Optimization of Siding Location for Single-track Lines with Non-Uniform Track Speed

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Abstract
Single-track with proper allocation of passing sidings (loops) is a substantial option to save investment and maintenance cost in both passenger and freight rail systems. Conventionally, railroads usually rely on both experience and simulation to determine siding locations to minimize train delay. The solutions identified are often effective, but this does not guarantee that all the potential options are taken into account or the best one is chosen. In this study, we proposed an optimal siding location model with the consideration of route speed variation to determine the optimal number and locations of sidings. The case study results show that this model can generate an optimal siding location plan for the planning of existing or new rail infrastructure. This tool can thus help railroads maximize the benefit gained from rail projects and provide a better service quality to customers.

Keywords
Single-track, Capacity, Decision Support System, Mixed Integer Programming

1 Introduction

The rail network in North America is facing capacity constraints due to increasing freight and passenger traffic demand (AASHTO (2007), AAR (2007)). Since rail congestion can potentially damage the efficiency and reliability of freight and passenger services, changes in operational strategies and rail infrastructure are necessary to maintain the performance of the network (Krueger (1999), Lai and Shih (2013)). Although operational strategies can be implemented with minimal capital expense, they only provide slight capacity increases. Considering that transportation demand is projected to be significantly increased in the long term, the rail network infrastructure must be expanded and upgraded to accommodate this change.

In North America, the same private freight railroads that operate freight services own the track infrastructure. Since the majority of freight mainlines are only single-track routes, projects to install double track have been the major capacity expansion method used by these railroads for the past 15 years (Shih et al. (2012)). This approach has been altered by recent changes in commodity flows and energy markets. With new demand from the shale oil and gas industry, railroads have shifted their capital to conduct passing siding (passing loop) projects on secondary or branch single-track lines with sparse numbers of sidings and historically low traffic density. For example, BNSF completed several siding projects related to energy industry development in 2012 (BNSF, 2012). Because of the large capital investment required for siding projects, the relationship between train delay reduction, traffic speed and siding project location has become an important research topic in North America (Atannasov et al. (2014), Dingler (2010), Dingeler (2013)). A general model of this the relationship can help the private railroads maximize benefits from their siding projects while minimizing the level of required investment.

Most of the operating rail lines in Europe are double-track and thus the capacity problem in European research has been less related to siding projects. This tradition is now changing due to the increasing construction cost of double track and the decreasing availability of government and public funding. Single-track is regarded as a solution
to minimize investment on lines with relatively low traffic demand (Petersen and Taylor (1987)). Some of the pending high-speed rail systems are planned to be single-track lines as well. A recent high-speed rail feasibility study in Norway suggested the use of single-track lines in suburban areas (VWI (2007)). The Spanish government also announced the use of single track on their most recent lines to satisfy the budgetary constraints (The Olive Press, 2014). In general, even though the single-track high-speed rail system in Europe is very different from the single-track freight rail system in the United States, the number and location of passing sidings still matters to their respective practitioners. The larger number of sidings on a single-track rail line, the more points for conflicting trains to meet and pass. Additionally, consideration of individual train speeds to develop an appropriate allocation of siding locations can help maximize the total number of available train paths for later train schedule planning.

Consequently, researchers have investigated methods to develop an effective capacity expansion plan based on siding projects. Petersen and Taylor (1987) aimed to find the best positions of longer sidings to accommodate mixed passenger and freight traffic by using a simulation approach. Pawar’s (2011) analytical models are capable of determining the length of long sidings for reducing potential meet delay. These two studies only focus on the effect of siding characteristics and do not consider the effect of heterogeneous traffic and track speed. Higgins et al. (1997) developed the first optimization model to determine optimal siding locations. The Higgins model depicts the relationship between train delay and the number and location of sidings while taking into account variation in train speeds. Despite this, the Higgins model only provides a rough solution to the problem since the model neglects some practical considerations such as siding capacity, construction cost of sidings, engineering constraints on siding locations and the existing pattern of sidings. To provide a better tool to solve the siding planning problem, an optimal siding location model (OSLM) is developed in this study. OSLM can determine the optimal number and locations of sidings based on both infrastructure and traffic characteristics.

2 Optimal Siding Location Model

Mince the model determines the optimal siding plan based on traffic characteristics, a part of OSLM is similar to capacity planning models and the other part is a train-dispatching model. As a result, the OSLM incorporates the capacity planning problem and train dispatching problem through a series of types of constraints. The first set of constraints avoids the conflicts between two adjacent trains by dividing them with minimum headways (Ahuja et al. (1993), Harrod (2008), Lamorgese and Mannino (2013)). Siding constraints avoid conflicts on sidings based on the length and the capacity of the sidings (Qiang and Kozen (2009)). The effect of train characteristics, composition and commercial schedule are also evaluated to measure the impact from traffic heterogeneity (Lai et al. (2010)). The possible number and location of prospective sidings must be enumerated first according to the existing corridor configuration. The constraints also dictate that the properties of the current track configuration, like the location of existing sidings and stations, must be maintained. Variation in average train operating speeds due to grade and curvature variation are considered as well to obtain a practical result for the reference of railroaders.

Figure 1 is the conceptual diagram for OSLM. The input parameters of the optimization model are composed of three types of data, rail infrastructure properties, traffic characteristics and operational parameters. Following the principle mentioned, the optimization framework uses these inputs to create two types of output, train paths and an optimal siding location plan.
The detailed input data required by OSLM are displayed in the list below. Traffic characteristics show the schedule and speed of traffic. Infrastructure properties are associated with existing track configuration. The effect of terrain (grade) and the track curvature along the line can be manifested in average speed limit of a line. Operational factors are the other environmental parameters which are related the train operation. These three different types of parameters are used by OSLM to generate an optimal siding location plan.

- Traffic characteristics:
  - Average train travelling speed profiles alone mainline toward each direction (kph)
  - Number, direction and type of each train (trains per hr)
  - Scheduled departure time for trains and commercial schedule for passenger trains (hr)
  - Lost time of acceleration and deceleration (hr)
  - Safety headway for adjacent trains (hr)

- Infrastructure properties:
  - Construction cost (any money unit)
  - Length of existing sidings and the corridor (km)
  - Speed limit of the sidings (kph)
  - Minimum siding spacing (km)
  - Location of existing sidings and stations (km)

- Operational parameters
  - Priority of trains (delay costs or arbitrary weights)
  - Turnout switching time (hr)
  - Budget (any money unit)

Most of the data in the list above can be used as direct inputs to OSLM but some of the infrastructural inputs need to be preprocessed. Figure 2 below is an example of the result from preprocessing. In this figure, the sign \( q \) stands for nodes, stations or sidings and the \( n \) in \( q_n \) is the no. of nodes or sidings. The sign \( p_n \) represents the segments in a line. From the number and location of existing nodes, the maximum number and relative location of prospective sidings can be determined. The maximum number of possible sidings between two existing nodes can be calculated by equation \( \left\lfloor \frac{d_p}{g} \right\rfloor - 1 \), where \( d_p \) is the segment length between two adjacent nodes and \( g \) is the minimum siding spacing. For example, the spacing between the first station and the first existing siding is 26 km.
Since the minimum siding spacing is assumed to be 8 km in this study, the maximum number of prospective sidings is \(\left\lfloor \frac{26}{8} \right\rfloor - 1 = 2\). Moreover, segments without any prospective sidings can be used to indicate the locations undesirable for adding sidings, like the segment p5 in Figure 3.

![Figure 2 Example of preprocessing process to infrastructure data](image)

### 3 Optimal Siding Location Model in Nonlinear Form

An optimization model for siding planning problems can be regarded as the integration of a capacity planning model and train dispatching model. By combining the two models, OSLM can generate an optimal siding location plan with minimized delay cost and late departure cost while keeping a set of practical constraints unviolated. The following is the notations list. It shows all the indices, sets and parameters used in this study:

**Indices:**
- \((i, j) \in N\) = indices referring to trains running through the line
- \((p, r) \in P\) = indices representing sections of the line
- \((q, s) \in Q\) = indices stand for sidings and stations (nodes)

**Sets:**
- \(b^+\) = set of any two trains with same direction
- \(b^-\) = set of any two trains with opposite direction
- \(\kappa\) = set of existing and prospective sidings
- \(\epsilon_i\) = set of origin to train \(i\)
- \(\eta^+\) = set of prospective sidings and station without sidings
- \(\eta^-\) = set of existing siding and stations with sidings
- \(k_i\) = set of the destination to train \(i\)
- \(\delta\) = set composed of all section \(p\) and the adjacent node \(q\) to enter the section
- \(\partial_p\) = set composed of all section \(p\) and the adjacent nodes \((q, s)\),
- \(\epsilon\) = set composed of all adjacent sections \((p, r)\)

**Parameters:**
- \(t_i^q\) = extra travelling time for train \(i\) to across siding \(q\) than the parallel section on mainline (hr)
- \(\tau_i^q\) = scheduled dwelling time for passenger train \(i\) on station \(q\)
- \(g\) = minimum siding spacing (mile)
Min W D A W D e
 

   
   

Equation (1) is the objective function of OSLM. It aims to minimize the total cost during the planning horizon, defined by the summation of meet and pass delay cost, and the late departure cost. Since \( W \) is the delay cost for different types of trains, this objective function reflects the business objective of North American railroads (Dingler, 2010; Dingler et al. 2011).

Objective: \( \text{Min} \sum_{i \in N} \sum_{q \in S_i} W^i (D^i_q - A^i_q) + \sum_{i \in N} \sum_{q \in S_i} W^j (D^j_q - e^j_i) \) (1)

This objective is subject to a set of constraints (equations 2~(22)), including train time constraints, minimum siding spacing constraints, commercial schedule constraints, meet and pass capacity constraints, track configuration constraints and environmental constraints. The train time constraints form the basic train dispatching mechanism in OSLM. They are listed in equation (2) to (7). The main purpose of these constraints is to separate the arrival or
departure time of two adjacent trains at each node with a reasonable headway. Equation (2) and (4) maintain an appropriate headway between the arrival time of any adjacent trains heading toward the same direction and equation (3) and (5) maintain a safe headway between the departure time of any two adjacent trains. Equation (6) and (7) keep a safety headway between two adjacent trains in opposite directions.

\[
M (1 - x_{ij}^q) + D_j^q \geq D_j^q + h_j^p + o_{i}^q \xi \quad \forall (i, j) \in b^+, i \neq j, (p, q) \in \delta \tag{2}
\]

\[
M (1 - x_{ij}^q) + A_j^q \geq A_j^q + h_j^p + o_{i}^q \xi \quad \forall (i, j) \in b^+, i \neq j, (p, q) \in \delta \tag{3}
\]

\[
M x_{ij}^p + D_j^q \geq D_j^q + h_j^p + o_{i}^q \xi \quad \forall (i, j) \in b^+, i \neq j, (p, q) \in \delta \tag{4}
\]

\[
M x_{ij}^p + A_j^q \geq A_j^q + h_j^p + o_{i}^q \xi \quad \forall (i, j) \in b^+, i \neq j, (p, q) \in \delta \tag{5}
\]

\[
M (1 - x_{ij}^p) + D_j^q \geq A_j^q + h_j^p + \xi \quad \forall (i, j) \in b^-, i \neq j, (p, q) \in \delta \tag{6}
\]

\[
M x_{ij}^p + D_j^q \geq A_j^q + h_j^p + \xi \quad \forall (i, j) \in b^-, i \neq j, (p, q) \in \delta \tag{7}
\]

Based on an existing passenger and freight train schedule, equation (8) and (9) are commercial schedule constraints which consider the effect of rail transportation demand. Equation (8) enforces that trains leave their origin within a given time range. Equation (9) ensures that all passenger trains get to stations according to the commercial schedule.

\[
e_i^+ \leq D_i^q \leq e_i^- \quad \forall i \in N, q \in \pi \tag{8}
\]

\[
\lambda_i^{+} \leq A_i^q \leq \lambda_i^{-} \quad \forall i \in N, q \in \kappa \tag{9}
\]

Equation (10) to (15) are meet and pass capacity constraints. Equation (10) uses the train stopping variable \( o_{i}^q \) to control the existence of extra train meet and pass delay. Equation (11) and (12) link the trains which stop on the same siding sequentially. Equation (13) can help avoid two trains stopping on the same siding. Moreover, equation (10) and (13) also work as a part of the commercial schedule constraints. The notation \( \tau_i^q \) in equation (10) and (15) guarantee the minimum dwelling time for passenger trains at stations. It works together with equation (9) and can help maintain the stopping pattern of passenger trains. Equation (14) is the siding length constraint. It enforces a train to use a siding only if the length of the siding is longer than the train. Equation (15) captures the lost time experienced by trains due to acceleration, deceleration, siding speed limit and turnout switching time while traveling through siding.

\[
M o_{i}^q \geq D_i^q - A_i^q + \tau_i^q \quad \forall i \in N, q \in Q \tag{10}
\]

\[
\Theta_{i}^q \geq o_{i}^q + o_{j}^q + x_i^p - 2 \quad \forall i \in N, j \in N, i \neq j, (p, q) \in \delta \tag{11}
\]

\[
3 \Theta_{i}^q \leq o_{i}^q + o_{j}^q + x_i^p \quad \forall i \in N, j \in N, i \neq j, (p, q) \in \delta \tag{12}
\]

\[
A_j^q \geq D_j^q + \zeta + h_j^p + M(1 - \Theta_j^q) \quad \forall i \in N, j \in N, i \neq j, q \in \kappa, (p, q) \in \delta \tag{13}
\]

\[
o_{i}^q \leq L_i^q \quad \forall i \in N, q \in \kappa \tag{14}
\]

\[
D_i^q \geq A_i^q + o_{i}^q (f_i + t_i^q + \zeta) + \tau_i^q \quad \forall i \in N, q \in Q \tag{15}
\]

The following constraints are the track configuration constraints. Constraint ensures the minimum siding spacing is displayed in equation (16). Equation (17) fixes the location of existing sidings at their locations. Equation (19) avoids trains to meet or pass at a non-existing siding. Equation (19) ensures all present sidings are in effective in OSLM.
\[ d_p \geq g - M(1 - \sum_{q \in \eta^p} z^q) \quad \forall p \in P \]  
(16)

\[ \sum_{r \in \{r < p\}} d_r = \varphi^q \quad \forall q \in \eta^-, (p, q) \in \delta \]  
(17)

\[ \sum_{i \in N} o_i^q \leq Mz^q \quad \forall q \in Q \]  
(18)

\[ z^q = 1 \quad \forall q \in \eta^- \]  
(19)

Equation (20) is the budget constraint and equation (21) ensures OSLM to complete the dispatching process within a given period. Equation (22) fixes the train running time between any two adjacent nodes according to the train operating speed profile. Since equation (22) contains a nonlinear function, the whole OSLM is a nonlinear model.

\[ \sum_{q \in \eta^-} Uz^q \leq B \]  
(20)

\[ A_i^q \leq E \quad \forall i \in N, q \in k \]  
(21)

\[ A_i^q - D_i^q = \int_{\zeta_i}^{\zeta_i^q} v_i(\zeta) d\zeta \quad \forall i \in N, (q, s) \in \mathcal{S}_p, p \in P \]  
(22)

Due to equation (22), OSLM becomes a nonlinear model. Next subsection will explain a method used to linearize OSLM.

4 Linearization of Optimal Siding Location Model

Non-linear optimization process didn’t always generate a converged optimal solution thus it is better to transform the mentioned non-linear OSLM into a linear OSLM. To linearize the non-linear term \[ \int_{\zeta_i}^{\zeta_i^q} v_i(\zeta) d\zeta \] in equation (22), a piecewise-linear function is used instead for travel time function. The piece-wise linear function approximates the original nonlinear train travel time function by linear sections thus can transform the original model into linear. Figure 4 provides an example of travel time approximation. In Figure 3, there are three linear sections for travel time profiles of eastbound trains. Their slopes and locations are related to original travel time profile which is affected by the terrain of the mainline. For example, the slope of section \( s \rightarrow a1 \) is larger than section \( a2 \rightarrow a3 \) since section \( s \rightarrow a1 \) to eastbound trains is an upgrade section and section \( a2 \rightarrow a3 \) is a level grade section. Another example is that eastbound train speed doesn’t slow down in section \( a3-e \) since it is a downgrade.

The transformed piecewise-linear function of travel time allows OSLM calculating the travel time of the \( p_i^3 \) segments by interpolation. First, the travel time to each node needs to be calculated. The edges of the line sections of speed profile in this study are defined as “speed edges”. The travel time from origin to any of the nodes (stations, sidings, or terminals) can be interpolated by the travel time from origin to each speed edge. In the example below, the accumulative travel time for node \( q1 \) can be interpolated by the accumulative travel time of \( a2 \) and \( a3 \) which are \( A_i^2 \) and \( A_i^3 \). After obtained the travel time of each node, the \( p_i^3 \) segment travel time can be calculated by the travel times of the adjacent nodes of segment \( p_i^3 \).
Based on the explained concept, the nonlinear model can be modified. The new indexes, sets and parameters required in the modified model are listed below:

**New indexes, sets and parameters**

(α,β) ∈ 𝐻 = speed zone

ε = pairs of adjacent line segments (p,r)

ω∗ = pairs of speed zone edges which are not adjacent to each other

Λα ∈ accumulative traveling time for train i to reach speed zone edge α

μα = milepost of speed zone edge α

Two new variables are added to the modified model for the purpose of travel time interpolation:

τip = interpolation coefficient of train i on segment p.

γ∗i = functional variable in interpolation process, ensure the interpolation speed edges are adjacent edges

In the modified model, the nonlinear travel time function is substituted by the piecewise-linear travel time function. Equation (23) shows the modified equation (22). The right-hand-side of equation (23) calculate segment travel time by subtracting the travel time of the entering nodes from the leaving node.

\[
A^q_i - D^q_i = \sum_{α ∈ H} \Lambda^q_i τ^q_i p - \sum_{α ∈ H} \Lambda^q_i \gamma^q_i α \quad ∀i ∈ k, (p,s) ∈ P
\]  

A group of interpolation constraints needs to be added as well to ensure the mechanism of travel time interpolation process.
\[
\sum_{\alpha \in H} \tau_{i}^{ap} = 1 \quad \forall i \in N, p \in P, \alpha \in H \tag{24}
\]
\[
\gamma_{i}^{ap} + \gamma_{i}^{bp} \leq 1 \quad \forall i \in N, p \in P, (\alpha, \beta) \in \omega \tag{25}
\]
\[
\sum_{\alpha \in H} \gamma_{i}^{ap} \leq 2 \quad \forall i \in N, p \in P, \alpha \in H \tag{26}
\]
\[
\tau_{i}^{ap} \leq \gamma_{i}^{ap} \quad \forall i \in N, p \in P, \alpha \in H \tag{27}
\]
\[
\sum_{\alpha \in H} \mu_{i} \tau_{i}^{ap} = \sum_{r \leq p} d_{r} \quad \forall i \in N, p \in P \tag{28}
\]

Equation (24) ensures the summation of interpolation coefficients is equal to 1. Equation (25) to (27) force the speed edges selected for travel time interpolation are adjacent edges. Equation (28) calculate the interpolation coefficients based on the ratio of distances from one node to its adjacent speed edges.

## 5 Case Study

To demonstrate the use of OSLM and the impact of speed variation from grade, we constructed three hypothetical scenarios with same vertical and horizontal alignment but different number of speed zones (Figure 4 & Table 1). The results from OSLM for the three scenarios can be used to display the importance of speed variation on the siding planning problem.

![Figure 4: Horizontal and vertical alignments, and speed zones of the hypothetical case](image)

---

9
Table 1 Train speed in speed zones (Unit: kph)

<table>
<thead>
<tr>
<th>Speed zones</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inter-city</td>
<td>Commuter</td>
<td>Freight</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EB</td>
<td>WB</td>
<td>EB</td>
</tr>
<tr>
<td>a1-a2</td>
<td>98</td>
<td>115</td>
<td>77</td>
</tr>
<tr>
<td>a2-a3</td>
<td>85</td>
<td>115</td>
<td>67</td>
</tr>
</tbody>
</table>

The values of the important parameters used in case study are listed in Table 2. The volume are assumed to be a potential number of hour traffic. The priority of passenger and freight trains are denoted by arbitrary weights. The departure time of trains is set to be evenly distributed during the peak hour without fleeting, i.e. no adjacent trains are the same type of train. The traffic volume is estimated to be 10 trains per peak hour according to the peak hour volume traffic for double-track sections in Lai et al. (2010). The question is how to effectively add new sidings to reduce rail traffic delay.

Table 2 Input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of each type of train</td>
<td>Intercity passenger trains: 2 trains/hr Commuter: 6 trains /hr Freight: 2 trains/hr</td>
</tr>
<tr>
<td>(passenger trains/intermodal/bulk)</td>
<td></td>
</tr>
<tr>
<td>Direction of trains (eastbound/westbound)</td>
<td>Evenly distributed in both directions</td>
</tr>
<tr>
<td>Priority of trains (weigh of each train type)</td>
<td>Intercity passenger trains: 5 Commuter: 3 Freight: 1</td>
</tr>
<tr>
<td>Required headway between two trains</td>
<td>1.5 mins</td>
</tr>
<tr>
<td>Stopping Pattern</td>
<td>Commuter stops 2 mins at the middle station</td>
</tr>
</tbody>
</table>

As mentioned in section 2, some of the infrastructure input data must be preprocessed before they can be used by the optimization model. There are three prospective siding projects that can be located any place between the two existing adjacent nodes without violate minimum siding spacing rule. Since there are two segments on this
hypothetical line, the total number of prospective sidings is six. Moreover, the budget constraint in this study only allow four prospective sidings to be selected in this study.

OSLM was coded into GAMS and solved by CPLEX. The scenario three of the model has 2,433 variables and 5,868 equations which is a large scale problem. The solution time ranges from 0.5 to 1 hours depending on the number of speed zones. Two types of outputs are generated from OSLM: the train dispatching result and the optimal siding location plan. Figure 5 shows the dispatching result of scenario three as an example. Since there are no conflicts and all the passenger trains meet their schedule, the train dispatching mechanism in OSLM is proved to be reasonable.

Figure 5 An example of train dispatching result from OSLM

Figure 6 shows the optimal siding location plans for scenario one to three. An extra scenario with no speed variation from grade (all trains travel with maximum average speed) is displayed together for comparison. Comparing the siding plan of the extra scenario to the three scenarios, the location of the first siding in the extra scenario is significantly different from the location of the first sidings in the three scenarios. This proves considering the speed variation from grade affects the location of sidings. For the siding location plans of the three scenarios, the siding locations of the scenario three are more centralized than the other two scenarios. Figure 7 shows the weighted total delay of the tested traffic under the obtained siding plan layouts with the effect of speed variation from grade. It implies that the inappropriate cuts of speed zones could deteriorate the quality of travel time approximation thus generated inferior solutions.

Figure 6 Siding location plans for extra and scenario one to three

![Diagram](image-url)
In North America, single-track lines with sparse number of sidings are expected to reach the limits of practical capacity due to changes in traffic patterns and growing demand. On the other hand, the share of single-track lines in the rail network is expected to be increased due to the increasing construction cost and decreasing government fund. The number and locations of sidings affect the performance of single-track lines. A decision support tool OSLM is developed in this study to help determine the optimal number and location of additional sidings. The case study demonstrates that OSLM with the ability to consider train speed variation can determine a more precise siding location plan than OSLM without. This tool is able to maximize the return on investment from capacity expansion projects for railroads and improve the service quality provided to customers. We are aiming to develop an effective solution algorithm for OSLM to consider multiple train schedule to increase the robustness of the solution quality. Another future direction would be incorporating the capability to determine the optimal length of sidings into OSLM.

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