Laboratory evaluation of under-ballast mat effectiveness to mitigate differential movement problem in railway transition zones

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ABSTRACT: Railway transition zones between open track and rigid structures such as bridges and culverts often experience differential movements due to abrupt changes in track stiffness and complicated dynamic loading effects. Such conditions commonly lead to track geometry issues, consequently increasing the magnitude of impact loads at these areas. This in turn leads to excessive degradation of ballast and other track components. Under-ballast mats have seen increased use in the last decades especially in the area of improved vibration mitigation and track stiffness reduction at bridge approaches due to their flexibility and damping capabilities. This paper presents results from an ongoing laboratory study aimed at investigating the effectiveness of under-ballast mats on the transient deformation behavior of track sections built over stiff substructures such as bridge decks and subjected to cyclic loading. According to the study findings, under-ballast mats can provide the necessary resiliency enhancements to track structure to mitigate differential settlements at transition zones by matching the vertical transient deformations of open track with those obtained over stiff support conditions.

1 INTRODUCTION

Railway transition zones characterize areas in the rail network where an abrupt change in track stiffness occurs; these include tunnels, at-grade crossings, special trackwork, and bridge and culvert approaches. Trains interacting with these track sections experience sudden variations in vehicle/track interaction forces (Dahlberg 2003). Railroad personnel have long reported these zones as problematic, engendering rapid development of track geometry issues and endangering a railroads efficiency due to increased maintenance requirements, delays, or slow orders (Briaud et al. 2006, Woodward et al. 2007, Banimahd et al. 2012, Sasaoka & Davis 2005, Jenks 2006, Lundqvist et al. 2006, Frohling et al. 2005, Hunt 1997). Moreover, maintenance and renewal expenses related to transition zones comprise a sizable share of a railroad's annual operating expenses with reported annual expenditures ranging from 110 to 200 million US dollars for European and North American railroads respectively (ERRI 1999, Hyslip et at. 2009, Sasaoka & Davis 2005).

Substantial research has been conducted to investigate railway transition zone problems and the mechanisms that drive its accelerated deterioration (Nicks 2009, Li & Davis 2005, Coelho et al. 2010, Varandas et al. 2011, Tutumluer et al. 2012, Mishra et al. 2012, Wang et al. 2015, Stark & Wilk 2015). Li & Davis (2005) attributed problems to three proposed major causes: (i) change in track stiffness leading to uneven track deflections under moving loads; (ii) differential settlement between approach and bridge sections; and (iii) geotechnical issues due to material quality, insufficient consolidation and compaction of the substructure and/or inadequate drainage. Sasaoka & Davis (2005) listed differential settlement, stiffness characteristic differences, and track damping properties as the most important parameters influencing transition zones problems. Although different authors attribute root causes of the problems in transition zones to different individual issues, an agreement exists as to the importance of track stiffness properties.

According to Li & Davis (2005), differential vertical movements of the track profile vary significantly between the approach and the structure. Due to differences in substructure conditions, the approach section undergoes higher deformations than the structure under loading conditions, resulting in the phenomenon referred to as differential movement. Reported driving mechanisms of differential movements include the abrupt changes in track stiffness and damping properties of the track structure and/or foundation, and settlements due to ballast degradation and/or subgrade and fill layers (Li & Davis 2005, Selig & Li 1994, Mishra et al. 2012, Sasaoka & Davis 2005, Nicks 2009, Tutumluer et at. 2012).

It is important to understand that the issues at transition zones are not singular to one phenomenon or component, but constitute a system problem that requires investigation holistically. Accordingly, differential movements instigate a negative feedback loop followed by plastic deformations of the approach, increased impact loads, ballast deterioration, and additional track settlements spawning continuously accelerated deterioration loops of the track, and of other components and/or structures.

The ballast is a vital component to the bearing capacity of railway tracks. As part of this composition, the ballast endures compounding effects of high impact loads stemming from heavy axle freight trains, most commonly applying heavy loads in the US, overcoming the uneven running surfaces of approach and structure and thus propagating vibrations through the track structure. Additional frictional wear of particles may also stem from the generated vibrations.

There are two primary mechanisms by which ballast particles degrade. First, attrition constitutes the deterioration of the surface texture and geometry of the ballast particles removing surface texture and angular characteristics of aggregates that are critical to the sustainability of the structural skeleton providing ballast with its load bearing capabilities and resistance to permanent deformation (Tutumluer & Pan 2008, Wnek et al. 2013, Lu & McDowell 2010). Second, breakage relates to the tensile failure of the ballast particles due to exceedingly high contact stresses between individual stones, resulting in material splitting (Selig & Waters 1994). Both of the above-mentioned mechanisms contribute to ballast fouling (Qian et al. 2014, Selig & Waters 1994, Selig et al. 1988).

For decades, railroads and researchers have explored the use of elastic resilient materials in the track structure. Three components have been mostly used to provide solutions as employed by railroads to manage the elastic properties of railway track; these are premium elastic fastening systems, under-sleeper pads (USP), and under-ballast mats (UBM).

UBMs are elastic rubber pads – usually manufactured using natural rubber, recycled tire rubber or Ethylene Propylene Diene Monomer (EPDM) rubber – installed below the ballast layer of a ballasted track structure or under the concrete slab in a slab track design. This component has long been employed as a vibration mitigation measure for both transit and freight environments, but most notably for the former. Still, in recent years, a consistent increase in the use of UBMs in freight railroad environments has provided opportunities to explore and report their potential effectiveness for mitigating transition zone problem and/or reducing ballast stresses (Indraratna 2016, Sol-Sanchez et al. 2014, Li & Maal 2015, Indraratna et al. 2014, Sol-Sanchez et al. 2015).

Published studies investigating the life cycle of UBMs are still limited (Wettschureck et al. 2002, Dold & Potocan 2013). Nevertheless, reports from tests conducted in sections of UBM retrieved from revenue service have demonstrated the capability of the component to retain its properties after many years in service (Wettschureck et al. 2002, Dold & Potocan 2013). Most of the conducted research into the topic is part of product development efforts of suppliers and results are not widely made available to the industry.

The UBMs are resilient pads that can provide additional resiliency to the track structure foundation and effectively dissipate more of the produced energy – manifested in the form of vibrations that propagate through the ballast structure – from the wheel-rail and/or tie-ballast interfaces that are associated with accelerated rates of ballast degradation (Sol-Sanchez et al. 2014). Kerr & Moroney (1993) traced the transition zone problem to the sudden changes in accelerations of the wheels and vehicles at these interfaces and cited key remediation methods aimed at reducing these changes, such as the reduction of vertical stiffness on the "hard" side of the transition. Despite the good potential of UBMs for addressing the abrupt changes in track stiffness and mitigating differential movement issues of problematic track transition zones, there is little to no documentation available in the literature.

2 OBJECTIVE AND SCOPE

The primary objective of this research effort has been to investigate the effectiveness of UBMs on the transient deformation behavior of track sections built over stiff substructures such as bridge decks, and subjected to cyclic loading. This study is part of a larger-scope, ongoing research effort at the University of Illinois at Urbana-Champaign (UIUC) aiming at evaluating and quantifying the overall performance of UBMs and their benefits to the track structure. As part of laboratory experiments conducted on a UBM sample, ballast vertical deformations were monitored under cyclic loading. Ballast degradation trends were quantified through laboratory sieve analyses with ballast gradations compared prior and subsequent to testing. This paper presents results of the laboratory tests conducted and compares them with field measurements of transient deformations obtained in bridge approach sites in North America from previously published literature.

3 MATERIALS

3.1 *Under-ballast mats*

A specific type of UBM named "Type A" and designed for freight traffic loading conditions was used in this study (see Figure 1). The sample comprised of a profiled mat bonded to a flat protective rubber layer with a synthetic fiber grid in between. Table 1 provides details of the sample geometry including its dimensions and thickness.

Table 1. Under-ballast mat sample properties.

Label	Mat Thickness		Sample Size	Construction
	Minimum	Maximum		
	mm (in.)		mm (in.)	
Type A	5 (0.197)	10 (0.394)	699x699 (27.5x27.5)	Profiled mat bonded to flat protective layer

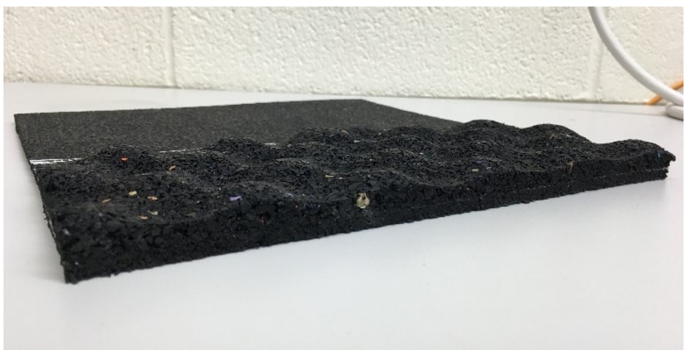

Figure 1. Type A under-ballast mat designed for freight traffic loading

3.2 *Ballast*

Ballast material used for this investigation originated from a quarry commonly used by a North American Class I railroad and was stored in a stockpile at the laboratory facility. The coarse aggregate material consisted of crushed granite with uniformly graded particle size distribution compliant with the American Railway Engineering and Maintenance-of-way Association (AREMA) No. 4A gradation recommendations. Figure 2 shows the original gradation for the ballast material employed along with the AREMA specified gradation limits for No. 4A ballast. To ensure the quality in the initial state of the sample, all ballast sampled from the stockpile was washed, oven dried for at least 48 hours and sieved. For the purpose of this research study, fines were considered as all particles smaller than 9.5 mm or passing the ⅜-in. sieve. Ballast material separated during the sieving process was recombined and mixed using the recommended practices from AASHTO T 248, mixing and quartering procedures from Method B were employed due to the large size of the sample.

4 LABORATORY EXPERIMENTATION

Laboratory tests performed as part of this study followed recommendations from the German Deutsches

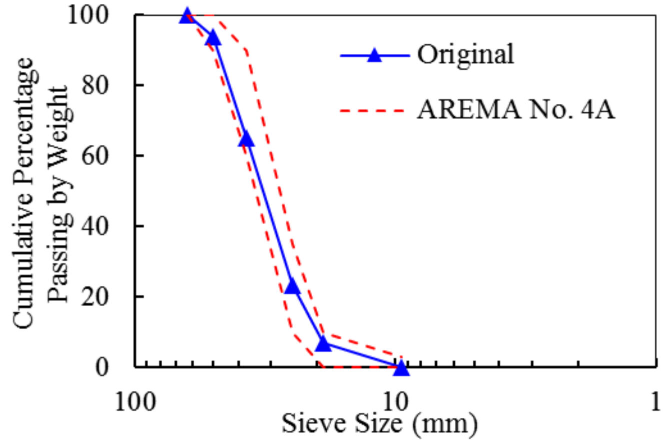

Figure 2. Particle size distribution of granite ballast material

Institut für Normung (DIN) 45673-5 standard (hereinafter referred to as DIN) for the determination of the mechanical fatigue resistance of under-ballast mat samples.

4.1 *Test setup*

Due to space constraints of the test frame available for testing, ballast box and loading plate had to be redesigned. A study was conducted on the recommendations and assumptions from the DIN based on information acquired by the researchers from personnel involved with its development and/or use. Hence, a new design was conceived with the intent to maintain most of the considerations of the original design, notably ballast depth, and pressures at the tie/ballast and ballast/mat interfaces. The newly designed apparatus consisted of a 30.5-cm (12-in.) diameter loading plate and a ballast box of 71 cm (28 in.) sides and 35.6 cm (14 in.) depth supporting a full 30.5-cm (12-in.) thick ballast layer section and capable of accommodating the thickest UBM sample available to the researchers at this time. Figure 3 shows the newly designed ballast box - named as the UIUC ballast box - and loading plate.

Figure 3. UIUC designed ballast box and loading plate

The applied loads and stresses at the plate/ballast interface in the setup were specified according to the DIN standard. To maintain the same stress levels of the DIN recommendations, the applied loads during testing were scaled based on the loading plate area used in this study. Details of the applied load levels are presented in the subsequent section.

The complete setup for testing was constructed using the UIUC ballast box, the UBM sample was placed on the bottom of the box over the flat steel bottom. Neoprene sheets, 6.35 mm (¼ in.) thick, were placed over the sidewalls, as specified by the DIN 45673-5, to provide elasticity to the ballast layer and better simulate particle confinement experienced in the field track conditions. Clean ballast was added and compacted for 60 seconds in three 10.2-cm (4-in.) lifts; an adjustable formwork vibrator attached to a steel plate (see Figure 4) provided a 4.4-kN (1000-lbf) compaction force at 60 Hz.

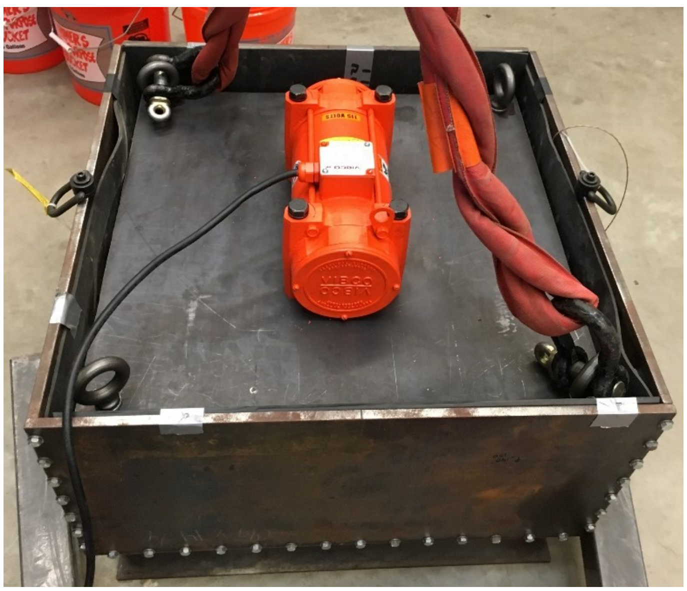

Figure 4. Vibratory plate used for ballast compaction

4.2 *Test procedures*

DIN recommended load levels served as basis for the determination of the loads to be used during the testing procedures. The prescribed fatigue load levels 1 and 2 for the UBM stiffness used were 75 kN (16.9 kips) and 100 kN (22.5 kips), respectively. These values were scaled based on the areas of the original design plate and UIUC loading plate in order to yield equivalent ballast stress levels of 265 kPa (38.5 psi) and 354 kPa (51.3 psi), respectively. Table 2 presents a summary the employed load levels for the sinusoidal loading procedures.

Table 2. Loading procedures employed.

Loading Type		Loading Range Minimum	Maximum	Sinusoidal Frequency	Number of Cycles
		kN (kips)	kN (kips)	Hz	
Fatigue	Level 1	1.8 (0.4)	19.6 (4.4)	5	$10x10^6$
	Level 2	1.8 (0.4)	25.8 (5.8)	5	$2.5x10^6$

The ballast box was placed in the testing frame. Displacements were measured using four potentiometers equally spaced along the perimeter of the loading plate attached to the 250-kN (55-kip) capacity vertical actuator (see Figure 1). Vertical transient displacement data were collected for ten consecutive cycles once every ten thousand cycles. Figure 5 presents the hysteresis loops representative of the system behavior under loading. After completion of 12.5 million loading cycles, the ballast material was carefully collected from the box and sieved for gradation check according to ASTM C136.

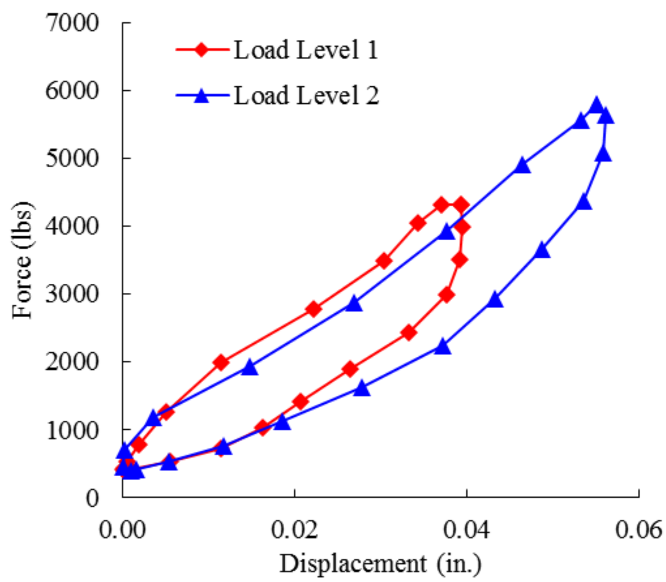

Figure 5. Hysteresis loops applied to the system

5 RESULTS AND DISCUSSION

To account for any possible tilting of the loading plate during the test, the analysis considered the average values obtained from the displacement data collected from all four potentiometers. Vertical transient deformation amplitudes were determined based on the maximum and minimum displacement values for every 10-cycle group collected. Figure 6 presents the results of this analysis. Sections of Figure 6 where amplitude results are omitted account for temporary instrumentation malfunction.

Figure 7 presents ballast particle size distribution obtained after the test in comparison with the original gradation. The two gradations showed very little variation. If any, the most noticeable changes occurred in sieve sizes 37.5 and 25.0 mm (1.5 and 1.0 in.) with

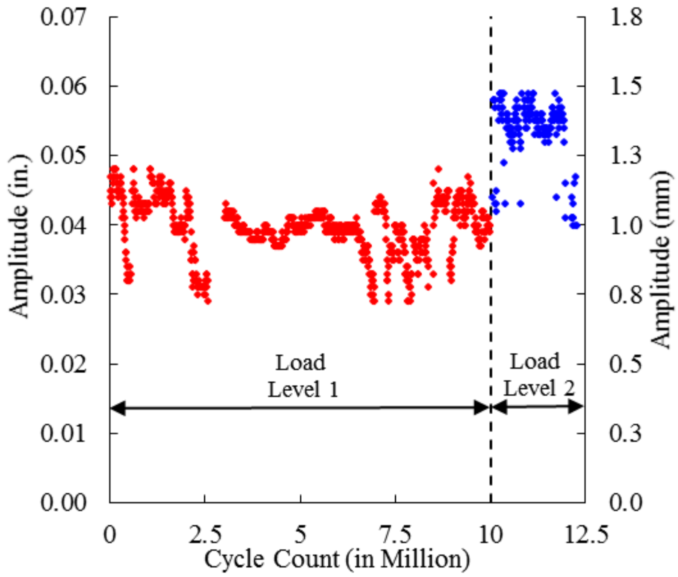

Figure 6. Loading plate vertical movement amplitudes

the largest difference being approximately of 1.3 percent only.

Visual assessment of the ballast after testing showed no noticeable particle breakage. This is also supported by particle size distributions as shown in Figure 7. Yet, fines that were not present in the clean original ballast were observed. The generation of additional fines in the ballast composition is believed to be due to relative movements between ballast particles as the system deforms under load, with large amplitudes of movement observed in the ballast surface as attested by Figure 6. Fines were generated as particle edges and corners chipped off, causing reductions in angularity, and the aggregate surfaces were subjected to frictional wear.

The analyses of the results presented in Figure 6 show slight variations in the vertical system movement amplitudes. The values varied between 0.7-1.2 mm (0.029-0.048 in) with an average of 1.0 mm (0.040 in) obtained for load level 1. For load level 2, values varied between 1.0-1.5 mm (0.040-0.059 in) and an average of 1.4 mm (0.054 in) was obtained. Moreover, during testing a clear vertical "bounce" or up-and-down movement of the ballast surface could be observed as the entire composition moved with the application of every load cycle.

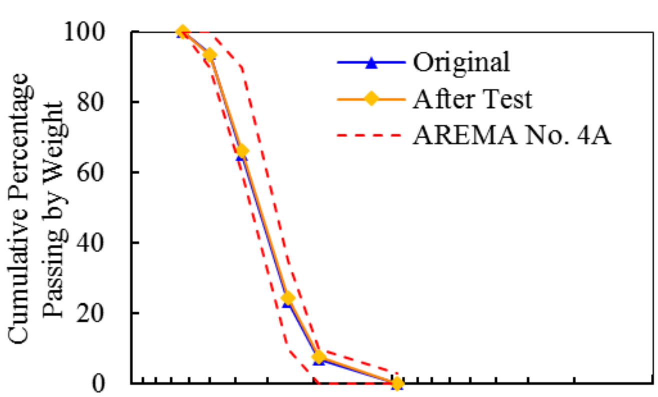

Figure 7. Particle size distributions of the ballast material before and after testing

In an effort to compare the laboratory results, typical ranges of field obtained vertical transient deformations at track transition zones were gathered from literature. There are various reports of field monitoring of track transient deformations (Coelho et al. 2010, Mishra et al. 2012, Mishra et al. 2014, Stark & Wilk 2015, Varandas et al. 2011). Mishra et al. (2012) recorded deflections of various layers (ballast, subballast, subgrade, etc.) of the track substructure and observed a maximum total transient vertical deformation of approximately 1.67 mm (0.066 in.) from the bottom of the sleeper using a multidepth deflectometer (MDD). Mishra et al. (2014) and Stark & Wilk (2015) employed the same apparatus and reported approximate total substructure transient vertical deformations of 1.85 mm (0.073 in), and 1.62 mm (0.064 in), respectively. Interestingly, it is encouraging to observe that the UBM sample tested was able to provide ballast layer deflection values during testing comparable to the open track values measured in the field. These results provide evidence to the potential of UBMs to increase track elasticity over rigid substructures. Ultimately, this equalization of transient deformations could lead to the deceleration of the previously mentioned track degradation negative feedback loop in transition zones.

Note that the measurements obtained by Mishra et al. (2014) from the use of the MDDs represent the movement of the bottom of the sleeper and so, may include displacement due to gaps developed between the tie and the ballast layer and the ballast migration and settlements resulting in "hanging ties". Additionally, it is necessary to emphasize the difficulty in replicating the exact field loading conditions in a laboratory setup. In field conditions, the tie/ballast contact is not always constant as the tie experiences uplifts between load applications (between axles), consequently there is a component of the measured displacements related to the deformation required before the ballast structure is mobilized, after which the true substructure deformations appear. Whereas in the laboratory experiments - in order to maintain the stability of the servo-hydraulic system - a minimum load of 1.8 kN (0.4 kips) was maintained throughout the entire duration of the test as recommended by DIN procedures. When evaluating Figure 5, note that the possible continuity of the hysteresis loop to zero load would provide additional measurements of displacement.

6 CONCLUSION

This paper presented results from laboratory experiments conducted as part of a larger-scope, ongoing research effort at the University of Illinois at Urbana-

Champaign (UIUC) aimed at evaluating and quantifying the overall performance of UBMs and their benefits to the track structure.

A test setup was developed based on recommended practices from the German DIN 45673-5 standard. An under-ballast mat (UBM) sample intended for freight applications was placed under a 30.5-cm (12-in.) ballast layer in a newly designed ballast box and subjected to 12.5 million repeated load applications. The employed ballast material was monitored for changes in particle size distribution, and the vertical deformations of the system were collected at regular 10,000 cycle intervals.

Test results from the sieve analysis conducted on the ballast material after completion of all loading cycles showed little signs of changes in gradation. Most of the observed deterioration was attributed to the relative movement between individual particles causing chipping off sharp corners and edges and frictional wear. Visual assessments after testing, which demonstrated the presence of fines yet no noticeable particle breakage, supported this finding.

Average vertical transient deformation amplitudes recorded throughout testing were in the order of 0.040 in. (1.0 mm) for load level 1 and 0.054 in. (1.4 mm) for load level 2. Amplitudes monitored during testing were compared to field measured values reported in the literature. Such comparison showed that the increased elasticity of the ballast structure due to the incorporation of the UBM over the rigid bottom of the box resulted in movements comparable to the field measurement values.

It is worth noting that, due to the explained differences in the tie/ballast contact between field and laboratory conditions, field results from transient deformations of track with UBMs installed in ballasted bridge decks could be larger than the laboratory obtained values, providing even closer transient deflection values to the ones obtained by field measurements.

Lastly, the development of thorough studies to investigate the life cycle of UBMs is of crucial importance for the widespread deployment of this component. Currently – as part of the larger scope of this research effort – a quantitative study of the life cycle of UBMs under different loading condition is underway and results will be reported in future publications.

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