Optimal Fueling Strategies for Locomotive Fleets in Railroad Networks

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Seyed Mohammad Nourbakhsh
Yanfeng Ouyang
UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

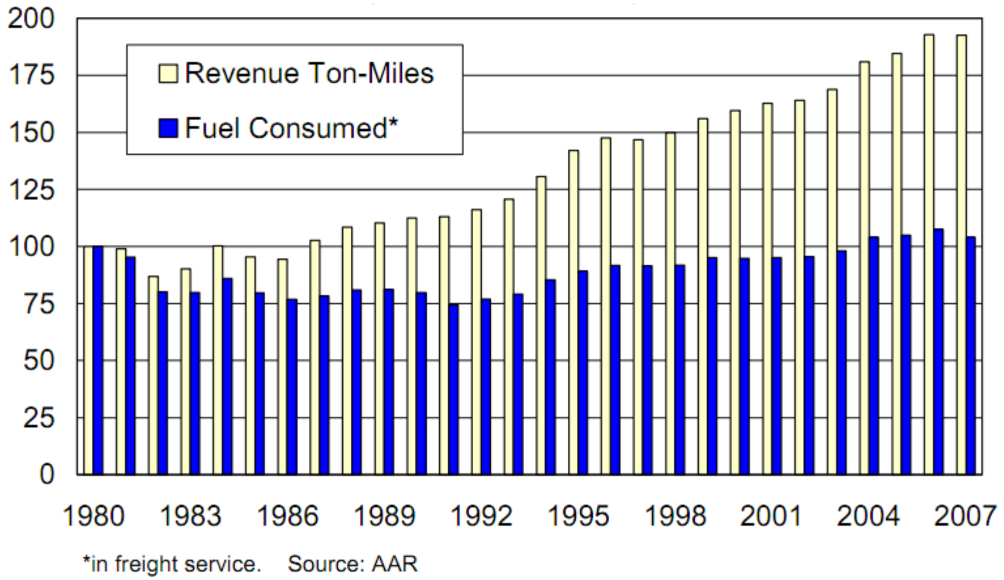
Outline

- Background
- Model Formulation
- Optimality Properties and Solution Techniques
- Case Studies
- Conclusion

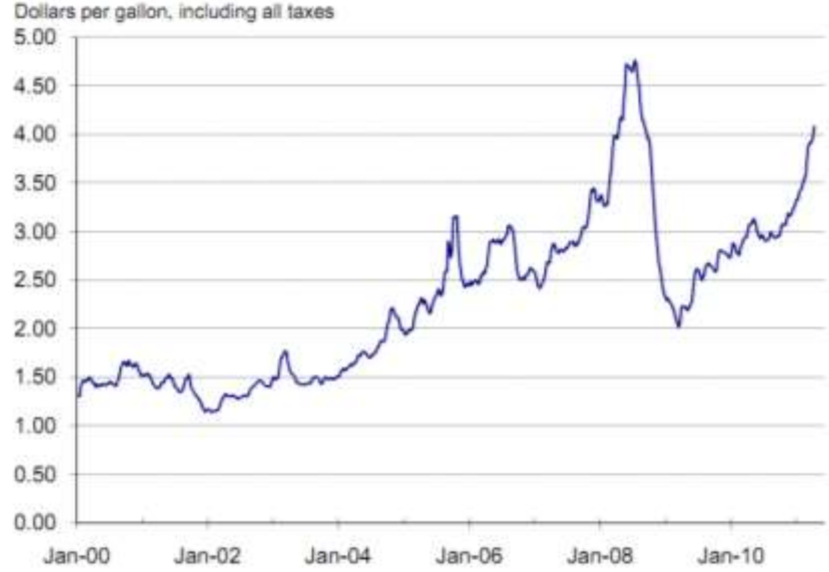
Fuel Price

- Fuel-related expenditure is one of the biggest cost items in the railroad industry

- Railroad fuel consumption remains steady

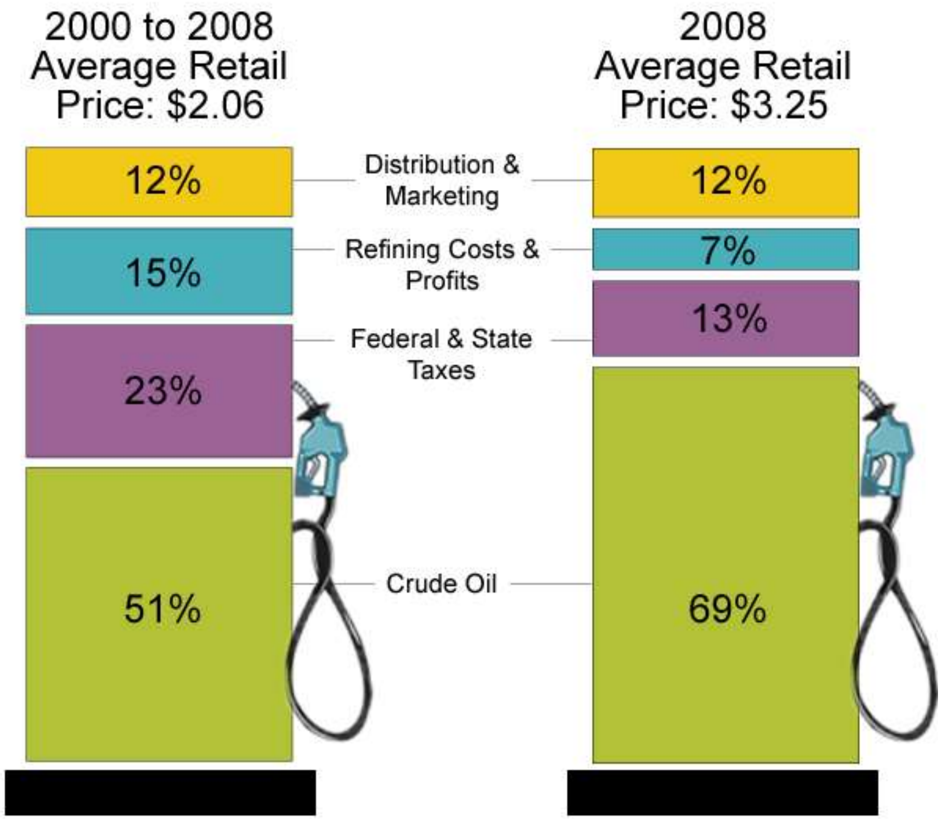


- Crude oil price sharply increases in recent years



Fuel Price

- Fuel (diesel) price influenced by:
  - Crude oil price
  - Refining
  - Distribution and marketing
  - Others

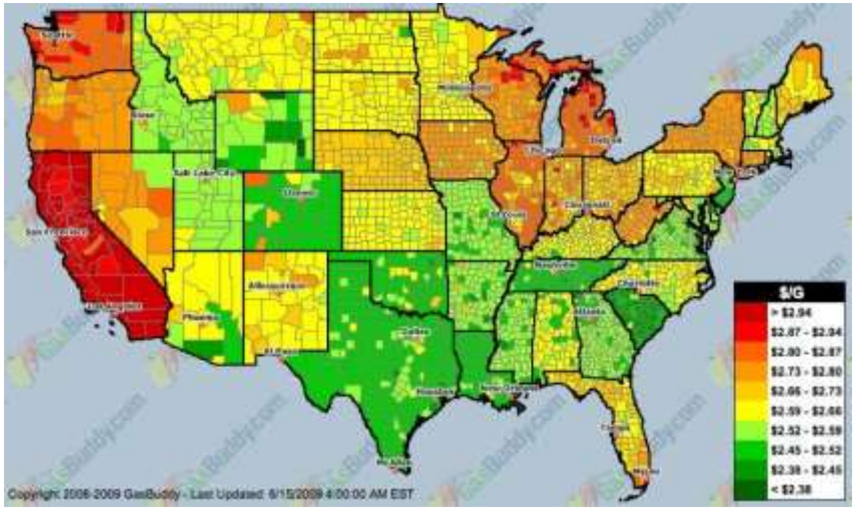


Source: Energy Information Administration.

Fuel Price

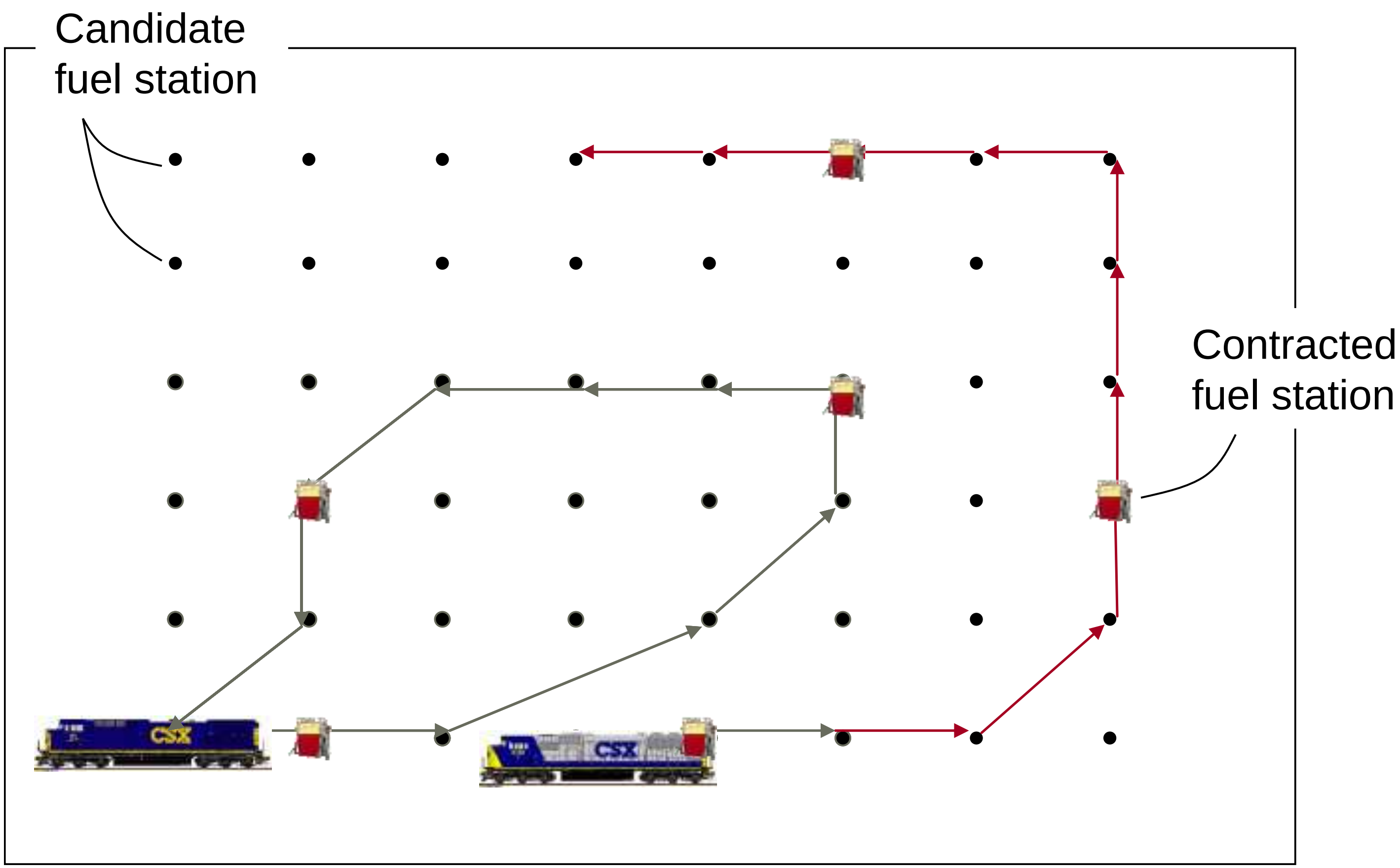
- Fuel price vary across different locations

- Each fuel station requires a long-term contractual partnership
  - Railroads pay a contractual fee to gain access to the station
  - Sometimes, a flat price is negotiated for a contract period



- US national fuel retail price, by county, 2009

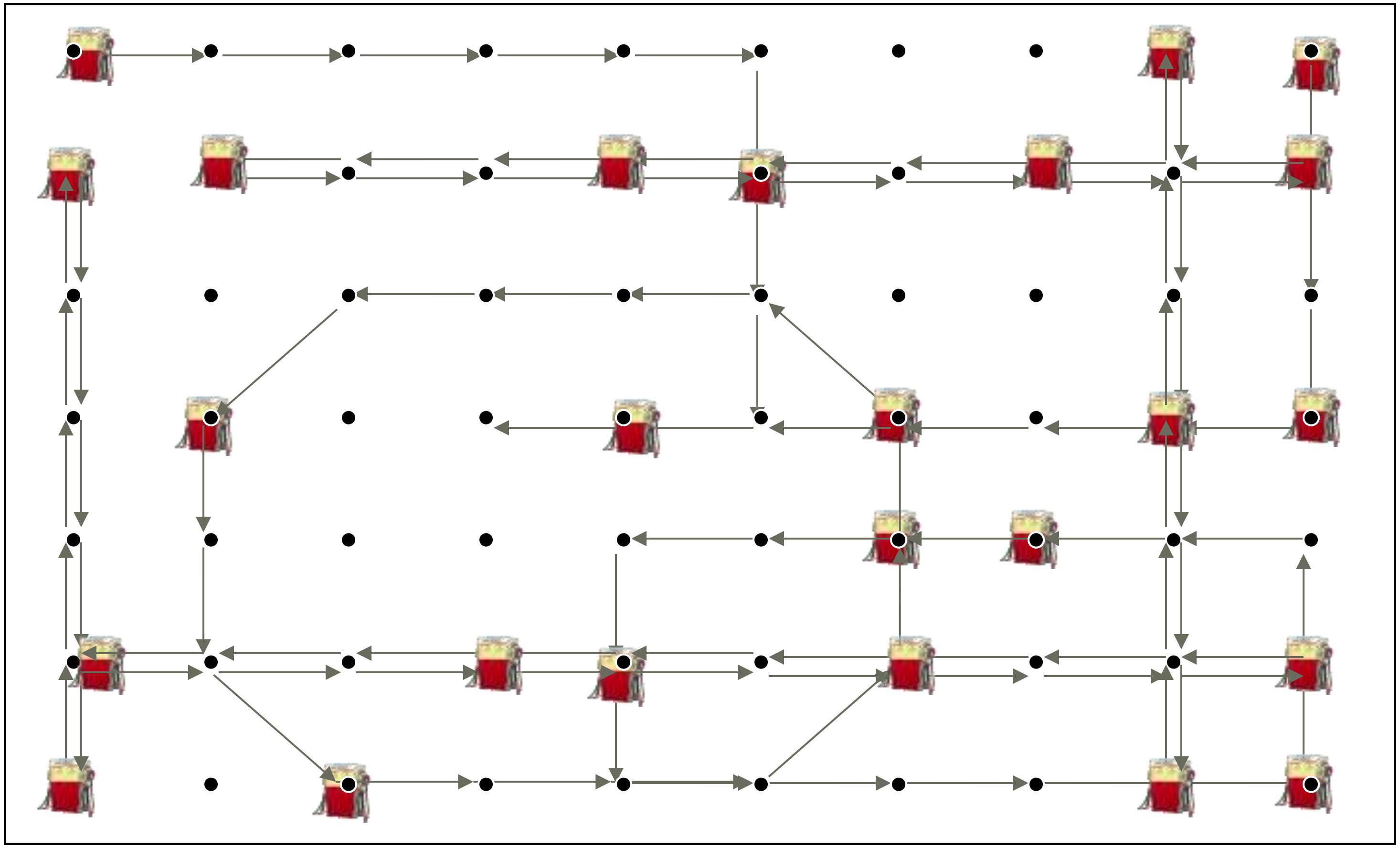
Locomotive Routes in a Network



Motivation

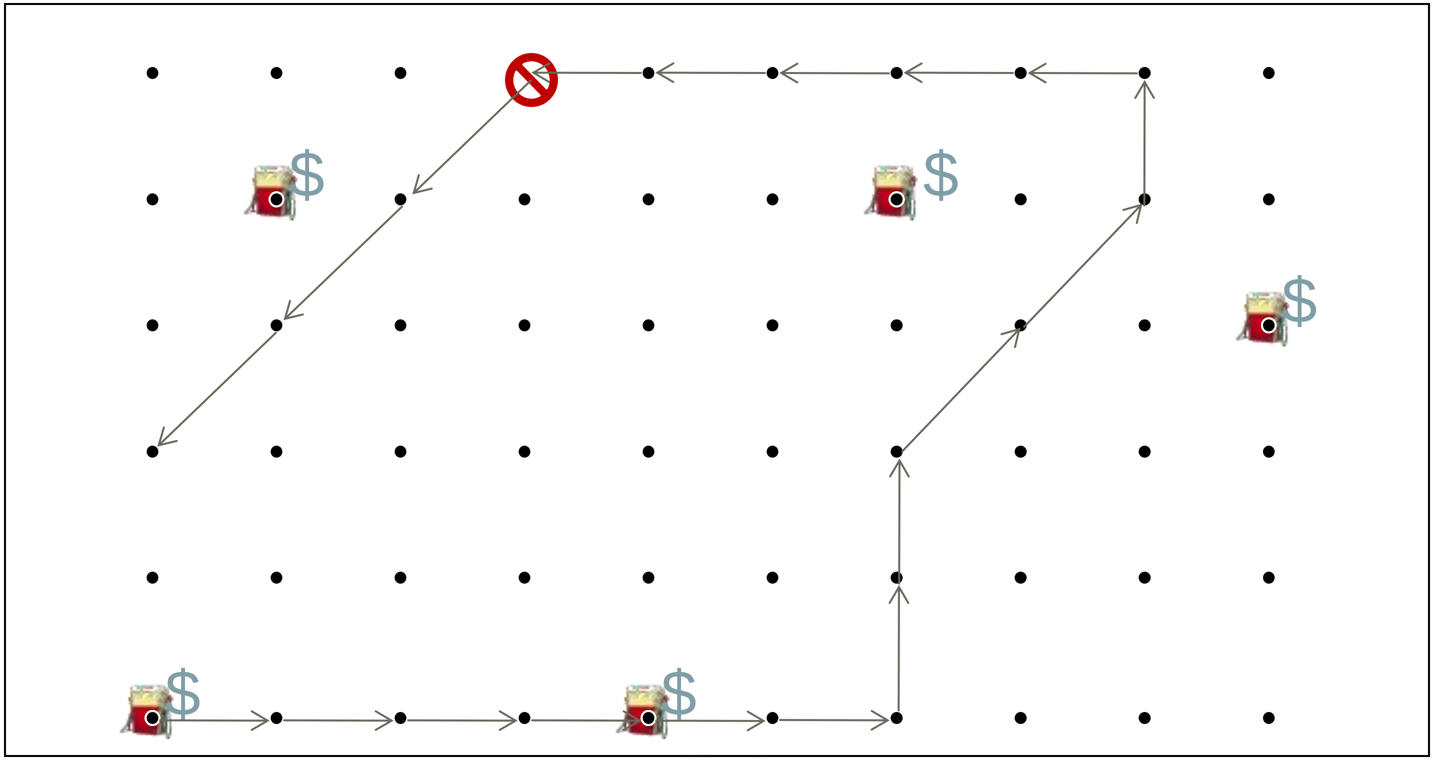
- Usage of each fuel station requires a contractual partnership cost
- Hence, should contract stations and purchase fuel where fuel prices are relatively low (without significantly interrupting locomotive operations)
  - In case a locomotive runs out of fuel, *emergency purchase* is available anywhere in the network but at a much higher price
  - Each fueling operation delays the train

The Challenge



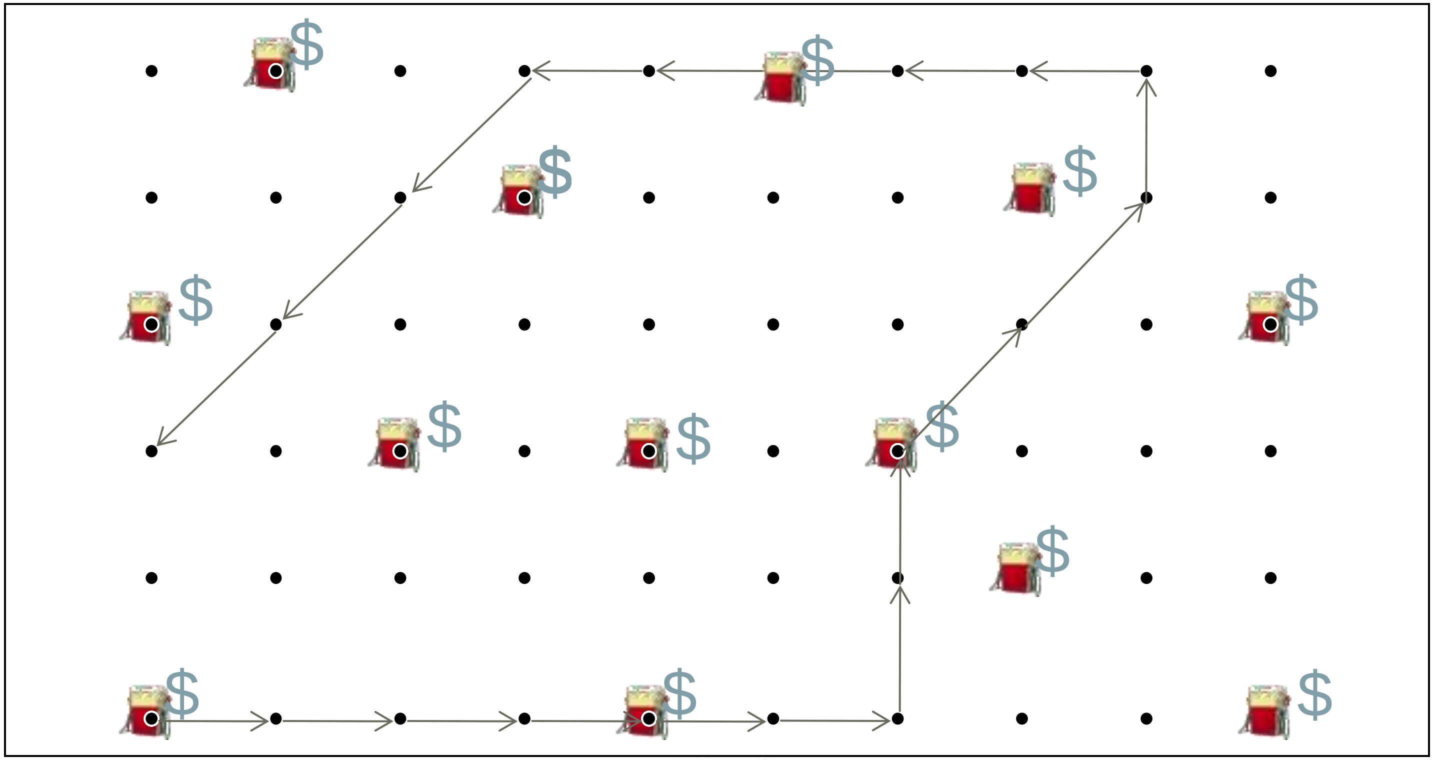
The Challenge

- Fuel cost vs. contract cost
  - Too few stations = high fueling cost (e.g., emergency purchase)



The Challenge

- Fuel cost vs. contract cost
  - Too many stations = high contracting costs



Problem Objective

- To determine:
  - Contracts for fueling stations
  - Fueling plan for all locomotives
    - Schedule
    - Location
    - Quantity
- To minimize:
  - Total fuel-related costs:
    - Fuel purchase cost
    - Delay cost
    - Fuel stations contract cost

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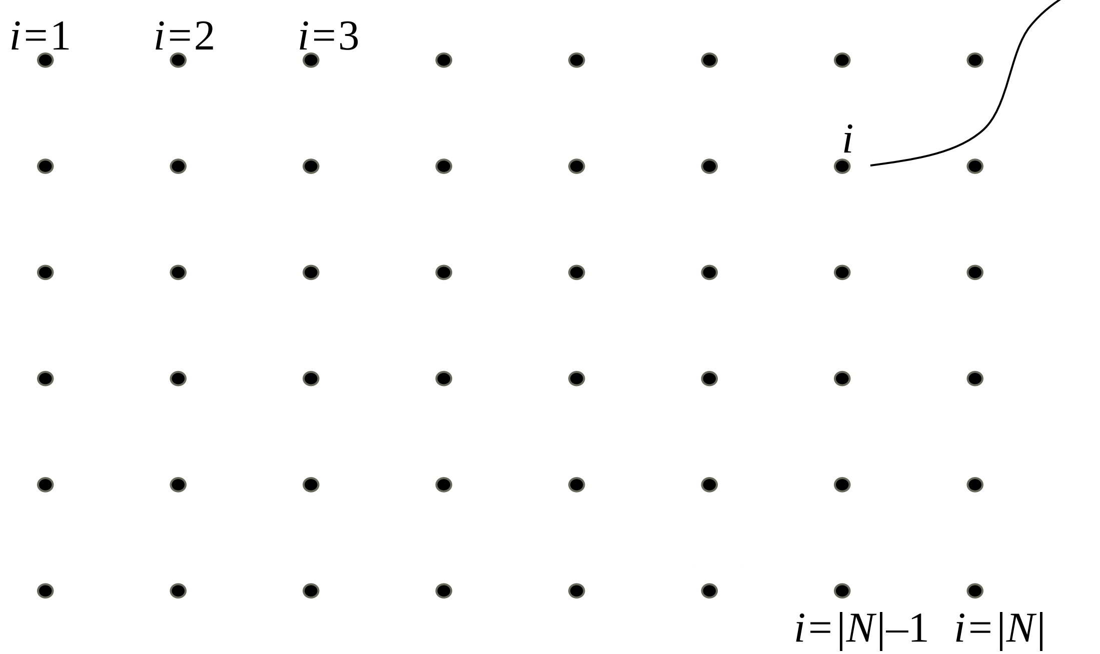
Notation

- Set of candidate fuel stations, $N = \{1,2,\ldots, |N|\}$
- Set of locomotives, $J = \{1,2,\ldots, |J|\}$
- Sequence of stops for locomotive j, $S_j = \{1,2,\ldots, n_j\}$, for all $j \in J$



For any location i

c_i = Unit fuel cost

a_1 = Delay cost per fueling stop

a_2 = Contract cost per fuel station per year

M_i = Maximum number of locomotives passing

p=Unit fuel cost for emergency purchase ($p>c_i$ for all i)

Notation

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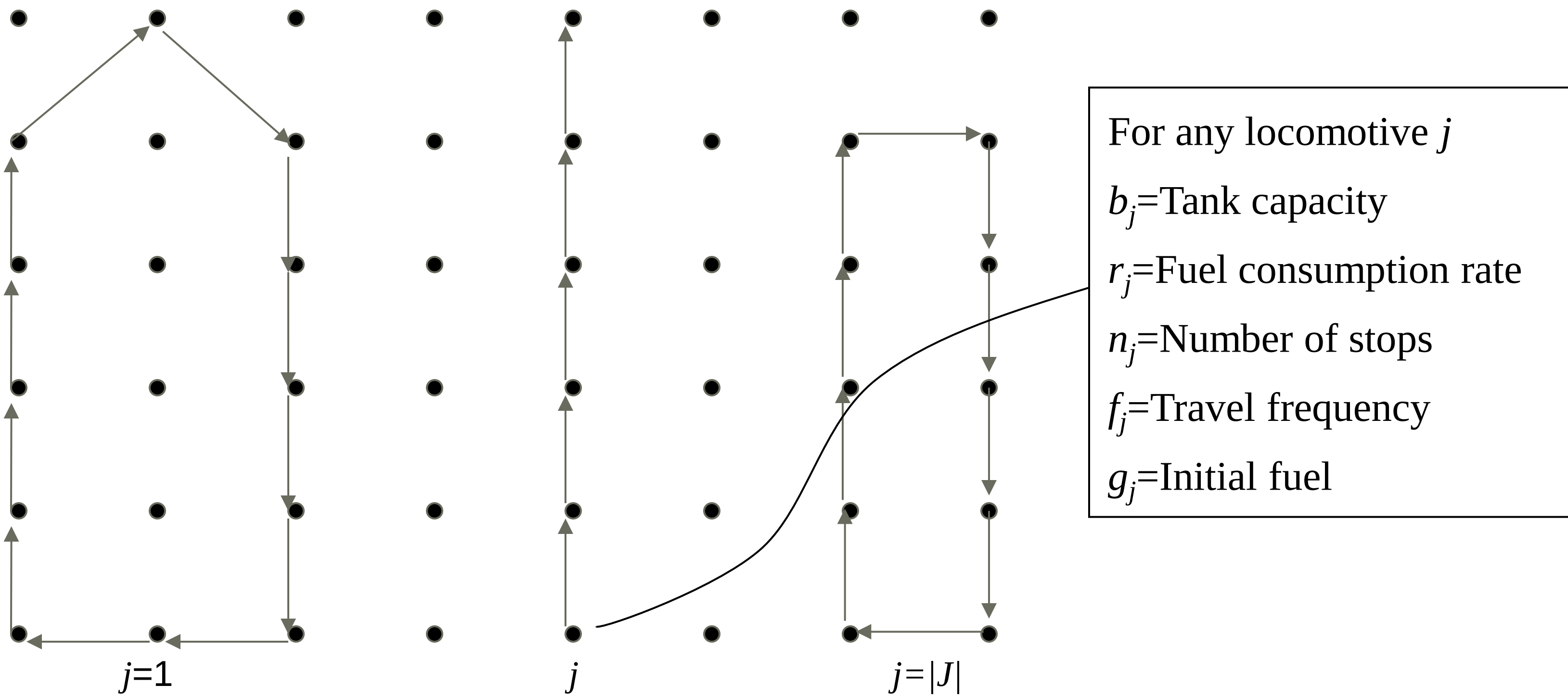

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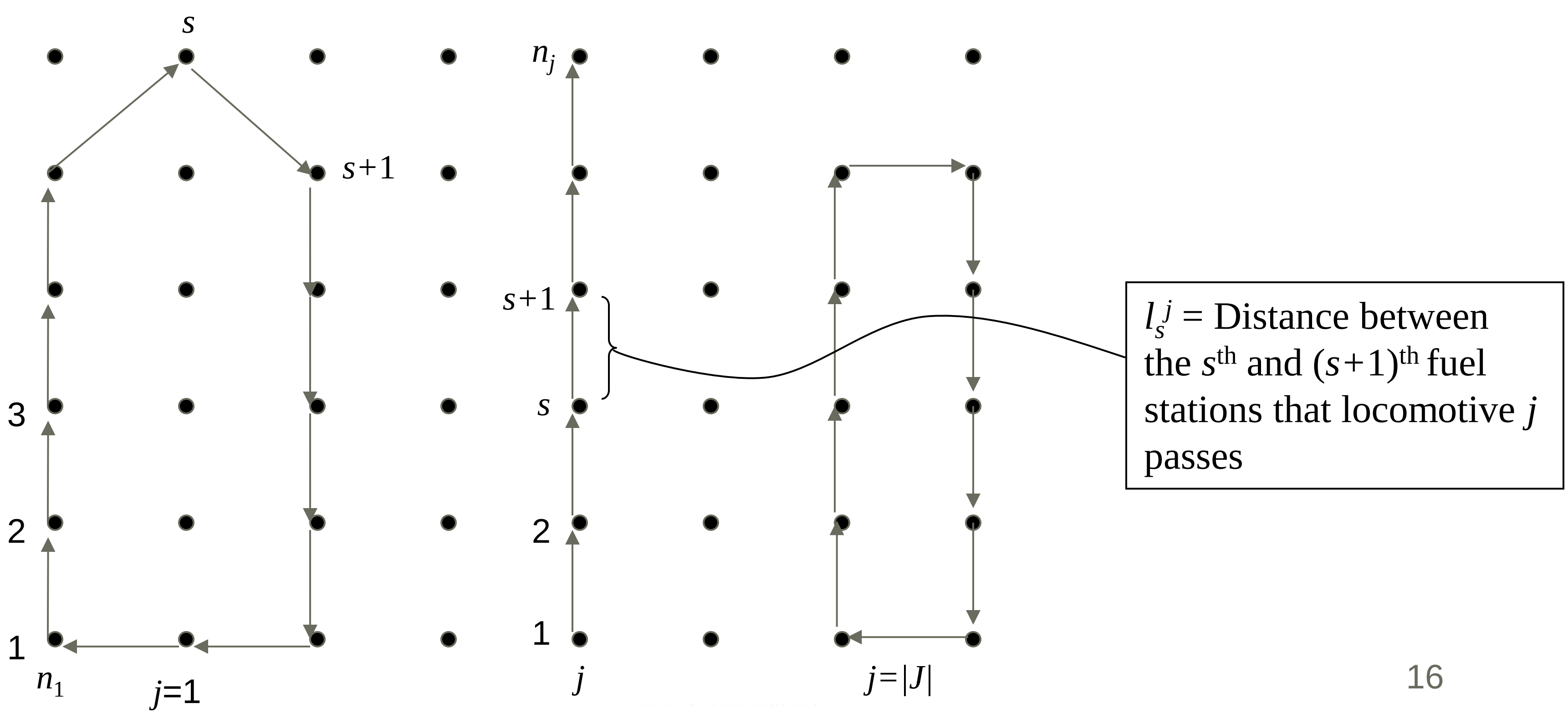

Decision Variables

- For each station, contract or not?
  - z_i = 1 if candidate fuel station i is contracted and 0 otherwise
- For each locomotive, where to stop for fuel?
  - x_s^j = 1 if locomotive j purchases fuel at its s^{th} station and 0 otherwise
  - y_s^j = 1 if locomotive j purchases emergency fuel between its s^{th} and $(s+1)^{th}$ station and 0 otherwise
- How much to purchase?
  - w_s^j =Amount of fuel purchased at stop s of locomotive j
  - v_s^j =Amount of emergency fuel purchased between the s^{th} and $(s+1)^{th}$ stations of locomotive j

Formulation

$$\min \sum_{j=1}^{|J|}\sum_{s=1}^{n_j} f_j[\sum_{i=1}^{|N|}(c_i q_{is}^j w_s^j) + (pv_s^j)] + \alpha_1 \sum_{j=1}^{|J|}\sum_{i=1}^{n_j} f_j(x_s^j + y_s^j) + \alpha_2 \sum_{i=1}^{|N|} z_i$$

Fueling cost + delay cost + contract cost

$$\text{s.t.} \quad g_j + \sum_{s=1}^{k}(w_s^j + v_s^j - r_j l_s^j) \geq 0 \ , \ \forall j \in J, \ \forall k = 1,2,\ldots,n_j - 1$$

$$\sum_{s=1}^{k}(w_s^j + v_s^j - r_j l_s^j) \geq 0 \ , \ \forall j \in J, \ k = n_j$$

Never run out of fuel

$$g_j + \sum_{s=1}^{k-1}(w_s^j + v_s^j - r_j l_s^j) + w_k^j \leq b_j \ , \ \forall j \in J, \ \forall k = 1,2,\ldots,n_j$$

Tank capacity never exceeded at fuel stations

$$w_s^j \leq b_j x_s^j \ , \ \forall j \in J \ , \ \forall s = 1,2,\ldots,n_j - 1$$

Must stop before purchasing

$$v_s^j \leq b_j y_s^j \ , \ \forall j \in J \ , \ \forall s = 1,2,\ldots,n_j - 1$$

Tank capacity never exceeded at emergency purchase

$$\sum_{j=1}^{|J|}\sum_{s=1}^{n_j} q_{i,s}^j x_s^j \leq M_i . z_i \ , \ \forall i \in N$$

Must contract fuel stations for usage

$$x_s^j, y_s^j, z_i \in \{0,1\} \ \ \forall j, \forall s = 1,2,\ldots,n_j - 1$$

Integrality constraints

$$w_s^j, v_s^j \geq 0, \forall j \in J, \forall s = 1,2,\ldots,n_j - 1$$

Non-negativity constraints

Problem Characteristics

- The MIP problem is NP hard…
  - Integration of facility location and production scheduling
- The problem scale is likely to be large
  - $2\sum_{j=1}^{|J|} n_j + |N|$ of integer variables, $4\sum_{j=1}^{|J|} n_j + |N|$ of constraints
  - For $|J|=2500$ locomotives each having $n_j=10$ stops among $|N|=50$ fuel stations, there are 50,050 integer variables and 100,050 constraints
- Commercial solver failed to solve the problem for real applications
- Hence, to solve this problem
  - Derive optimality properties to provide insights
  - Develop a customized Lagrangian relaxation algorithm

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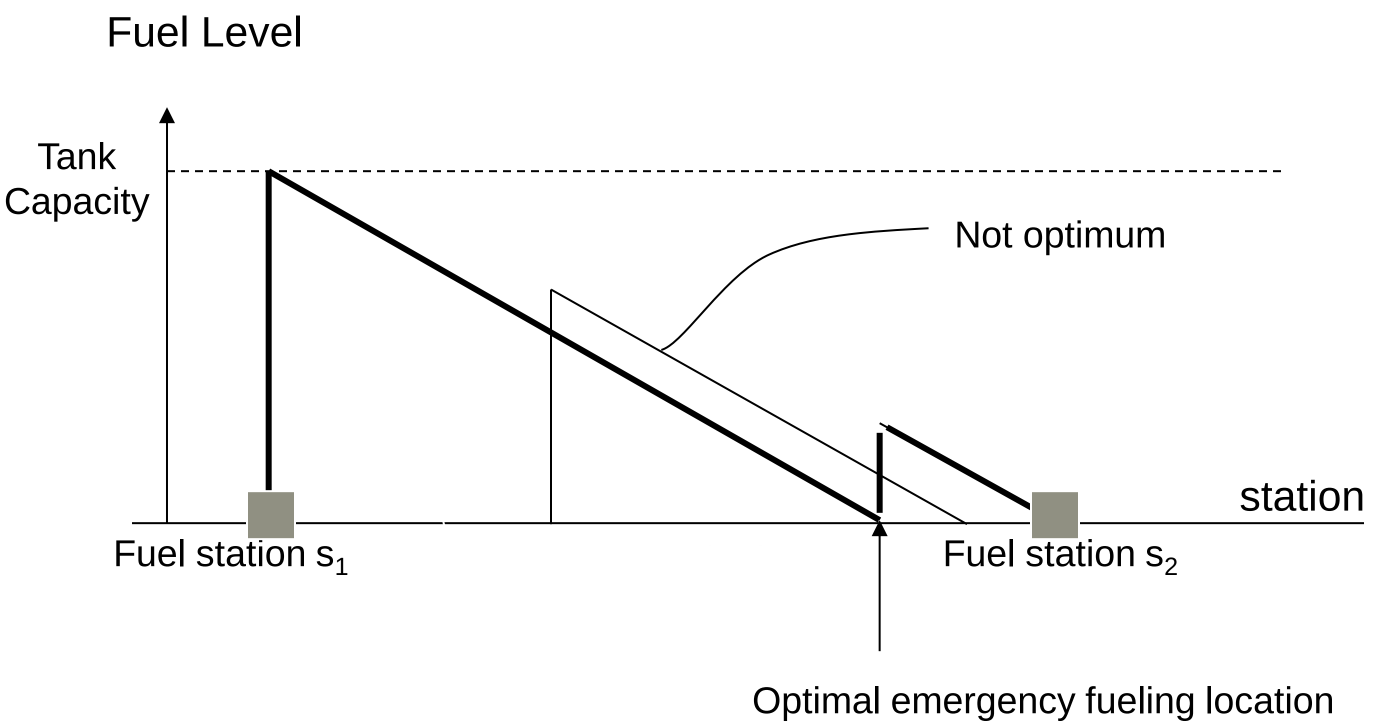
Theoretical Findings

Optimality Condition 1

There exists an optimal solution in which a locomotive stops for emergency fuel only when the locomotive runs out of fuel.

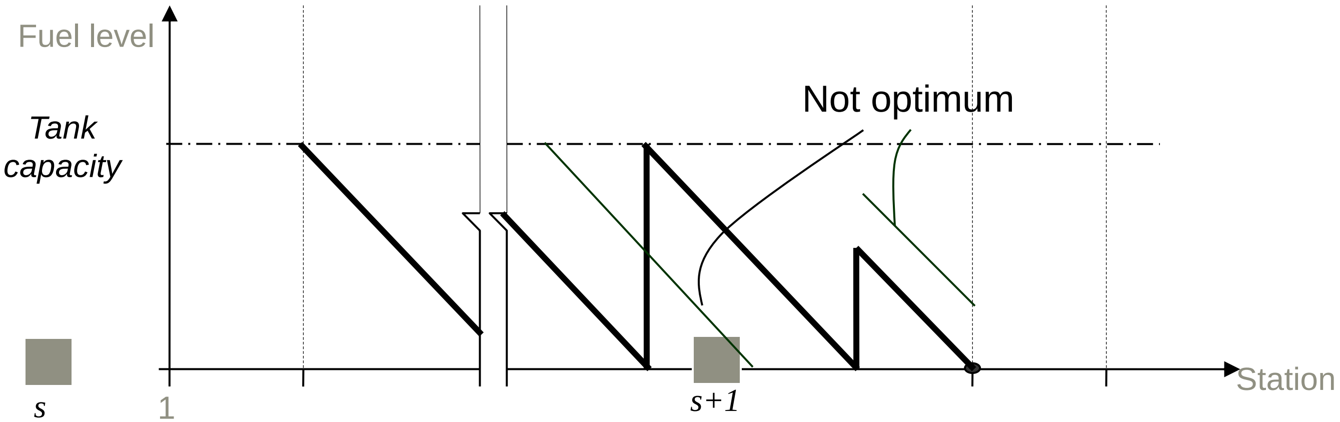


Theoretical Findings

Optimality Condition 2

There exists an optimal solution in which a locomotive purchases emergency fuel only if the following conditions hold:

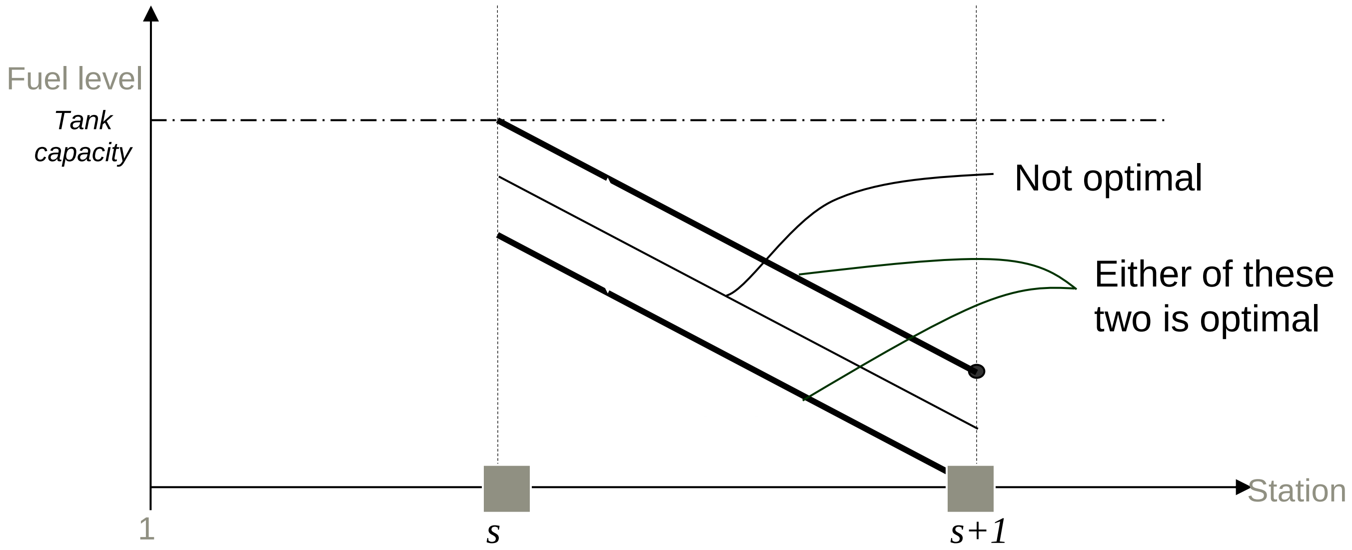
- Its previous fuel purchase (from either an emergency or fixed station) must have filled up the tank capacity
- If the next fuel purchase is at a fixed stations, then the purchased fuel should be minimum; i.e., the locomotive will arrive at the next station with an empty tank



Theoretical Findings

Optimality Condition 3

If a locomotive purchases fuel at two fixed fueling stations s_1 and s_2 (not necessarily adjacent along the route) but no emergency fuel in between, then there exists an optimal solution in which the locomotive either departs s_1 with a full tank, or arrives at s_2 with an empty tank.



Lagrangian Relaxation

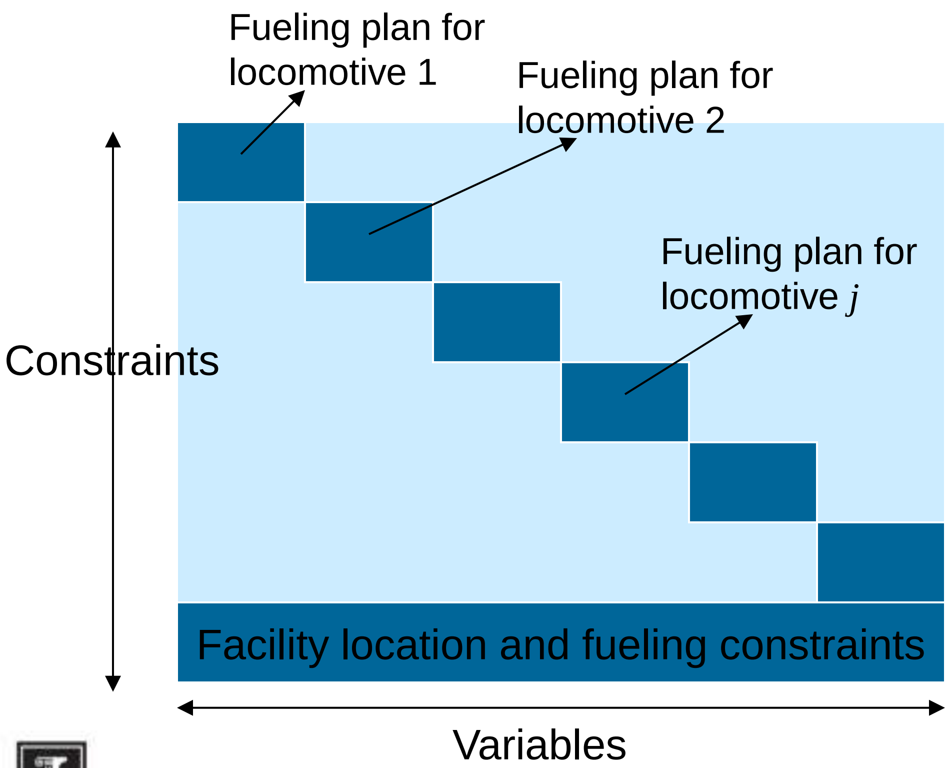
- Relax hard constraints:

$$\sum_{j=1}^{|J|}\sum_{s=1}^{n_j} q_{i,s}^j x_s^j \le \sum_{j=1}^{|J|} M_i z_i \ , \ \ \forall i \in N$$

- Then add them to the objective function with penalty:

$$\sum_{i=1}^{|N|} u_i \left(\sum_{j=1}^{|J|}\sum_{s=1}^{n_j} q_{i,s}^j x_s^j - \sum_{j=1}^{|J|} M_i z_i\right)$$

Structure of the constraints:



Formulation of Relaxed Problem

$$\min \sum_{j=1}^{|J|}\sum_{s=1}^{n_j} f_j\left[\sum_{i=1}^{|N|}(c_i q_{is}^j w_s^j) + (p v_s^j)\right] + \alpha_1 \sum_{j=1}^{|J|}\sum_{i=1}^{n_j} f_j (x_s^j + y_s^j) + \alpha_2 \sum_{i=1}^{|N|} z_i \quad + \sum_{i=1}^{|N|} u_i \left(\sum_{j=1}^{|J|}\sum_{s=1}^{n_j} q_{i,s}^j x_s^j - \sum_{j=1}^{|J|} M_i z_i\right)$$

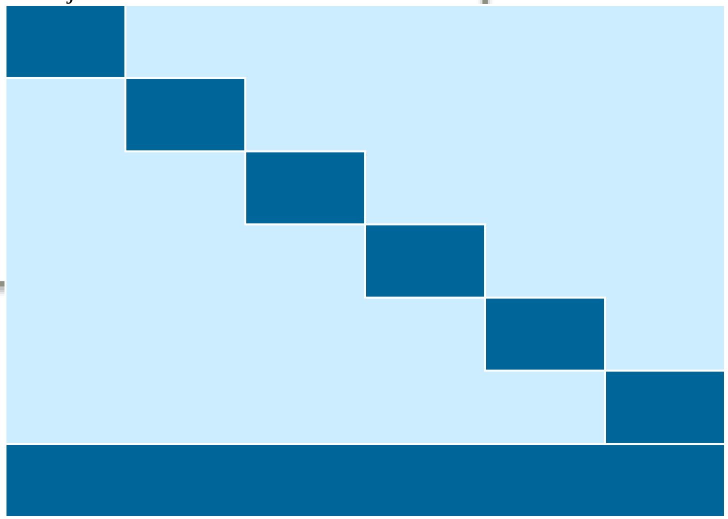
$$\text{s.t.} \quad g_j + \sum_{s=1}^{k}(w_s^j + v_s^j - r_j l_s^j) \ge 0 \ , \ \forall j \in J, \ \forall k = 1,2,\ldots,n_j - 1$$

$$\sum_{s=1}^{k}(w_s^j + v_s^j - r_j l_s^j) \ge 0 \ , \ \forall j \in J, \ k = n_j$$

$$g_j + \sum_{s=1}^{k-1}(w_s^j + v_s^j - r_j l_s^j) + w_k^j \le b_j \ , \ \forall j \in J, \ \forall k = 1,2,\ldots,n_j$$

$$w_s^j \le b_j x_s^j \ , \ \forall j \in J \ , \ \forall s = 1,2,\ldots,n_j - 1$$

$$v_s^j \le b_j y_s^j \ , \ \forall j \in J \ , \ \forall s = 1,2,\ldots,n_j - 1$$



$$x_s^j, y_s^j, z_i \in \{0,1\} \ \forall j, \forall s = 1,2,\ldots,n_j - 1$$

$$w_s^j, v_s^j \ge 0, \forall j \in J, \forall s = 1,2,\ldots,n_j - 1$$

Relaxed Problem

- After relaxing hard constraints the remaining problem could be decomposed into sub-problems
  - Each sub-problem solves the fueling planning for each locomotive

$$\text{relaxed objective } = \sum_{j=1}^{|J|} z_j(\mathbf{u}) + \sum_{i=1}^{|N|} z_i.(\alpha_2 - u_i M_i)$$

where $z_j(\boldsymbol{u})$ is optimal objective function of j^{th} sub-problem

Sub-problem for the jth Locomotive

min $$z_j(\mathbf{u}) = \sum_{s=1}^{n_j} f_j[\sum_{i=1}^{|N|}(c_i q_{i,s}^j w_s^j) + p v_s^j] + \alpha_1 \sum_{s=1}^{n_j} f_j(x_s^j + y_s^j) - \sum_{i=1}^{|N|} u_i \sum_{j=1}^{|J|} \sum_{s=1}^{n_j} q_{i,s}^j x_s^j$$

s.t. $$g_j + \sum_{s=1}^{k}(w_s^j + v_s^j - r_j l_s^j) \geq 0 \ , \ \forall k = 1,2,\ldots,n_j - 1$$

$$\sum_{s=1}^{k}(w_s^j + v_s^j - r_j l_s^j) \geq 0 \ , \ k = n_j$$

$$g_j + \sum_{s=1}^{k-1}(w_s^j + v_s^j - r_j l_s^j) + w_k^j \leq b_j \ , \ \forall k = 1,2,\ldots,n_j$$

$$w_s^j \leq b_j x_s^j \ , \ \forall s = 1,2,\ldots,n_j - 1$$

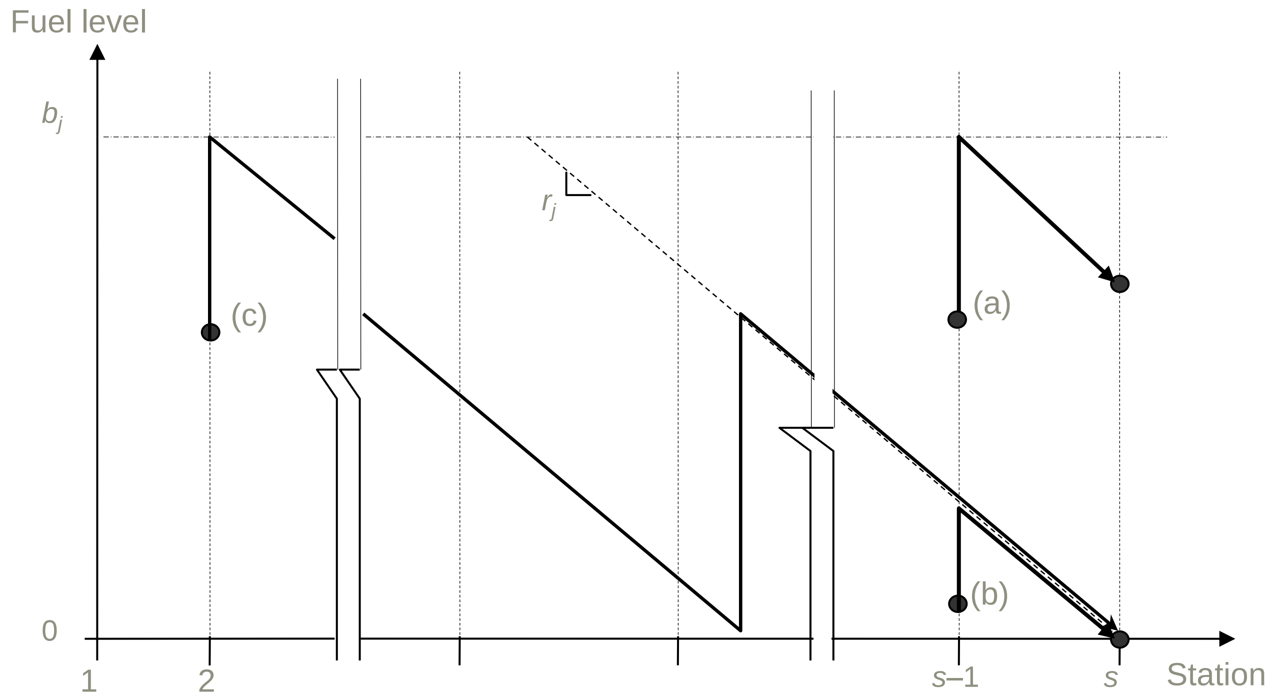
$$v_s^j \leq b_j y_s^j \ , \ \forall s = 1,2,\ldots,n_j - 1$$

$$x_s^j, y_s^j, z_i \in \{0,1\} \ , \forall s = 1,2,\ldots,n_j - 1$$

$$w_s^j, v_s^j \geq 0 \ , \ \forall s = 1,2,\ldots,n_j - 1$$

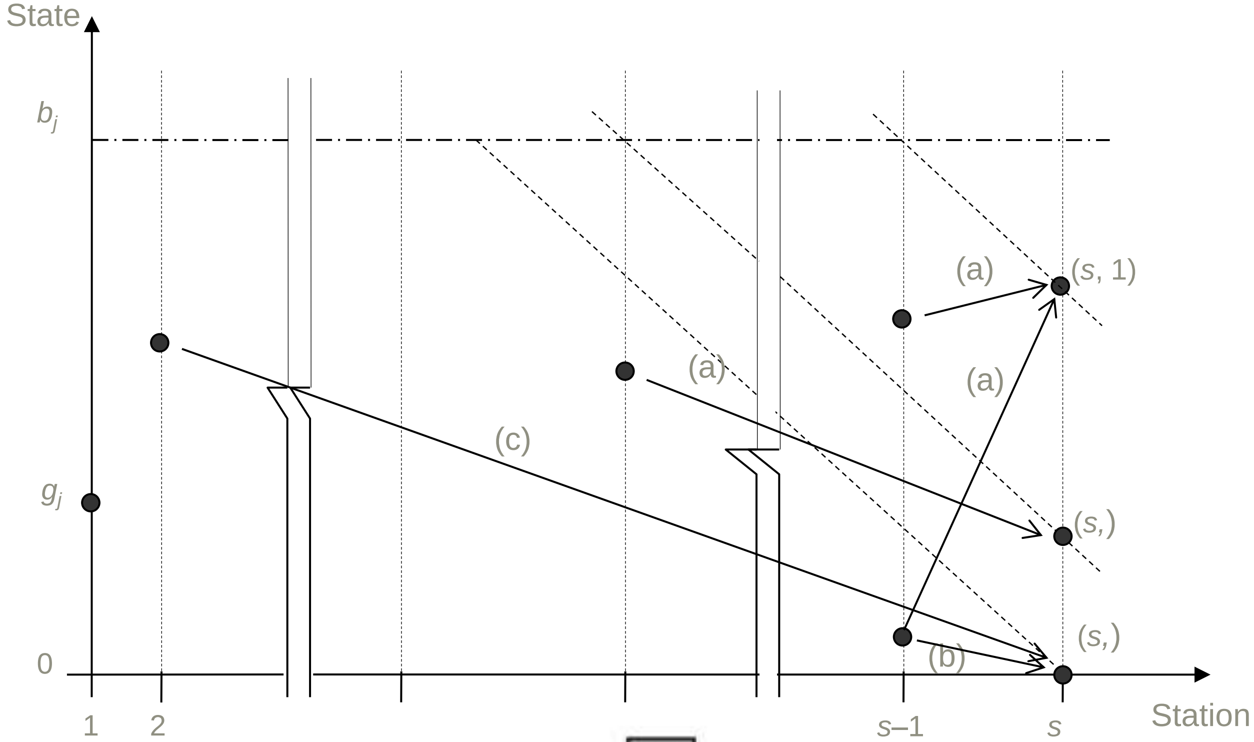
Sub-problem for Individual Locomotive

- Three types of possible "optimal" fuel trajectory
  - Type a: From one station to nonzero fuel at another station
  - Type b: From one station to zero fuel at another station, without emergency purchase
  - Type c: From one station to zero fuel at another station, after one or more emergency fuel purchases



Shortest Path Method

- We find a way to apply a simple shortest path method to solve the sub-problem



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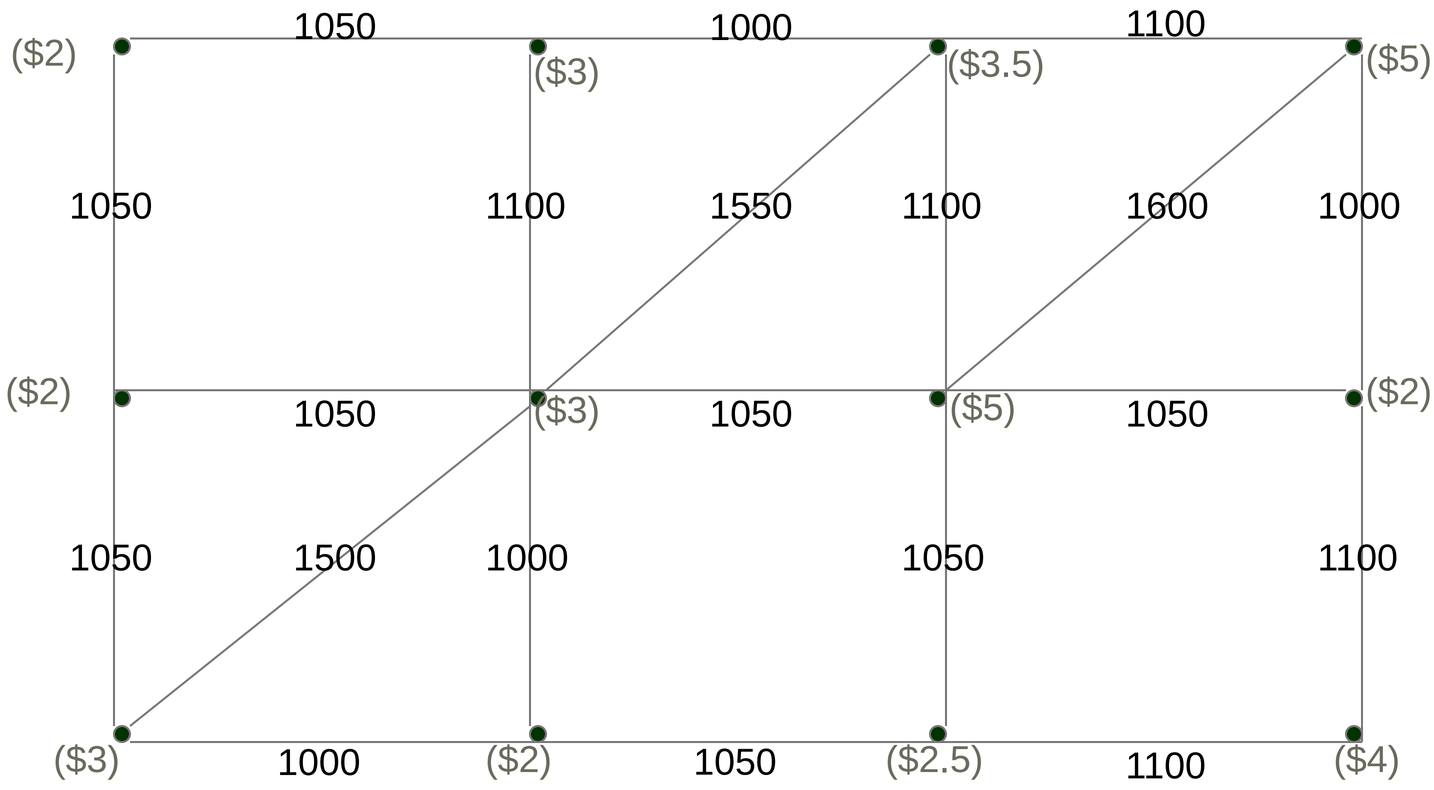
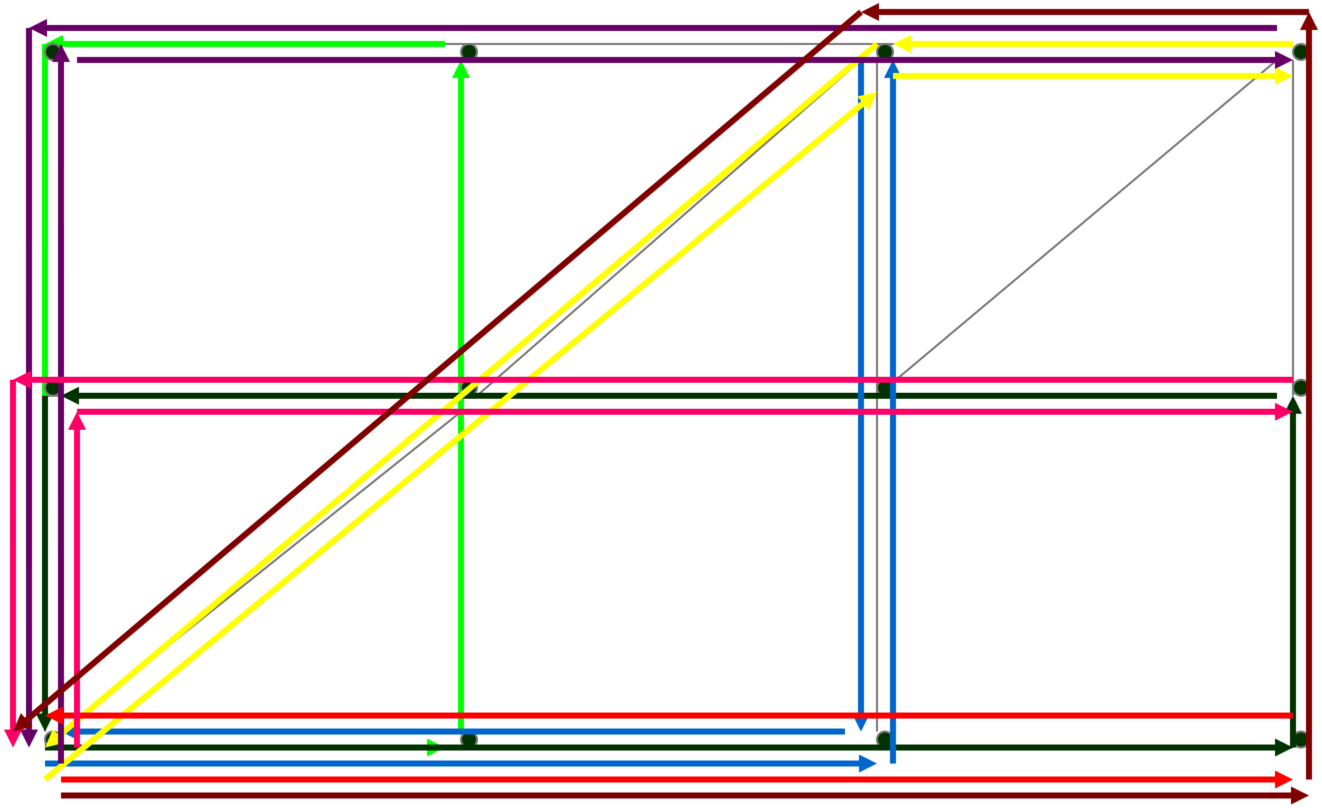
Test Case

Network Information



Test Case

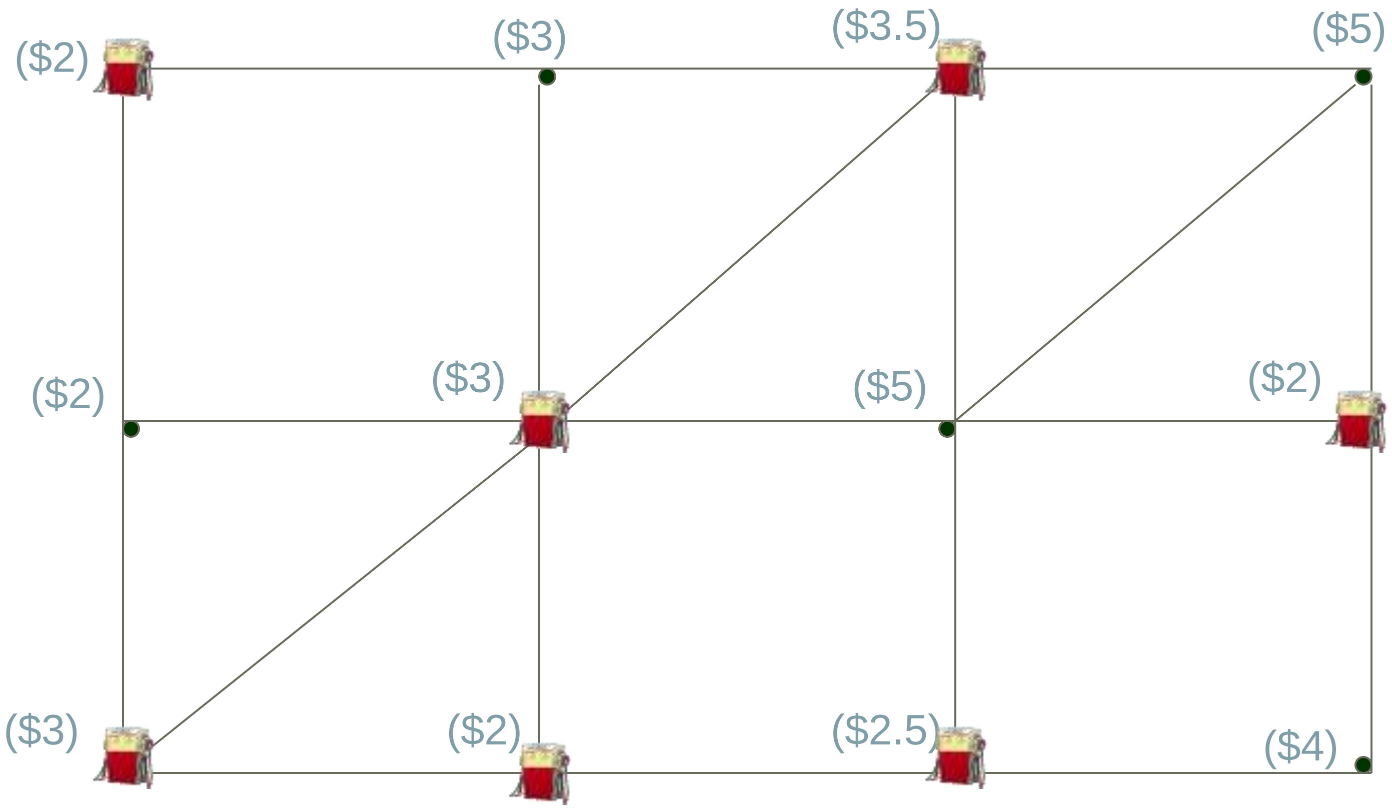
Locomotive Route Information



Test Case

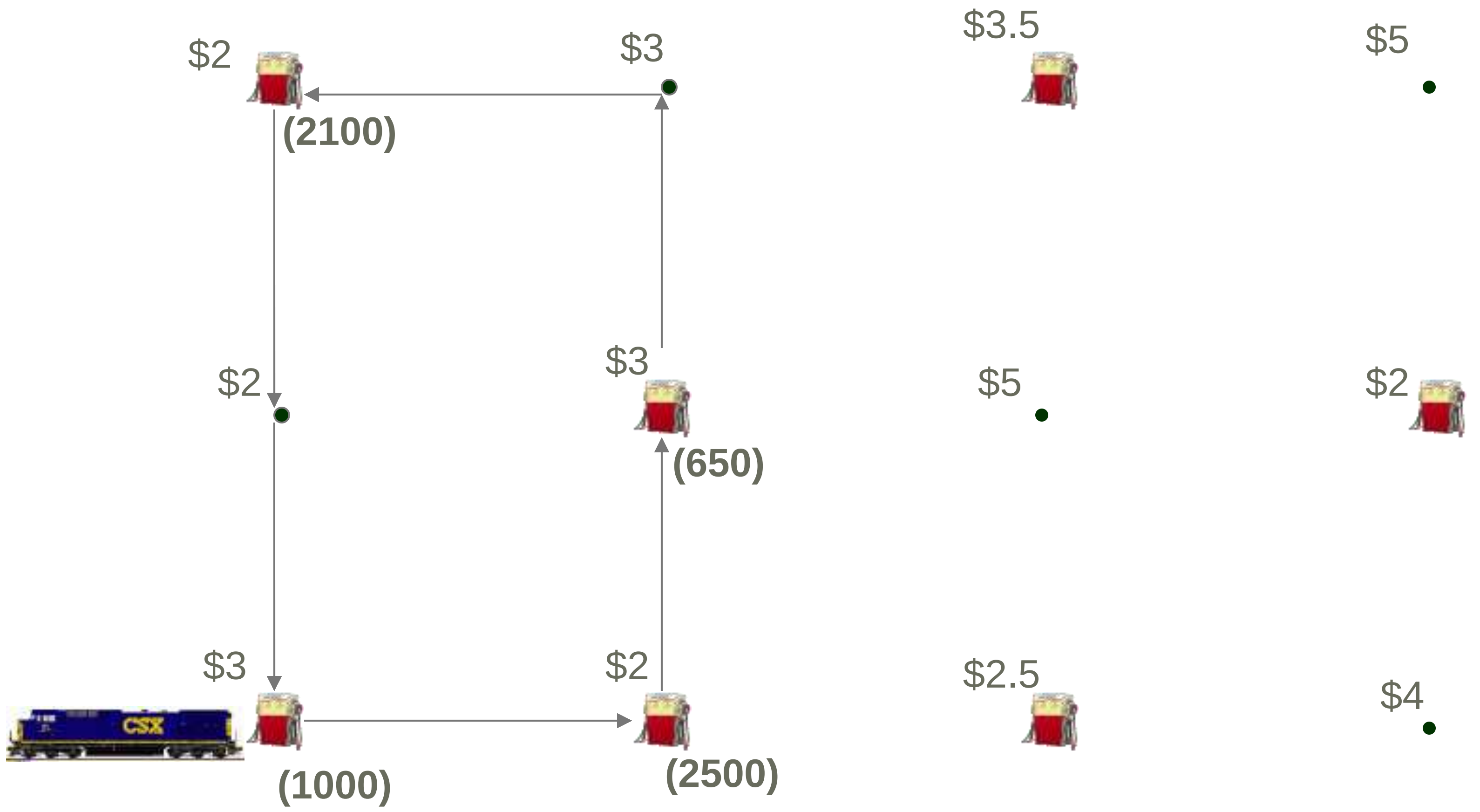
- 12 nodes, 8 locomotives
- α_1=100, α_2=10,000
- Tank capacity=2500
- Different fuel price for fixed stations between $2 to $5 and $7 for emergency
- Frequency assumed 1 for all locomotives
- Consumption rate assumed 1 for all locomotives

Optimal Fuel Stations

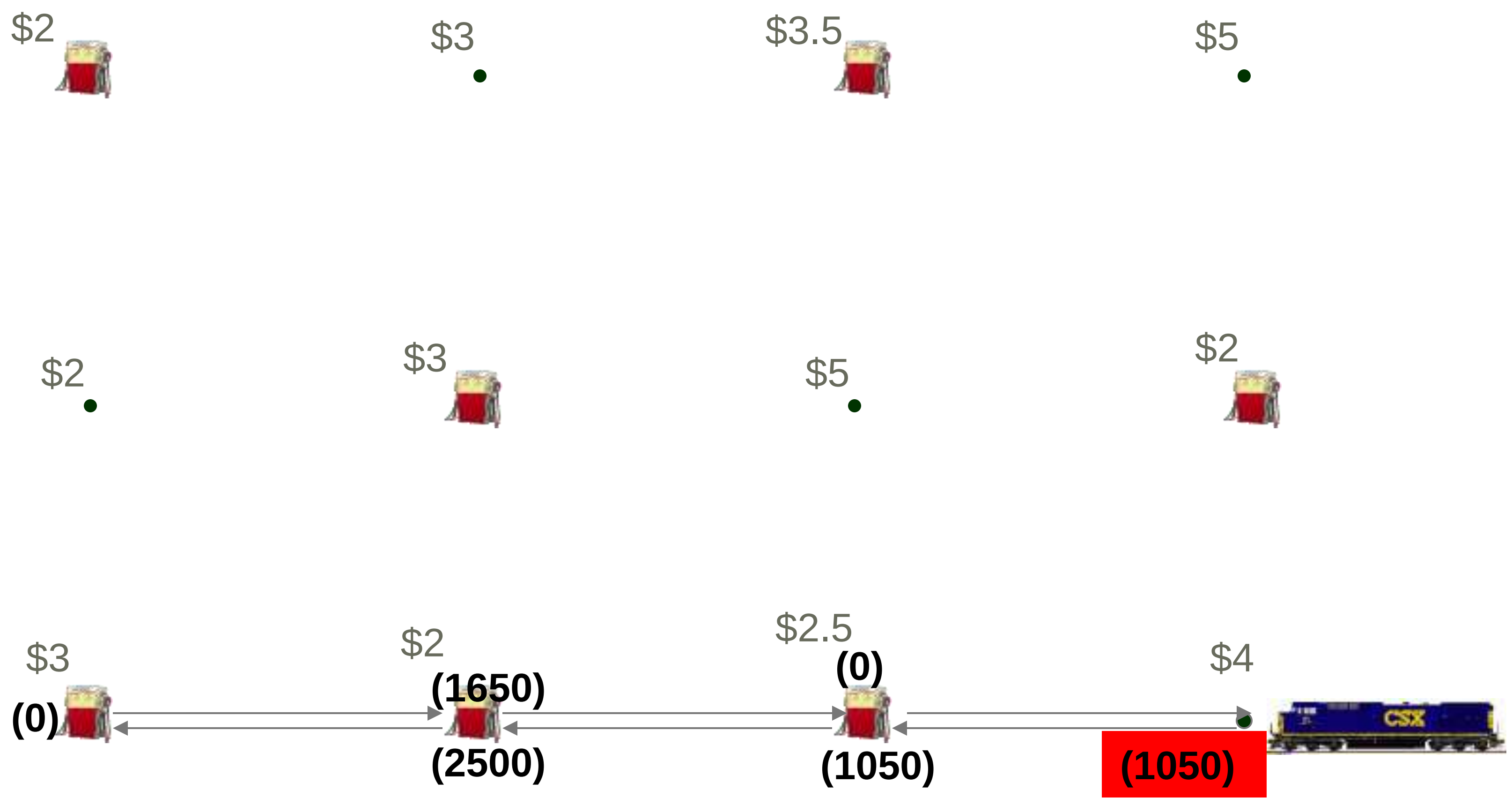


Optimal Fueling Plan: Locomotive 1

Optimal Fueling Plan: Locomotive 2



Optimal Fueling Plan: Locomotive 8



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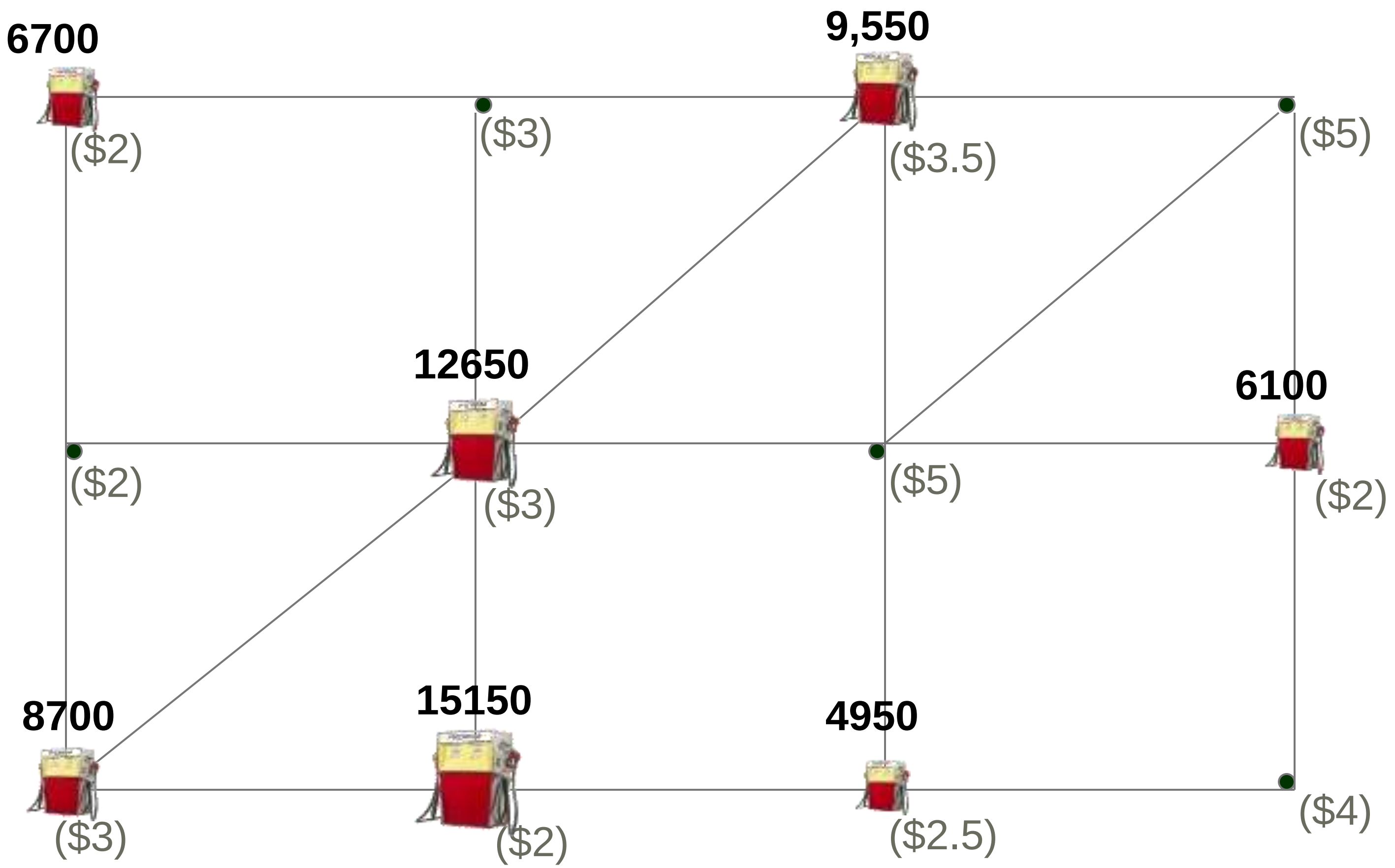
Total Fuel Consumption



Real World Case Study

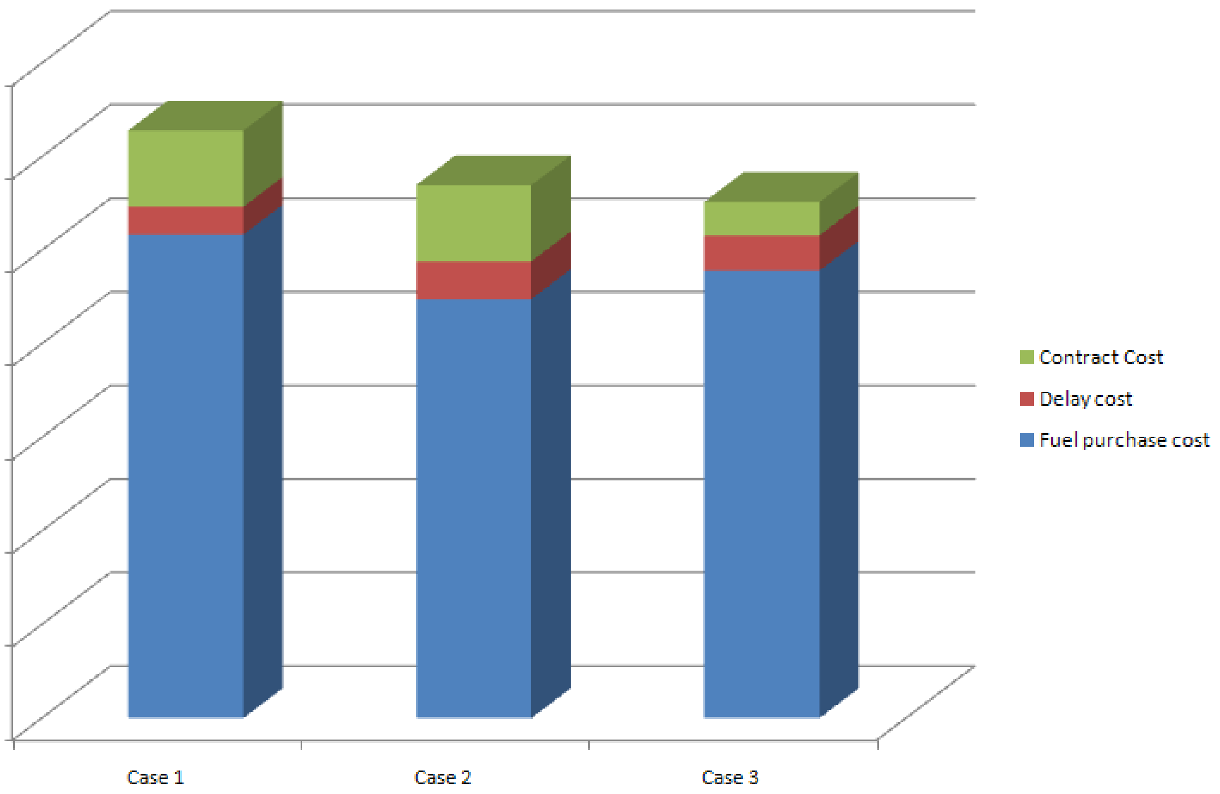
- Full railroad network of a Class-I railroad company
- 50 potential fuel stations
- Thousands of predetermined locomotive trips (per week)
- Fuel price from $1.9 - $3.0 per gallon with average $2.5 per gallon
- Tank capacity 3,000 - 5,000 gallons
- Consumption rate 3 - 4 gallons per mile
- Contracting cost of fuel stations $1 - $2 billion per year

Real World Case Study

- Algorithm converges after 500 iterations in 1200 CPU seconds
- The optimality gap was less than 6%
- This model can efficiently reduce the total cost of the system

Real World Case Study

- Solution 1: Bench-mark (current industry practice)
- Solution 2: Optimal fueling schedule using all current stations
- Solution 3: Global optimum (using an optimal subset of stations)



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Conclusion

- A Mixed Integer Programming (MIP) model
  - Integrates fuel schedule problem and station (location) selection problem
  - Considers fuel cost, delay, and fuel station contracting costs
- LR and other heuristic methods are developed for large-scale problem with good computational performance
- We developed a network representation and shortest path method for solving scheduling sub-problems
- This problem was later used as a competition problem at INFORMS Railway Applications Section (RAS)

Thank you.

nourbak1@illinois.edu
yfouyang@illinois.edu