A Hybrid Logistic Regression/Neural Network Model for the Prediction of Broken Rails

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Abstract

Broken rails are the leading cause of major derailments on North American railroads, including the most frequent cause of hazardous materials releases. Railroads in the US average 126 mainline broken rail derailments per year with an average track and equipment cost of $275,000 per incident. More importantly, the number of mainline broken-rail-caused derailments has increased from 98 in 1996, to 139 in 2005; therefore, efforts to reduce their occurrence are increasingly important. The purpose of this study was to examine the factors that influence the occurrence of a broken rail to improve the quantitative understanding of how they contribute to the likelihood of such an event. The objective was to develop an accurate, predictive tool that will enable railroads to identify locations with a high probability of broken rail occurrence. The factors that were considered included rail characteristics, infrastructure data, maintenance activity, operational information, and rail testing results. To analyze the broken rail factors two modeling techniques were applied, one using statistical regression and the other employing an artificial neural network (ANN). Both approaches have relative strengths and weaknesses, and for that reason, a hybrid model was also developed. The results of this study will enable railroads to more effectively allocate resources to prevent or mitigate the occurrence of broken rails.

Introduction

The mitigation of broken rail derailments and service failures is an increasingly important topic for US freight railroads. The number of broken rail occurrences has increased in recent years and may be due to several factors, including an increase in traffic levels and heavier axle loads. Typically, broken rails are caused by the undetected growth of either internal or surface defects on the rail. The prediction of fracture growth within a rail once a defect is detected has been examined previously [1,4,8]. However, the majority of broken rail events occur at locations where a defect has not been detected. This is due to both the rapid growth of defects under load as well as the high cost of defect detection techniques. Additionally, previous work on this topic has been conducted to examine factors that lead to a broken rail event [3,7]. The factors previously evaluated include rail and traffic characteristics. The purpose of this study was to examine and quantify the factors that may influence the likelihood of a broken rail event. The factors that were considered include rail characteristics, infrastructure data, maintenance activity, operational information, and rail testing results.

Two modeling techniques were applied to analyze the possible factors that lead to broken rail events; one uses statistical regression and the other employs an artificial neural network (ANN). Both of these modeling techniques have been used extensively in engineering applications for the purpose of failure classification and prediction. Additionally, a hybrid model combining both statistical regression and neural network techniques was developed. Previous work has shown that hybrid ANN/logistical regression models outperform purely statistical approaches in other fields, but this approach has not previously been applied to the prediction of broken rails [12].

The objective of this analysis was to develop a predictive tool that will enable railroads to accurately identify locations at high risk for broken rail occurrence so they can take the proper preventive measures. Broken rail events are often addressed using a reactive approach; the results of this study will enable railroads to develop a more preventive approach that will provide both safety and economic benefits.

Service Failure Data

In order to develop a predictive model, it is desirable to consider as many factors as possible that might affect the occurrence of broken rails. A previous study conducted an in-depth analysis of possible track and traffic factors [2] and in our study we considered these, as well as an additional group of factors. From the standpoint of rail maintenance planning it is just as important to determine which factors are correlated with broken rails, as it is to determine which are not. Therefore our analysis included a wide-range of possible variables where data was available. This included track and rail characteristics such as rail age, rail curvature, track speed, grade, and rail weight. Also, changes in track modulus from the presence of infrastructure such as bridges and turnouts have a potential effect on rail defect growth and therefore were examined. Additionally, maintenance activities are included that can reduce the likelihood of a broken rail occurrence, such as rail grinding and tie replacement. Finally, track geometry and ultrasonic testing for rail defects are used by railroads to assess the condition of track and therefore the results of these tests are included as they may provide predictive information about broken rail occurrence.

A major North American railway provided relevant information regarding the location of broken rail service failures. A service failure is defined as an incident where a track was taken out of service and trains could not proceed due to a broken rail or rail flaw. A service failure does not include incidents where trains are halted due to a rail that is found to be badly worn or damaged. Additionally, a service failure implies that the broken rail was detected in any number of ways (signal system, track inspector, train crews, etc.) before a train was able to proceed to the location of the broken rail. Whereas, a broken rail derailment is defined as a broken rail location that is undetected and causes a train to derail. Previous work has shown that a significant statistical relationship exists between the likelihood of broken rail derailments and the likelihood of broken rail service failures [2].

A database was developed from approximately 24,000 route miles of mainline trackage for a major North American railroad covering the four-year period, 2003-2006. The data available included specific locations for all service failures occurring across the network. The railroad experienced 12,685 service failures, as defined previously, during the four-year period. Additionally, rail characteristics, infrastructure data, maintenance activity, operational information, and track testing results were linked to each of these service failures. Specifically, the following 28 variables were included in the analysis:

- Rail weight
- Rail type (bolted or welded)
- Age of rail
- Maximum allowable speed
- Annual number of trains
- Annual number of tons
- Accumulated tons on rail
- Annual number of cars
- Average tons per car
- Average dynamic tons per car
- Annual number of wheel passes
- Occurrence of a internal defect
- Occurrence of a geometric defect
- Severity of a geometric defect

- Degree of curvature
- Length of curve
- Degree of superelevation in curve
- Percent rise of grade
- Length of grade
- Recent tie replacement or tie work
- Presence of a bridge
- Presence of a culvert
- Presence of a tunnel
- Presence of a diamond
- Presence of a turnout
- Presence of a grade crossing
- Curve rail grinding activity
- Out-of-face rail grinding activity

The objective of this analysis was to determine track segments that have a high likelihood of experiencing a broken rail event. The railroad's network was divided into 0.01-mile segments (or approximately 53 feet) and the location of each service failure recorded. The initial dataset

was comprised of the 12,685 0.01-mile track segments that experienced a service failure during the study period. For the case of modeling rare events it is common to sample all of the rare events and compare these with a similar sized sample of instances where the event did not occur [5]. Therefore an additional 12,685 0.01-mile segments that did not experience a service failure during the four-year period were randomly selected from the railroad's network. Additionally the non-failure locations were given a random date within the four-year time period for use in evaluating certain variables, such as the recent occurrence of an internal defect. Therefore, the dataset used in the remainder of this analysis included 25,370 total segment locations, each with a particular date, from the railroad's network.

Previous Service Failure Classification Model

The most relevant work completed on this topic was a study that was conducted with the purpose of predicting service failures based on relevant track and traffic data [3]. The outcome of this study was a multivariate statistical model which was able to quantify the probability of a service failure at any particular location based on a number of track and traffic related variables. This was a statistical model developed based on logistic regression techniques using available service failure data. The model's classification equation that was developed is as follows:

$$P_{SF2} = \frac{e^u}{(1+e^u)} \tag{1}$$

$$U = Z + .059A + .025AC - .00008A^2C^2 + 5.101\tfrac{T}{S}$$
$$+ 217.9\tfrac{W}{S} - 3861.6\tfrac{W^2}{S^2} + .897(2N-1) - 1.108\tfrac{P}{S} \tag{2}$$

where,

P_{SF2} = probability that a service failure occurred at a particular location during a two-year period
Z = -4.569, model specific constant
A = rail age (in years)
C = curvature of track (in degrees)
T = annual traffic (in million gross tons)
S = rail weight (in pounds per yard)
W = annual number of wheel passes (in millions)
P = dynamic wheel load (in tons)
N = presence of turnout (1 if present, 0 otherwise)
L = weight of car (in tons)
V = track speed (in miles per hour)

An optimal probability threshold for Equation (1) was determined to be 0.5 to classify each location. The data included used a total of 1,903 service failures from a two-year time period from May 1998 to May 2000. This model was found to classify locations correctly with 87.4% accuracy when using a dataset that was composed of half failures and half non-failures. This model was not tested against any "unseen", or validation, data at the time it was developed. Therefore, the next step was to test the previous model against the current service failure data.

The most current two years of data along with the corresponding parameters was selected. During the time period of 2005 to 2006, the railroad experienced 6,613 service failures. These service failures, as well as 6,613 random non-failure locations, were entered into the above model in Equations (1) (2). Only the parameters that were included in the previous model were evaluated for this analysis. Again, using a probability threshold of 0.5 it was determined that the previous model classified 7,247 of the 13,226 cases correctly (54.8%). However, the new optimal probability threshold was found to be 0.1 with an accuracy of 57.2%, thus minimizing the number of false negatives. Table 1 shows a summary of these results. It is clear that the

previous model has limited predictive power for current service failure locations; therefore a new classification model was developed.

Probability Threshold	Correct Predictions	Accuracy	False Positives	False Negatives
0.1	7566	57.2%	29.8%	13.0%
0.2	7500	56.7%	24.0%	19.3%
0.3	7402	56.0%	19.7%	24.3%
0.4	7338	55.5%	16.8%	27.7%
0.5	7247	54.8%	14.0%	31.2%
0.6	7070	53.5%	11.1%	35.4%
0.7	6931	52.4%	8.0%	39.6%
0.8	6807	51.5%	5.1%	43.5%
0.9	6663	50.4%	2.1%	47.5%

Table 1: Results of testing previous service failure model with current data (13,226 cases)

Statistical Classification Model

The first new classification model that was developed to predict service failure locations used the same logistic regression technique as the previous work. However, unlike the previous work, more possible factors influencing crack growth were included to develop the model, such as infrastructure data, maintenance activities, and track testing results. The logistic regression technique is a discrete choice model that produces an output value of the probability of failure. The determined probabilities are then used to classify each case as either failure or non-failure. A statistical regression equation is calculated based on any and all significant input parameters to determine the probability of failure.

A step-wise selection method is used to determine which parameters are significant for prediction. The step-wise method first adds the most significant variable to the model. In the second step, the step-wise method adds the variable that, when evaluated in conjunction with the first variable, produces the greatest improvement. This process continues until no additional variables meet a 0.05 significance level for entry into the model. Additionally, at each step the model will remove any previously entered variable if its removal does not decrease the predictive power. This model was developed using the complete four-year time period of service failures and non-failure locations as well as all available rail, infrastructure, maintenance activity, operational, and track testing data. The developed logistic regression equation was:

$$P_{SF2} = \frac{e^u}{(1+e^u)} \tag{3}$$

$$U = Z - 0.0486S - 1.32R - .00362A - .0447V + .0000520F + .0313T - 0.000150H$$
$$- .0542L + .0487P - .487W + 1.60I + .689G + .0501C + .0637E - 1.42 \times 10^{-6}J \tag{4}$$
$$- .107M + 1.62B + .157K + 3.01D + .980N + .361X + .778O + .589Q$$

where,

P_{SF2} = probability that a service failure occurred at a particular location during a four-year period
Z = 6.318, model specific constant
S = rail weight (in pounds per yard)
R = rail type (1 if welded, 0 if bolted)
A = rail age (in years)
V = track speed (in miles per hour)
F = annual number of trains (total, both directions)
T = annual traffic (in million gross tons)

H = accumulated tons on rail since rail was installed (in millions)
L = weight of car (in tons)
P = dynamic wheel load (in tons)
W = annual number of wheel passes (in millions)
I = presence of a ultrasonic defect in the last three years (1 if present, 0 otherwise)
G = presence of a geometric defect in the last three years (1 if present, 0 otherwise)
C = curvature of track (in degrees)
E = superelevation of track (in inches)
J = length of grade (in feet)
M = recent tie replacement or tie work in last three years (1 if present, 0 otherwise)
B = presence of a bridge within 200 feet of track segment (1 if present, 0 otherwise)
K = presence of a culvert 200 feet of track segment (1 if present, 0 otherwise)
D = presence of a diamond within 200 feet of track segment (1 if present, 0 otherwise)
N = presence of a switch or turnout within 200 feet of track segment (1 if present, 0 otherwise)
X = presence of a grade crossing within 200 feet of track segment (1 if present, 0 otherwise)
O = out-of-face rail grinding activity performed on track segment (1 if present, 0 otherwise)
Q = curve rail grinding activity performed on track segment (1 if present, 0 otherwise)

The new model contains more variables which contribute to the likelihood of a broken rail as compared to the previous model. This is due to the fact that the previous model examined only 11 possible prediction factors; whereas the new model evaluated 28 different factors. Additionally, the previous model allowed for two term interaction; whereas this model was limited to single variable interaction due to available computing power. Advanced models, such as artificial neural networks, are also developed and evaluated in the next section which incorporate all possibilities of multiple variable interaction. The accuracy of prediction for this model was 66.3% at an optimal threshold of probability of 0.50. A summary of the predictive power of the model for varying levels of probability is shown in Table 2.

Probability Threshold	Correct Predictions	Accuracy	False Positives	False Negatives
0.1	12721	50.1%	49.8%	0.0%
0.2	13298	52.4%	47.0%	0.5%
0.3	14711	58.0%	39.0%	3.0%
0.4	16545	65.2%	23.3%	11.5%
0.5	16822	66.3%	12.8%	20.9%
0.6	16485	65.0%	7.2%	27.9%
0.7	15825	62.4%	3.8%	33.8%
0.8	14787	58.3%	1.5%	40.2%
0.9	13402	52.8%	0.3%	46.9%

Table 2: Results of new service failure model with current data (25,370 cases)

The new model increased the accuracy of classification for the most recent service failure data by 11.5% over the previous model. Therefore, the development of a new model with the inclusion of additional possible factors leading to service failures increased the model's predictive ability. In particular the first five terms, or most significant factors, entered into the model were: 1) presence of an ultrasonic defect, 2) rail type, 3) annual MGTs, 4) average tons per car, and 5) presence of a geometric defect. Of these five terms neither the presence of ultrasonic or geometric defects had been included in the previous model. Additionally, the presence of infrastructure, such as bridges, grade crossings, and diamonds, were not previously evaluated, yet have influence in the new statistical model.

Artificial Neural Network Classification Model

The second new classification model that was developed is an artificial neural network (ANN). Artificial neural networks have been used in various studies of event prediction, in particular classifying future events into either failure or non-failure. The use of ANNs has been shown to

be a more powerful alternative to logistic regression models in various applications. A previous study conducted, developed a neural network model for predicting bankruptcy failure of firms based on limited financial data. The study concluded that the neural network developed showed a higher level of prediction accuracy and robustness over previous modeling techniques [6].

ANNs are a computational tool that have the ability to "learn" mathematical relationships between a series of input variables and their respective output value. ANNs are an interconnected group of neurons that have the ability to change its structure based on information that flows through the network. The main parts of an ANN are the inputs, neurons, hidden layer, output, and node connections. The input layer of the ANN is comprised of the various input parameters into our service failure classification problem. The neurons in the hidden layer are mathematical equations relating the connected nodes. The node connections are weighted connections between various nodes. The creation of nodes, equations, and connection weights are determined by computer learning techniques. Finally, the output node is connected to the hidden layer and the computer produces an output value. Using ANNs, only a classification of failure or non-failure is determined.

With the development of ANNs as an alternative to logistic regression for prediction studies, research has been conducted to explore the differences in the two techniques [10]. There are many advantages and disadvantages to the use of artificial neural networks as a classification tool. The most important advantage of neural networks is the ability to detect complex non-linear relationships between input and output variables. The hidden layers and neurons as well as the node connections allow ANNs to have non-linear relationships between the input values, nodes, and output value. Another advantage is that ANNs can detect all possible interactions between input variables. The previous statistical model that was developed only evaluated single term interactions. The inherent design of a neural net by computer software evaluates and considers every possible interaction and power. Finally, ANNs have the advantage that they can be developed and evaluated using different learning techniques and different objective functions. This allows the creator of the neural network the ability to try different techniques to determine the optimal classification model.

An artificial neural network model was developed for classifying track segment locations as either failure or non-failure. The same service failure data, as well as non-failure locations, that were used to develop the logistic regression model were used again for construction of the neural network. However, only 15,999 randomly selected cases could be analyzed due to limitations of the software. The software used for construction of the neural network was "NeuroShell Classifier" developed by Ward Systems Group, Inc [11]. Using the data from the four-year study period, a neural network was developed using back-propagation. The ANN classified 67.7% of the cases correctly, an improvement of 1.4% over the previous logistic regression model. This model classified 12.7% false positives and 19.6% false negatives (Table 3).

ANN Model Type	Accuracy	False Pos.	False Neg.	Computation Time (sec)	Hidden Neurons
ANN	67.7%	12.7%	19.6%	38	76
Logit-ANN	67.5%	13.0%	19.5%	34	71
Plogit-ANN	67.9%	12.8%	19.2%	39	77

Table 3: Summary of results for three developed artificial neural network classification models

The number of hidden neurons constructed for this neural network was 76. An ANN is constructed by computer software by adding hidden neurons one by one until the optimal network is determined. An optimal network and the optimal number of neurons are determined between a balance of model accuracy and generalization. A network that generalizes well is one that is able to provide good results for data not used to train the neural net. In other words,

the software attempts to produce a neural network that is both accurate and robust for unseen cases.

Hybrid Classification Models

The final two classification models that were developed are logistic regression / ANN hybrid models. One of the disadvantages of ANNs when compared to logistic regression models is that ANNs frequently have difficulty analyzing systems that have a high number of parameters due to the large amount of time taken to learn the system as well as possibly over-fitting the model during the initial learning phase. Previous work has shown that the development and use of hybrid models between ANNs and logistic regression techniques improves the classification performance when compared to previous techniques [9,12].

Two common types of hybrid models for ANNs and logistic regression have been developed in previous studies. The first hybrid model type is constructed by the use of logistic regression for the pre-selection of variables based on their significance for the model. Only the factors included in the initial logistic regression model are then considered into the development of the ANN. This model type is defined as a Logit-ANN model. The second common hybrid model type is constructed by using the logistic regression model to calculate the probability of failure and then adding that value as an additional input variable into the ANN. This type of model is defined as a Plogit-ANN model. The two hybrid models produce advantages over the previous techniques. The hybrid models decrease initial learning time for the ANN, meaning that more cases can be devoted to optimizing the network instead of learning the network. Additionally, the hybrid models condense information for very large problems that lead to a decrease in learning time which can be a significant factor for very large datasets.

Both hybrid models were developed using the previous logistic regression model that was developed in this analysis. The first hybrid, Logit-ANN, is constructed by first pre-selecting the input variables using the logistic regression technique. 23 of the 28 input factors included in this analysis were considered to be factors that influenced the occurrence of a service failure. Only these 23 factors were then used to construct the ANN. The Logit-ANN hybrid model was determined to be 67.5% accurate. The second hybrid model, Plogit-ANN, is constructed by using the logistic regression model to calculate the probability of failure for each case. This value is then added to the input variables for construction of the ANN. The Plogit-ANN hybrid model was found to be the most accurate model with a 67.9% correct classification rate (Table 3).

The Plogit-ANN hybrid model performed better than any other developed classification model, including the logistic regression technique. It is important to note that the Logit-ANN hybrid model preformed worse than the stand-alone ANN, meaning the simple ANN model considered additional variables significant that the previous statistical model did not. Finally, the three artificial neural network models only had about a 1 to 2% higher level of prediction over the traditional statistical method.

Prospective Service Failure Prediction Model

As stated previously, the use of artificial neural network models have some inherent disadvantages when compared to traditionally used techniques such as logistic regression. One disadvantage is that ANNs do not give a value for the probability of an outcome. In this study the ANN only produces an output of either failure or non-failure. This is a significant disadvantage when compared to the logistic regression model developed that can estimate the probability of failure for each case using the developed equation. Finally, another disadvantage of neural networks is that they are "black box" models, meaning that the inner workings of a neural network cannot be easily reproduced. The ability to explain what the model is doing and why is thus limited. Additionally, a user cannot evaluate the possible relationships between input variables for an ANN model. Overall the neural network models evaluated did not greatly

increase the classification accuracy of service failure prediction. Therefore, the final prospective service failure model is based on the logistic regression model developed.

The classification models decribed above are retrospective models created using a dataset in which half the records had a service failure and half did not. A transformation is needed to develop a prospective model that can be used to predict the location of service failures. Previous work has shown how the transformation can be done using a logistic regression model [3,5]. The transformation was completed with adjustment of the model specific constant, Z, to reflect the average service failure probability across the entire system. During the four-year period, there were 12,685 service failures on the railroad that were classified according to which of the 0.01-mile segments they occurred on. The railroad maintains approximately 24,000 main line route miles of track and thus has a total of approximately 2.4 million 0.01-mile-long segments. Therefore, the probability that a service failure will occur on any particular segment during a similar four-year period is then 0.00529. This probability was converted into a new model-specific constant using the log-odds operator:

$$Z^* = Z + \ln\left(\frac{P_{SF_{AVG}}}{1 - P_{SF_{AVG}}}\right) = 6.318 + \ln\left(\frac{.00529}{1 - .00529}\right) = 1.081 \tag{5}$$

$$U = 1.081 - 0.0486S - 1.32R - .00362A - .0447V + .0000520F + .0313T - 0.000150H$$
$$- .0542L + .0487P - .487W + 1.60I + .689G + .0501C + .0637E - 1.42\text{x}10^{-6}J \tag{6}$$
$$- .107M + 1.62B + .157K + 3.01D + .980N + .361X + .778O + .589Q$$

$$P_{SF2} = \frac{e^u}{(1 + e^u)} \tag{7}$$

where,

P_{SF2} = probability that a service failure occurred at a particular location during a four-year period
Z = 6.318, model specific constant
Z^* = 1.081, adjusted model constant

The previous model specific constant, Z, is replaced in the logistic regression equation by the adjusted constant, Z^*, as shown in Equation (6). Therefore, Equation (7) represents the prospective service failure model, with updated value for U, for the prediction of service failures during a four-year period. This equation can be used to determine specific locations with a high likelihood of a service failure, as well as used to determine the overall service failure rate for a specific line. The annual service failure rate can be determined by calculating the four-year service failure rate and dividing by 4, assuming a time-linear distribution of service failures.

Conclusions

Four service failure classification models were developed to assist in the prediction of broken rail events. Service failure data from the railroad's entire network were used for a four-year time period. A previous service failure classification model, using logistic regression, that incorporated only track and traffic characteristics, was evaluated and determined to have limited predictive ability for current service failure data. A new logistic regression model was developed that included additional factors such as infrastructure data, maintenance activities, and rail testing results. The logistic regression model was determined to be 66.3% accurate for classifying track segments. An artificial neural network model was also developed to classify cases as either failure or non-failure. Additionally, two logistic regression ANN hybrid classification models were developed based on the current service failure data. Each of the three advanced models performed slightly better than the traditional logistic regression technique. Finally, the logistic regression model was transformed into a prospective prediction model based on the overall probability of service failures.

The models developed in this study may assist railroads to more effectively allocate resources to prevent or mitigate the occurrence of broken-rail events. The models presented here can be implemented in two different ways involving maintenance planning. The first approach is for short-term maintenance assistance, such as determining specific track segments to monitor closely or repair. The second approach is for long term maintenance planning and scheduling. The two most common mitigation techniques for broken rails are rail grinding and rail replacement. Both of these activities require long lead times for planning and have high associated costs. Railroads can use the models presented here to assist them in planning the location of these maintenance activities more effectively.

Future Work

This analysis presented areas of future exploration on the topic of predicting broken rails. The previous analysis included many of the available variables that are possibly responsible for the growth of defects and the occurrence of broken rails. However, some additional factors that could be considered include climatic data for track locations and track inspection frequency. Climate effects, especially in areas of continuously welded rail where the rail is in high tension, may have an effect on the growth of rail defects as well as the occurrence of broken rails. Also, evaluating the track inspection frequency may be found to have a correlation with the likelihood of a broken rail event. Finally, this analysis presented classification models based on statistical and neural network techniques. Future work may be possible to examine other classification models for the prediction of service failures.

Acknowledgements

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