follows:

$$R = P_R \times P_C \times C \quad (1)$$

where:

R = annual risk of transporting a hazardous material

$P_R$  = annual rate of a release accident

$P_C$  = probability of a particular release scenario occurring

C = consequence level (defined here as the number of people affected)

The overall risk analysis framework employed in this study is summarized by the decision diagram in Figure 1. The diagram provides an overview of the principal inputs affecting the risk calculation, and the relationship between them. Figure 2 shows a decision tree with specified variables and outcomes used in the risk analysis model we developed. Table 1 characterizes and defines the principal elements of the model, and the individual sections that follow explain each one in more detail.

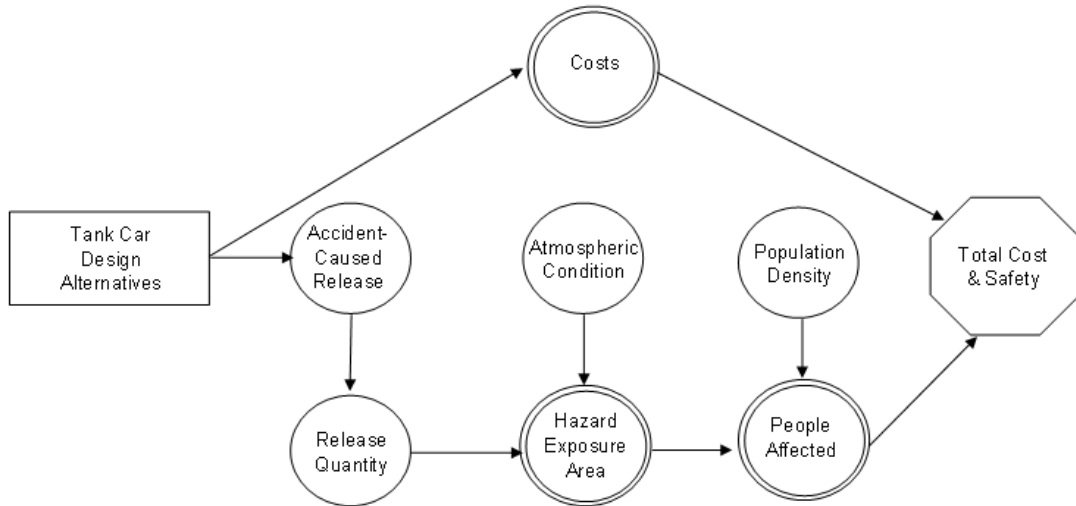


Figure 1: Decision diagram for consideration of alternate tank car designs

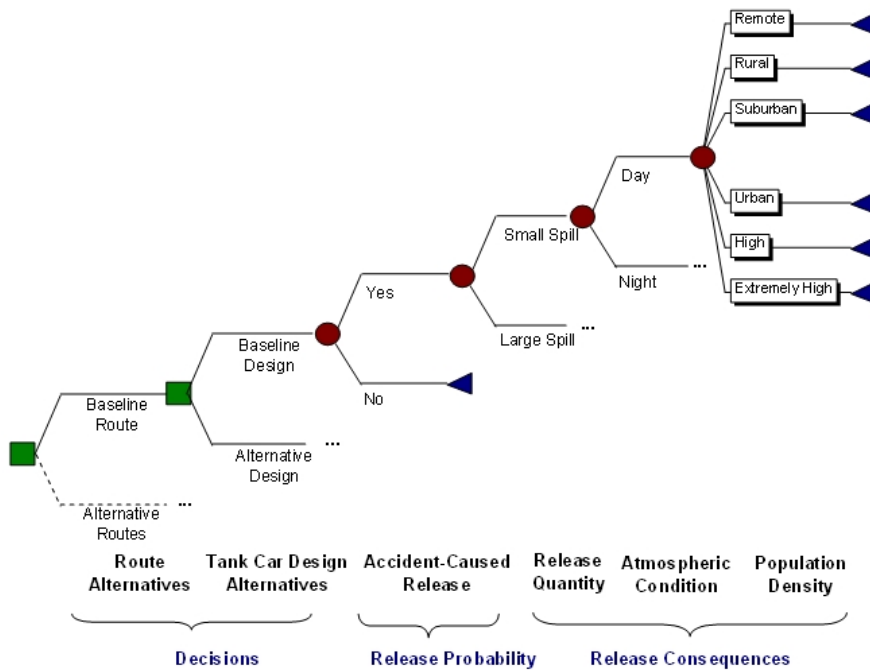


Figure 2: Generic decision tree used for alternate-design tank car consideration

The rectangle in Figure 1 represents the decision variable regarding replacement of current tank cars with alternate design cars. Nodes in the diagram represent specific uncertainty events for

each step in the risk analysis process. These include the likelihood of an accident-caused release, which is directly influenced by the damage-resistance of a tank car's design. Release quantity and atmospheric conditions are two key variables affecting the consequence model used in this analysis. These two variables affect the size of the exposure area as indicated by the double-circle node in Figure 1 with arrows from the release quantity and atmospheric condition uncertainty nodes. Together with population density distribution, the hazard area determines the number of people affected. The decision diagram also indicates that there is a cost associated with the decision of replacing the current baseline tank car design. The octagon denotes the overall decision objectives where safety is considered along with cost.

Risk Element	Definitions
Tank Car Design Alternatives	Transport Chlorine using the baseline 263,000 lb. GRL DOT-105A500W tank cars or in tank cars with enhanced top fittings protection equivalent to 286,000 lb.-GRL DOT-105J600W. In the case of Anhydrous Ammonia, the decision is between using the baseline 263,000 lb. GRL DOT-112J340W or the equivalent to a 286,000 lb. GRL DOT-112J500W with enhanced top fittings protection.
Accident-Caused Release	Annual rate of release that accounts for accident rate, tank car conditional probability of release in an accident, annual number of shipments and differential capacity.
Release Quantity	Quantity of release from a tank car. Classified as either a small spill with 5% or less of the car's capacity, or a large spill with more than 5% of the car's capacity using the scenarios considered in the U.S. Department of Transportation (DOT) Emergency Response Guidebook (ERG).
Atmospheric Condition	Atmospheric conditions during day or night time scenarios considered in the U.S. DOT ERG.
Population Density	Population density distribution along the U.S. rail network.
Hazard Exposure Area	Hazard exposure area (square miles) per the chemical-specific exposure model in the U.S. DOT ERG.
People Affected	Number of people that need to be evacuated or sheltered in-place as recommended in the U.S. DOT ERG

Table 1: Definitions of various terms used

### Tank Car Safety Design Optimization

Table 2 summarizes the baseline tank car safety design features for Chlorine and Anhydrous Ammonia. The conditional probability of release given that a car is derailed in an accident,  $P_{RIA}$  is calculated for each baseline design using the regression model in Treichel et al 2006 [16]. Tank car light or empty weight and capacity were estimated using a tank car size and weight program [13].

<b>Commodity</b>	<b>Head Shield Type</b>	<b>Head Thickness (in.)</b>	<b>Shell Thickness (in.)</b>	<b>P<sub>RIA</sub></b>	<b>Nominal Capacity (gal.)</b>
Anhydrous Ammonia	Full-Height	0.6080	0.6080	0.0788	33,016
Chlorine	None	0.7870	0.7870	0.0509	16,043

Table 2: Baseline tank car designs

There are three principal sources of release from tank cars in TIH service: the tank head, shell, and top fittings (Figure 3). These are also the locations on the car that are candidates for design enhancements to reduce the likelihood of release if the car is derailed in an accident. A maximum 286,000 lb. gross rail load (GRL) was assumed for the enhanced cars compared to 263,000 lb. GRL for the baseline cars. The higher maximum GRL for the enhanced-safety car helps offset the weight penalty imposed by addition of extra safety features such as a thicker tank head or shell.

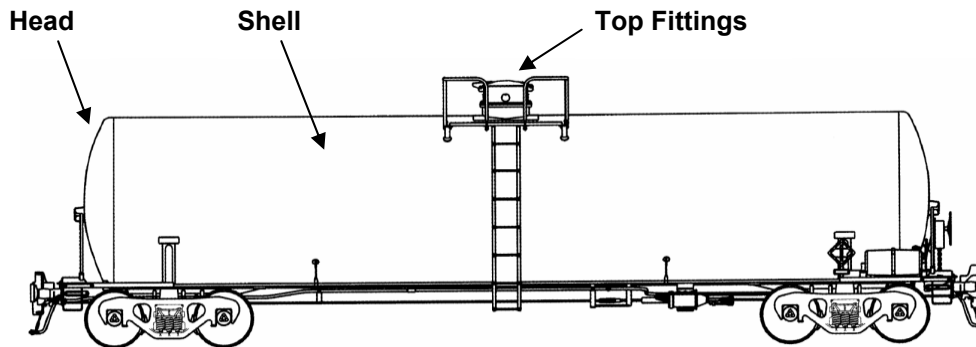


Figure 3: An example of a DOT pressure-specification tank car in TIH service

Enhanced top-fittings protection was included as part of the new tank car design analysis. We assumed that the new design reduces the likelihood of a release from the top fittings by 50% compared to the baseline performance of the current chlorine car top fittings protection. This assumption was a conservative estimate based on analysis of a new design developed by TrinityRail (authorized under U.S. DOT Special Permit 14167) that found that the rollover velocity that caused top-fittings failure was 2.6 times higher for the enhanced-design fittings compared to the baseline, chlorine-car-design for top-fittings protection [8].

The conditional probability of release, given that a tank car is derailed in an accident,  $P_{RIA}$ , and the tank car weight, was enumerated for each possible configuration of alternate design tank car using a 45-by-45 matrix of shell and head thicknesses (considered in increments of 1/16 of an inch). In addition to the increases in the tank head and shell thickness, the effect of 1/2-inch, full-height, head shields was also considered for the enhanced tank car design alternatives. The conditional probability of release and the light weight of a car were used as the proxy for safety and cost, respectively. Mathematical representation of the two objective functions and the weight constraint were considered as follows:

$$F_{m,n} \propto RRO_j \quad (2)$$

where:

- $F_{m,n}$  = vector of objective functions  $m$  and  $n$
- $m$  = percent increase in light weight
- $n$  = percent reduction in the conditional probability of release
- $RRO_j$  = risk reduction option or safety design combination
- $Cap + LW \leq GRL$
- Cap = tank car capacity
- LW = tank car light weight
- GRL = gross rail load
- $LW \propto RRO_j$

An example of the vector space for the reduced probability of release and increased weight for all possible alternative designs for the Chlorine tank cars is plotted in Figure 4. The Pareto efficient frontier represents the set of solutions that give the greatest reduction in the conditional probability of release with the least increase in light weight.

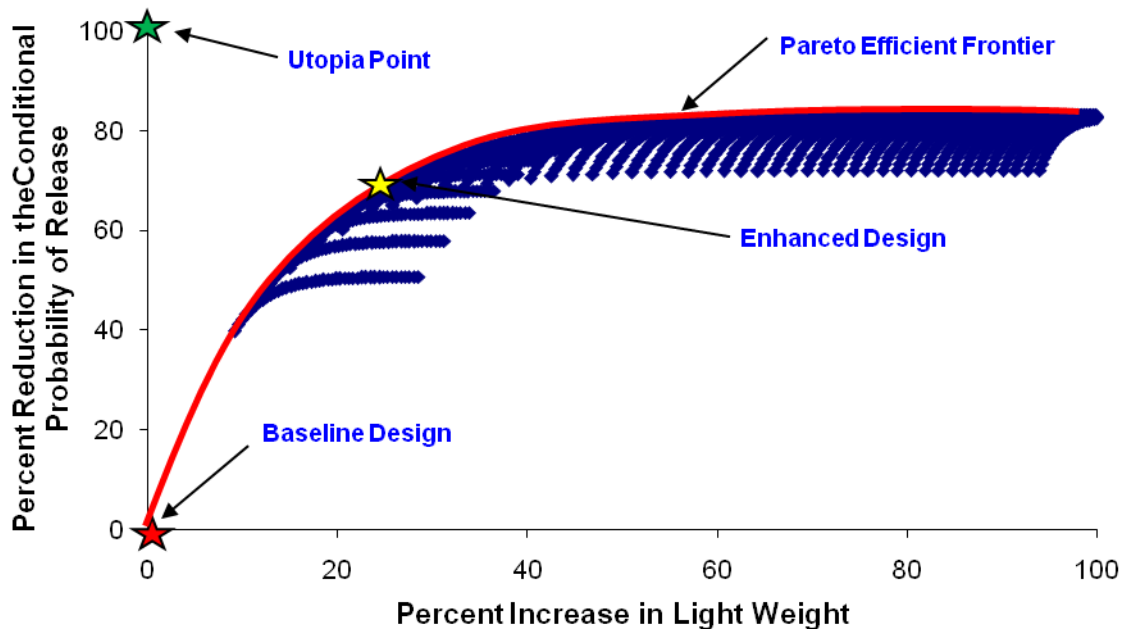


Figure 4: Tank car safety design optimization vector space for Chlorine

Ideally, the probability of release would be reduced 100 percent, with no loss in capacity as represented by the solution indicated as the utopia point. However, this is infeasible with conventional tank car materials and designs. Thus, a compromise solution may be determined by identifying the solution on the efficient frontier that is closest to the utopia point [14]. The utopia point method is often used in game theory and vector optimization to identify an optimal solution using the compromise solution concept [20, 21]. If the per-unit change in the two conflicting objective functions is valued equivalently by the decision-maker then the utopia point approach simply identifies the solution on the Pareto-efficient frontier that minimizes the Euclidean distance,  $N(x)$  between the frontier and the utopia point, as defined in Equation (3) [9]. If some differential weighting of the relative value of the two conflicting objectives is appropriate, the formula can be modified to reflect this.

$$N(x) = |F(x) - F^o| = \left\{ \sum_{m,n}^i [F_i(x) - F_i^o]^2 \right\}^{1/2} \quad (3)$$

where:

$F(x)$  = vector of objective functions for a point  $x$  in a decision space  
 $F^o$  = vector representing the utopia point

The overall optimization objective can be summarized mathematically as follows:

$$\text{Minimize } N_{m,n} = |F_{m,n} - F_{m,n}^o| = \left\{ \sum_i^2 [F_{m,n_i} - F_{m,n_i}^o]^2 \right\}^{1/2} \quad (4)$$

where:

$N_{m,n}$  = Euclidean distance from a point in a decision space to the utopia point  
 $F_{m,n}$  = vector of objective functions for a pair of vectors  $m$  and  $n$

$F_{m,n}^o = [0, 100]$  = vector representing the utopia point

$m$  = percent increase in light weight  $\forall$  **RRO**

$n$  = percent reduction in the conditional probability of release  $\forall$  **RRO**

where:

**RRO** = combination of risk reduction options

Table 3 summarizes the enhanced tank car safety design features for Chlorine and Anhydrous Ammonia as determined using the utopia point method.

Commodity	Head Shields Type	Head Thickness (in.)	Shell Thickness (in.)	$P_{RIA}$	Capacity (gal.)
Anhydrous Ammonia	Full-Height	1.2330	0.9205	0.0206	35,467
Chlorine	Full-Height	1.4745	1.0995	0.0147	16,537

Table 3: Enhanced tank car designs determined using the utopia point method

The optimization model presented offers insight into the tradeoffs involved in enhancing tank car safety design. Tank car designs identified using the utopia point method provided theoretical designs for consideration. However, the tank head and shell thicknesses identified using this technique had to be modified to take into account practical considerations related to material properties and availability, as well as fabrication constraints. Furthermore, there is greater uncertainty in the tank car damage resistance estimates for thicknesses beyond the bounds of the data in the regression analysis of tank cars involved in accidents that were used to develop the probability figures [16]. Consequently the AAR developed a set of performance-based standards for enhanced tank car designs for TIH service that took these factors into account [2]. They also published a set of design standards that conformed to the performance standard (key elements summarized in Table 4). The risk analysis described in the following sections of this paper uses these examples as the alternate tank car designs.

Commodity	Head Shield Type	Head Thickness (in.)	Shell Thickness (in.)	$P_{RIA}$	Capacity (gal.)
Anhydrous Ammonia	Full-Height	1.0300	0.8900	0.0231	33,581
Chlorine	Full-Height	1.1360	0.9810	0.0190	17,160

Table 4: Example alternate tank car designs that meet the AAR performance standard

### Accident-Caused Release Rate

The annual rate of release,  $P_R$ , is estimated by multiplying the conditional probability of release for a particular tank car design,  $P_{R|A}$ , times the rate that the car is expected to be derailed in a mainline accident,  $P_A$ , and the number of car miles.

$$P_R = P_{R|A} \times P_A \times M \times S \quad (5)$$

where:

- $P_R$  = annual accident-caused release rate
- $P_{R|A}$  = conditional probability of release from a tank car derailed in a mainline accident
- $P_A$  = tank car derailment rate per mile traveled
- $M$  = number of car-miles
- $S$  = shipment multiplier =  $\text{Cap}/\text{Cap}'$ 
  - Cap = nominal gallon capacity of a baseline tank car
  - Cap' = nominal gallon capacity of a tank car with enhanced safety design

Anderson & Barkan [1] conducted a detailed analysis of FRA accident data for use in hazardous materials transportation risk analysis. They determined that the average rate that a railcar is expected to be derailed in an FRA-reportable mainline accident ( $P_A$ ) was approximately once per 7.8 million miles traveled, or  $1.28 \times 10^{-7}$  per mile. Based on the assumption that the distribution of different FRA track classes along lines with Chlorine and Anhydrous Ammonia traffic is not significantly different than the distribution along the overall U.S. rail network, the national average ( $P_A$ ) was used in this analysis.

The number of annual U.S. shipments for Chlorine and Anhydrous Ammonia were estimated using the AAR's railcar movement database, TRAIN II [15], for the year 2005. These figures are multiplied by the average car-miles per shipment for Chlorine and Anhydrous Ammonia calculated using the 2004 STB waybill sample [18] to get the number of car miles.

Table 5 summarizes the shipment information and the annual rate of release,  $P_R$  estimates for the baseline and alternate tank car designs.

Commodity	Annual U.S. Shipments	Average Distance per Shipment (miles)	Annual Release Rate, $P_R$	
			Baseline Tank Car	Alternate Tank Car
Anhydrous Ammonia	52,065	655.0	0.3440	0.0991
Chlorine	36,478	712.2	0.1692	0.0591

Table 5: Annual release rate calculation for the baseline and alternate design tank cars

### Hazard Exposure Model

The Department of Transportation Emergency Response Guidebook (ERG)'s hazard exposure model was used in this analysis [19] to estimate the consequence of a release of TIH materials. The affected areas in the guidebook were determined from a statistical model that used sophisticated emission rate and dispersion models, historical release incidents, meteorological observations in North America and current toxicological exposure guidelines [3, 19]. The area is estimated by adding the protective action area and half of the initial isolation zone defined in the ERG for a specific chemical. In principle this defines the area in which population must be evacuated and/or sheltered in-place. Thus the risk metric used in this analysis is the number of people likely to be affected if emergency response personnel conform to the recommendations of the U.S. DOT ERG. The area is calculated for four different scenarios as specified by the ERG (Table 6).

<b>Spill Size</b>		
<b>Atmospheric Condition</b>	<b>Small</b>	<b>Large</b>
Day	0.011	0.162
Night	0.011	1.962

a) Anhydrous Ammonia

<b>Spill Size</b>		
<b>Atmospheric Condition</b>	<b>Small</b>	<b>Large</b>
Day	0.041	2.286
Night	0.641	21.196

b) Chlorine

Table 6: Affected area (miles<sup>2</sup>) for spills of a) Anhydrous Ammonia and b) Chlorine based on DOT ERG recommendations

It was assumed that transportation of these materials is equally likely to occur during the day or night and thus, a 0.5 probability was assigned to these two atmospheric conditions. The proportion of “large” vs. “small” releases was determined using the quantity lost distribution for pressure tank cars derailed in mainline accidents [16]. Releases of 5% or less of a car’s capacity were classified as small spills and comprised 22.1% of pressure tank car spills, and releases greater than 5% were classed as large spills accounting for 77.9%.

### Population Exposure

The 2004 Surface Transportation Board (STB) waybill sample [18] was used to determine major routes involving specific chemicals. Each waybill record represents origination and destination (O-D) information as well as all intermediate railroads involved in a shipment. This information was used with PC\*MILER-Rail, a routing, mileage and mapping software for the North American rail network developed by ALK-Technologies, to determine the practical route involving the O-D points. Point locations for a specific waybill record from PC\*MILER-Rail were then exported to ArcGIS, a geographic information system (GIS) software from ESRI used for spatial analysis to create the route over the rail network map from the DOT [10]. A spatial buffer was created over the route to represent the worst-case release scenario. The size of the buffer is based on the maximum downwind distance from the DOT Emergency Response Guidebook [19].

The exposure buffer for each TIH chemical was overlaid on the U.S. census tract map from ESRI Data & Maps [6] to estimate the proportion of different population density levels [17] along Anhydrous Ammonia and Chlorine routes (Tables 7a and 7b).



Maximum Population Density (people/mile <sup>2</sup> )	Population Class <i>k</i>	Proportion Rail Mileage, P(Pop <sub><i>k</i></sub> )	Average Population Density (people/mile <sup>2</sup> )
20	Remote	0.676	10
100	Rural	0.229	60
1,000	Suburban	0.079	550
3,000	Urban	0.011	2,000
10,000	High	0.005	6,500
> 10,000	Extremely High	0.000	10,000

a) Anhydrous Ammonia

Maximum Population Density (people/mile <sup>2</sup> )	Population Class <i>k</i>	Proportion Rail Mileage, P(Pop <sub><i>k</i></sub> )	Average Population Density (people/mile <sup>2</sup> )
20	Remote	0.617	10
100	Rural	0.267	60
1,000	Suburban	0.091	550
3,000	Urban	0.015	2,000
10,000	High	0.008	6,500
> 10,000	Extremely High	0.001	10,000

b) Chlorine

Table 7: Distribution of different population density levels potentially affected by spills along a) Anhydrous Ammonia and b) Chlorine routes

**Possible Release Consequences**

The set of different release consequences was determined by multiplying the exposure areas (Table 6) by different average population densities (Table 7). Table 8 summarizes each of the possible consequence levels considered for different release scenarios.

Population Class <i>k</i>	Atmospheric Condition <i>j</i>	Spill Size <i>i</i>	
		Small	Large
Remote	Day	0	2
	Night	0	20
Rural	Day	1	10
	Night	1	118
Suburban	Day	6	89
	Night	6	1,079
Urban	Day	21	325
	Night	21	3,925
High	Day	69	1,055
	Night	69	12,755
Extremely High	Day	106	1,623
	Night	106	19,623

a) Anhydrous Ammonia

Population Class <i>k</i>	Atmospheric Condition <i>j</i>	Spill Size <i>i</i>	
		Small	Large
Remote	Day	0	23
	Night	6	212
Rural	Day	2	137
	Night	38	1,272
Suburban	Day	22	1,257
	Night	352	11,658
Urban	Day	81	4,572
	Night	1,281	42,392
High	Day	264	14,859
	Night	4,164	137,774
Extremely High	Day	406	22,861
	Night	6,406	211,961

b) Chlorine

Table 8: Estimated population potentially affected by rail shipments of  
a) Anhydrous Ammonia and b) Chlorine

### Risk Estimates

The product of accident-caused release rate and the weighted sum of all possible consequence levels and the probability of each level gives an annual risk estimate of transporting the TIH materials in baseline versus alternate-design tank cars. The use of enhanced tank car designs can potentially reduce the annual risk of transporting Anhydrous Ammonia and Chlorine by approximately 71% and 65%, respectively (Table 9). In addition to the point estimates of average risk, an understanding of the distribution of risk outcomes is often useful for risk management decisions. Use of risk profiles allows comparison of the distribution of risk for the baseline tank cars compared to the alternative designs for each material (Figures 5a and 5b).

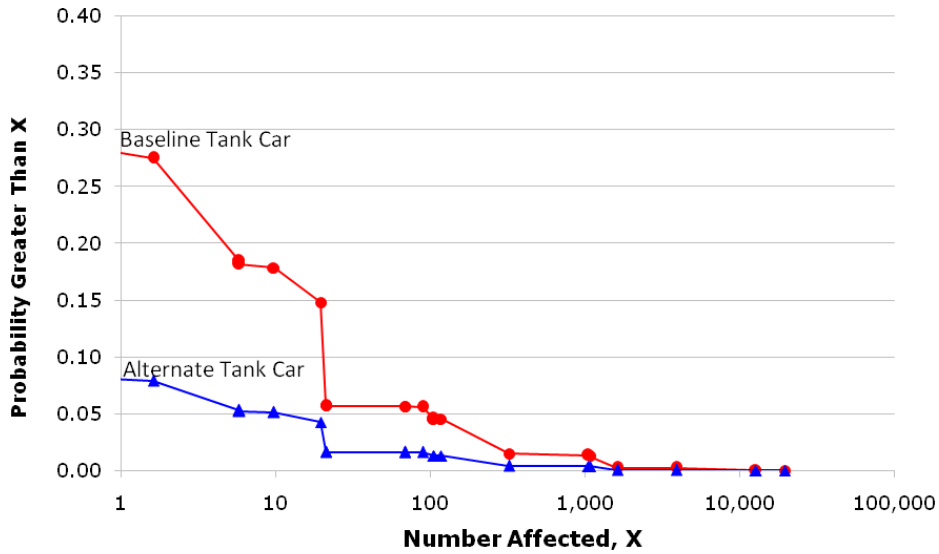
	Expected Number of People Affected	Percent Reduction (%)
Baseline Tank Car	35	0.00
Alternate Tank Car	10	71.43

a) Anhydrous Ammonia

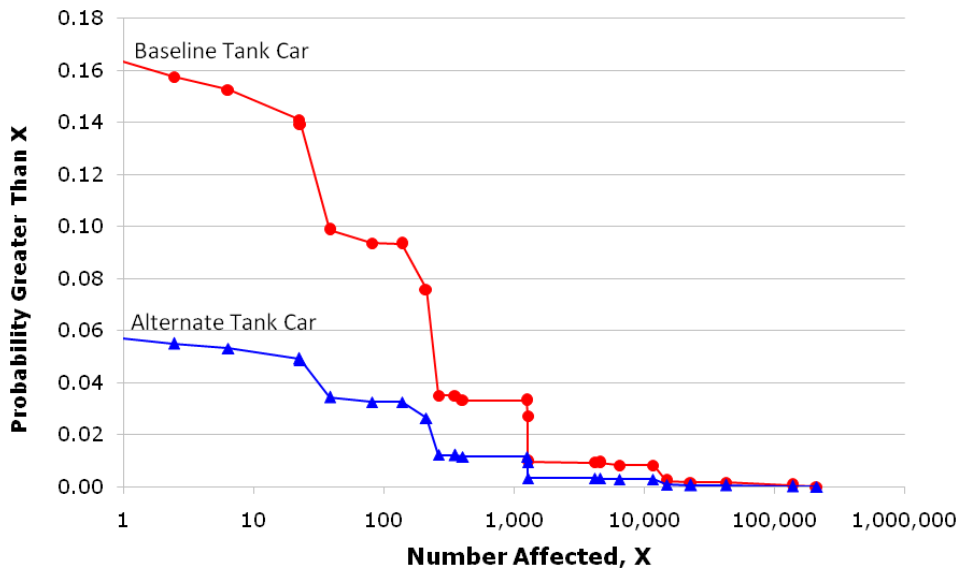
	Expected Number of People Affected	Percent Reduction (%)
Baseline Tank Car	259	0.00
Alternate Tank Car	90	65.25

b) Chlorine

Table 9: Annual risk estimates for a) Anhydrous Ammonia and b) Chlorine



a) Anhydrous Ammonia



b) Chlorine

Figure 5: Annual risk profiles for baseline and alternate design tank cars for rail transportation of a) Anhydrous Ammonia and b) Chlorine

### Discussion & Conclusions

This paper presents a model to optimize the safety design of tank cars used to transport TIH materials and a quantitative risk assessment of rail transport of these materials in North America and the reduction in risk if alternate design tank cars are used. The conditional probability of release and car weight was used as the proxy for safety and cost, respectively. The concept of Pareto optimality was introduced in determining possible alternate tank car safety designs.

In addition to TIH chemical tank cars, the optimization model can be used to identify possible safety design enhancements for all classes and designs of tank car and the risk model can be adapted to address other hazards such as flammability or environmental damage. The methodology and results described in this paper were used to assist the North American railroad industry in its development of new specifications for TIH tank cars.

A comprehensive financial cost model is being developed to estimate the cost of replacing the fleet of tank cars for a specific service with enhanced-design cars. The risk, costs and benefits of specific chemicals and tank car designs will be explicitly considered to determine the optimal tank car design for each different hazardous material.

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