

Superelevation Optimization for Mixed Freight and Higher-Speed Passenger Trains

Authors

C. Tyler Dick, P.E.
Senior Research Engineer
Rail Transportation and Engineering Center -
RailTEC
University of Illinois at Urbana-Champaign
205 N. Mathews Ave.
Urbana, Illinois 61801
Phone: (217) 300-2166
Email: ctdick@illinois.edu

Luv Sehgal
Graduate Research Assistant
Rail Transportation and Engineering Center -
RailTEC
University of Illinois at Urbana-Champaign
205 N. Mathews Ave.
Urbana, Illinois 61801
Phone: (217) 904-5962
Email: lsehgal2@illinois.edu

Conrad J. Ruppert, Jr.
Senior Research Engineer
Rail Transportation and Engineering Center -
RailTEC
University of Illinois at Urbana-Champaign
205 N. Mathews Ave.
Urbana, Illinois 61801
Phone: (217) 300-2132
Email: ruppertc@illinois.edu

Shashad Gujran
Graduate Research Assistant
Rail Transportation and Engineering Center -
RailTEC
University of Illinois at Urbana-Champaign
205 N. Mathews Ave.
Urbana, Illinois 61801
Phone: (217) 721-7874
Email: gujran2@illinois.edu

Number of Words = 5,426 + 8 figures/tables @ 250 each = 7,426

ABSTRACT

Selecting a superelevation design on curves is a complex process on mixed-use corridors where trains operate over a range of speeds. Since a given curve has one unique equilibrium speed, a range of curving speeds results in both underbalance and overbalance operating conditions which impact track safety, maintenance, ride comfort and network capacity. As the demand for higher-speed passenger trains increases, the disparity between freight and passenger operations also increases. Higher-speed passenger trains use improved suspensions, lowered center of gravity and tilting technology allowing them to operate at higher underbalance through curves. At the same time, the operation of heavy-axle-load freight trains at lower speeds results in overbalance conditions. These conditions can be exacerbated on grades where in-train forces can increase or decrease required superelevation. On mixed-use rail corridors, the result is a bi-modal distribution of train speeds on curves. Differences in train speeds can be magnified by rail traffic congestion or local site conditions that cause certain types of trains to travel well below the maximum allowable track speed. Selection of a single optimal value for curve superelevation under these conditions is not straightforward. To aid in making informed curve superelevation decisions on mixed-use rail corridors, this paper aims to synthesize industry knowledge and tools used to evaluate superelevation for different curve scenarios into a superelevation optimization framework. The framework is presented as both a graphical approach and a mathematical model that can be applied to select superelevation design parameters for multiple curves on a rail corridor.

INTRODUCTION

The basic physics of establishing balanced superelevation on curves for a single type of rail vehicle operating at a constant speed is well-documented in railway texts and design literature (1). However, the situation becomes more complex when a curve is subject to trains operating over a range of speeds. When operating at speeds above and below the balancing speed for the superelevation on the curve, rail vehicles may impart additional vertical and lateral loads into the track infrastructure with safety, maintenance and ride comfort implications. Although this is not a new phenomenon, the range of maximum freight and passenger train speeds on a railway curve in North America, has historically been relatively narrow. Many conventional passenger trains only operate 10 to 15 mph faster than priority freight trains on curves due to limits on cant deficiency, vehicle suspensions and other railroad design and operating parameters.

Recent trends towards higher-speed passenger rail service have increased the disparity in current and planned maximum train operating speeds on many rail corridors. The higher-speed passenger trains often use improved suspensions, lowered center of gravity and tilting technology to operate at higher cant deficiencies and faster speeds on curves. At the same time, efforts to seek even greater economies of scale have led to the operation of heavy-axle-load freight trains of increasing length and total weight. These long, heavy freight trains often, and on grades in particular, operate below maximum allowable freight train speeds. On shared or mixed-use corridors, the result of these operational trends can be a bimodal distribution of train speeds on curves. Selection of a single optimal value for curve superelevation under these conditions is not straightforward.

To aid in making informed curve superelevation decisions on mixed-use rail corridors, this paper first reviews the basic physics of superelevation design. Next, the paper synthesizes knowledge on curving at various levels of unbalanced superelevation and other factors influencing selection of superelevation. Finally, this information is used to develop a framework and mathematical model for optimizing superelevation on curves subject to disparate train speeds.

PHYSICS OF SUPERELEVATION

Basic Quasi-Static Curving

The basic physics of superelevation are well understood and presented in numerous textbooks and other references, with Elkins (2) as one example. Moody (3) presents an excellent summary of the basic physics and equations governing superelevation and unbalance on railway curves.

To negotiate a circular curve at constant speed, a rail vehicle must be subject to a centripetal force acting inwards towards the center of the curve. The magnitude of this force is a function of the sharpness (radius or degree) of the curve, speed of the rail vehicle and the mass (weight) of the rail vehicle. The centripetal (or curving) force is created by a combination of the rail vehicle wheels reacting against the rails and superelevation (or banking) on the curve. Superelevation raises the outside rail on the curve by rotating the track structure about the inside rail. The force of gravity acts down the resulting incline to pull a rail vehicle towards the center of the curve, helping to create the necessary centripetal curving force.

For a given radius (degree) curve, the centripetal force required by a rail vehicle operating at a given speed will be exactly equal to the force of gravity acting laterally down track inclined at a certain superelevation (measured by the difference in elevation between the low and high rail on the curve). Under this combination of speed and superelevation, the rail vehicle is in equilibrium. The corresponding amount of superelevation is termed the *equilibrium superelevation* (or *balanced superelevation*) for that combination of speed and degree of curve. The equilibrium superelevation (E) can be calculated by Equation 1 where V is the speed of the rail vehicle, R is the radius of the curve and g is the acceleration due to gravity (1).

$$E = \frac{4.9V^2}{gR} = \text{Equilibrium Superelevation} \quad (1)$$

By substituting the value of g , the arc definition of degree of curve for R and making adjustments for units, Equation 1 can be transformed into Equations 2 and 3 relating degree of curve, maximum track speed and superelevation (1).

$$V_{\max} = \sqrt{\frac{E_a + E_u}{0.0007D}} \quad (2)$$

$$E_a = (0.0007DV_{\max}^2) - E_u \quad (3)$$

In Equation 2 and 3, V_{\max} is the maximum operating speed in miles per hour. The parameter D is the degree of curve as defined by a 100-foot chord. E_a and E_u are defined as follows:

- *Actual Superelevation* (E_a) is the actual difference in elevation between the high and low rails on a curved segment of track expressed in inches. This is the amount of superelevation that is actually installed in the track.
- *Unbalanced Superelevation (or Cant Deficiency)* (E_u) is the difference between the actual superelevation and the superelevation required to create equilibrium conditions for the considered combination of speed and degree of curve. Cant deficiency exists when a rail vehicle travels through a curve at a speed greater than the equilibrium speed of that curve (given the actual superelevation and degree of the curve). Expressed in inches, cant deficiency is also referred to as “underbalance”.

For a rail line with uniform traffic, superelevation on each curve can be designed for the maximum allowable track speed using Equation 3. For reasons discussed later in the paper, the design speed is usually set such that trains operate with one or more inches of cant deficiency. Thus the actual installed superelevation is usually one or more inches less than the superelevation required to obtain equilibrium conditions for the design speed.

Consideration of In-Train Forces on Ascending and Descending Grades

One shortcoming of this classic approach is that it only looks at quasi-static forces acting on a single rail vehicle. If the rail vehicle is considered as part of a train, in-train buff and draft coupler forces become a factor in superelevation design.

When negotiating a curve, the couplers at either end of a railcar move to an angle relative to the longitudinal centerline of the railcar. When in-train buff and draft forces act in line with this angled coupler position, they create both a longitudinal force along the rail centerline and a lateral force. Typically, the coupler angle is very small and the lateral force is negligible. However, for heavy-haul freight trains negotiating curves on steep grades, the lateral component of the in-train forces can become large enough to aid or work against the desired curving motion (4-6). When this additional force is considered in the classic quasi-static condition, the amount of superelevation required to obtain equilibrium for a certain speed on a particular curve changes (7).

Draft Forces on Ascending Grade

On an upgrade, assuming all motive power is at the front of the train, draft (or tension) coupler forces act at an angle to the railcar centerline in both directions (Figure 1a). The lateral component of these draft forces acts to pull the railcar inwards towards the center of the curve. As shown by the vector addition in Figure 1a, the draft forces help to achieve the necessary centripetal curving force and decrease the amount of force required from superelevation. With less force required, less superelevation is necessary to balance the train speed on the curve than in the quasi-static case that neglects in-train forces.

If the speed is slow, draft forces are large and superelevation is set according to Equation 3, the combined coupler and superelevation forces may overcompensate for the speed of the train on the curve. To balance the system, additional outward lateral forces are required from reactions through the wheel/rail interface, potentially increasing wear on the low rail.

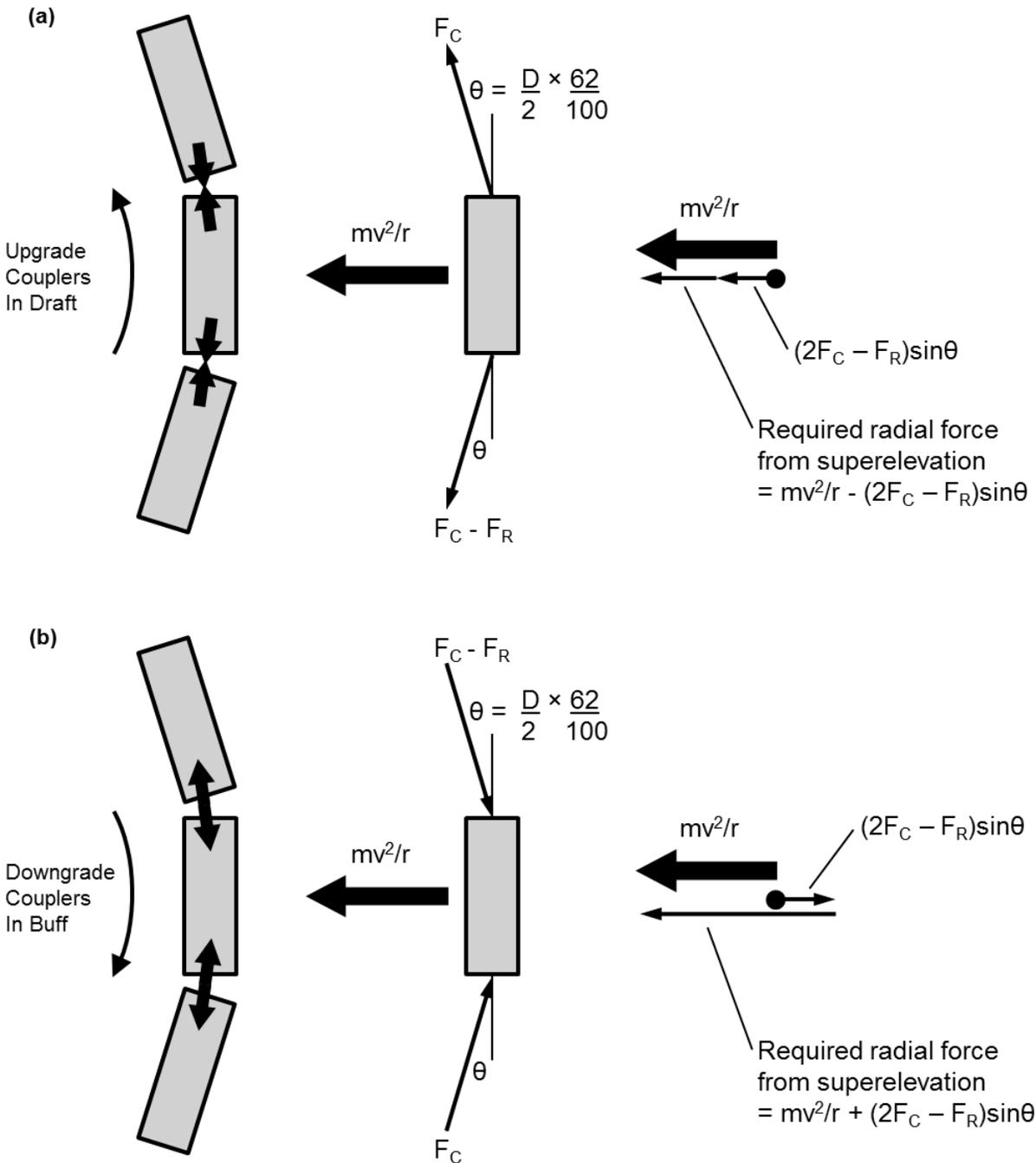


Figure 1: Influence in-train forces on required superelevation for (a) upgrade and (b) downgrade conditions at the same train speed

Buff Forces on Descending Grade

On a downgrade, under similar assumptions, draft (or compression) coupler forces act at an angle to the railcar centerline (Figure 1b). The lateral component of these buff forces acts to push the railcar outwards away from the center of the curve. As shown by the vector addition in Figure 1b, the buff forces act against the necessary centripetal curving force and increase the amount of force required from superelevation. Thus more superelevation is required to balance the train speed on the curve than the original quasi-static case where in-train forces are neglected.

The need for additional superelevation is important in this case because trains descending grades are more likely to be travelling closer to the track design speed. At these speeds, the actual superelevation may not be sufficient to adequately balance the combined coupler and centripetal curving forces. To balance the system, additional inward lateral forces are required, potentially increasing rail wear on the high rail. In extreme cases, the lateral forces may cause the high rail to roll over, leading to a derailment.

Equivalent Superelevation

Using the approach outlined in Figure 1, for a given railcar in a train, the in-train forces can be translated into a resulting lateral force component. The magnitude of the lateral force varies along the length of the train; the in-train forces are of greatest magnitude for the first railcar behind the locomotives and are of smallest magnitude for the last railcar in the train.

Using Equations 1 and 3, the lateral component of in-train forces acting on a railcar can be further transformed into the equivalent amount of superelevation required to provide equal force on a given degree curve. The value of equivalent superelevation is independent of train speed but varies with railcar position in the train (or absolute magnitude of in-train forces), grade direction (upgrade with draft forces or downgrade with buff forces) and degree of curve (Figure 2). Positive values of equivalent superelevation on ascending grades indicate where the in-train forces cause the railcar to behave in the cant excess (overbalance) condition as if extra superelevation were installed. Negative values of equivalent superelevation on descending grades indicate where the in-train forces cause the railcar to behave in the cant deficiency (underbalance) condition as if less superelevation were installed than actually is. The values in Figure 2 were calculated for a 100-car train of loaded 286,000-pound railcars. It is assumed that all locomotives are at the front of the train and the entire train is on a 1-percent ascending grade or 1-percent descending grade.

As described above, equivalent superelevation takes its extreme values for the first railcar in the train. On an 8-degree curve, depending on the direction of the grade, just over five inches of superelevation must be added or subtracted to restore the quasi-static curving conditions for the first railcar. The effect decreases as the degree of curve decreases. The final railcars in the 100-car train do not experience large enough in-train forces to alter superelevation requirements.

The form of Figure 2 suggests two complications to superelevation design on grades. The first is that within an individual train, each railcar may have its own equilibrium superelevation. This makes it nearly impossible to achieve a consistent value of cant deficiency for the entire train. The second complication is that upgrade and downgrade trains are likely to have very different equilibrium superelevation requirements. For a 100-car train negotiating a 8-degree curve on a 1-percent grade, the first railcar in a downgrade train may require 10 inches more superelevation (combined actual and unbalanced) than the first railcar in an upgrade train. It is unlikely that a single superelevation design can serve these trains equally well, even if they are operating at similar speed.

To address this difficulty, software tools have been developed to assist with design of curve superelevation on steep grades where many different types of loaded and empty trains are operating (8). Roney (9) has shown that effective placement of distributed power and optimized superelevation provide gains in the areas of train speed and lateral loads on curves in mountainous terrain. Oldknow (10) has demonstrated in his experiments on two heavy haul freight rail lines that the ascending grade does not dictate the superelevation and speed requirements; descending grades and other factors must also be considered.

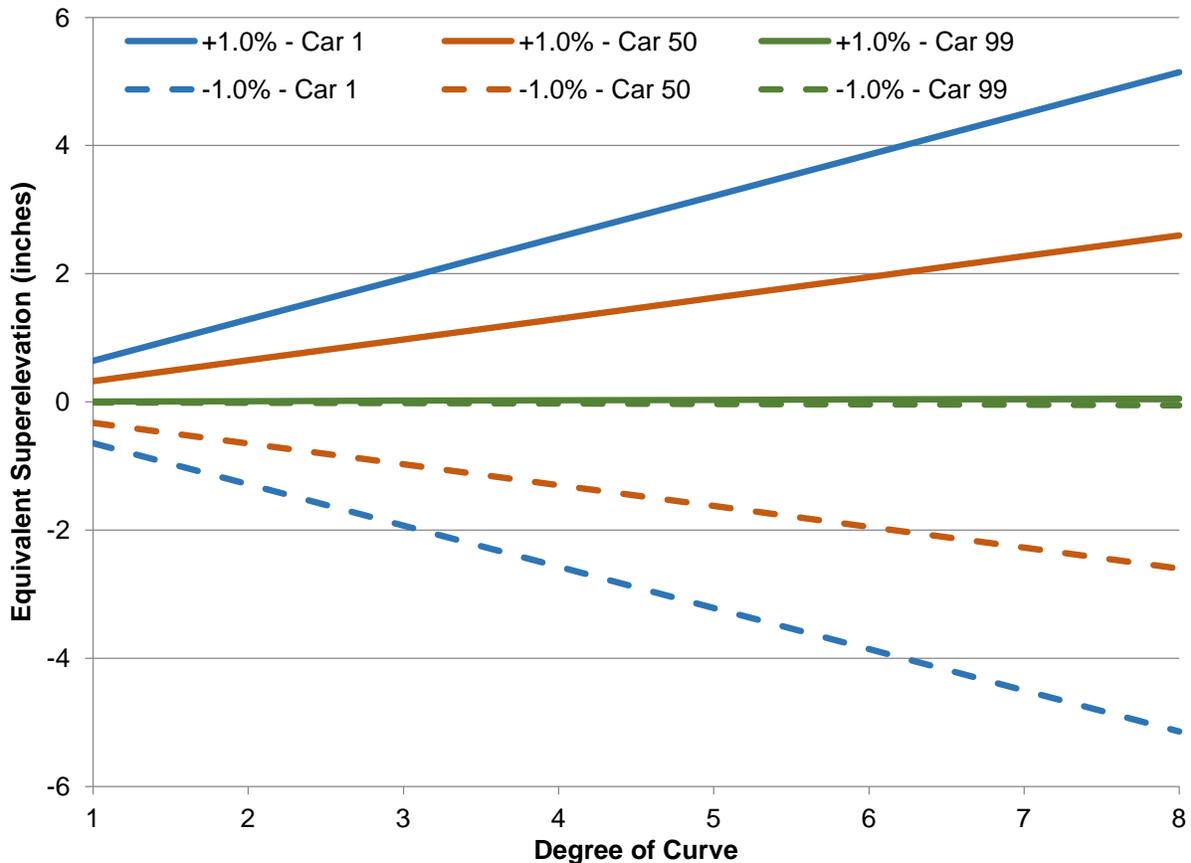


Figure 2: Equivalent superelevation due to in-train forces at different positions in a 100-car train on a 1-percent downgrade or upgrade

TRAIN SPEED FOR CALCULATION OF SUPERELEVATION

A consequence of Equation 2 is that for a given combination of degree of curve and actual superelevation, there is a single unique train speed for which equilibrium conditions exist. Any trains that operate over the route at different speeds will traverse the curve with varying amounts of cant deficiency (positive E_u) or cant excess (negative E_u). The track design engineer is faced with the task of selecting an appropriate train speed for superelevation design. It is common practice to design superelevation for the maximum allowable track speed. However, there are a number of factors that complicate this task, as described in the following sections.

Mixed-Use Corridors

When multiple train types operate on a rail corridor, several maximum train speeds may be specified in the timetable. When these maximum train speeds cover a range of values, selecting one to govern the superelevation design is non-trivial. While most freight trains operate between 40 and 60 mph, an increasing number of passenger trains are qualified by the FRA to operate at higher cant deficiencies and speeds from 70 to 110 mph on curves (11). While conventional 60 and 70 mph passenger train speeds largely overlapped (or were contiguous with) those of freight trains, on mixed-use corridors with higher-speed passenger trains, the distribution of train speeds is distinctly bi-modal (Figure 3). The majority of passenger trains are travelling much faster than the maximum freight train speed and over 60 mph can separate the most frequent freight and passenger train speeds. As will be demonstrated later in the paper, it is difficult to adequately cover this wide range of train speeds with a combination of actual superelevation and cant deficiency.

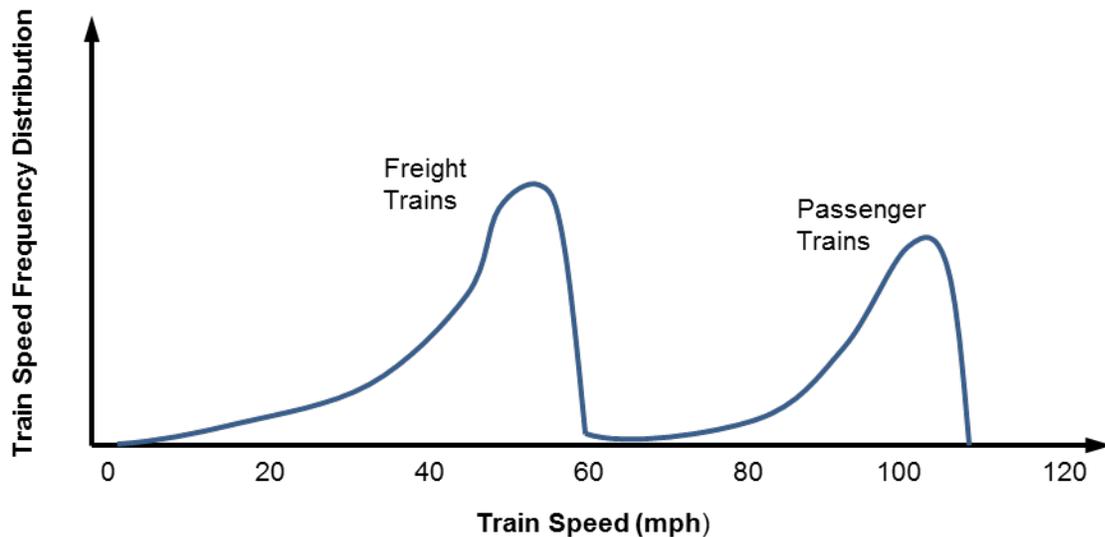


Figure 3: Example distribution of train speeds on mixed-use freight and passenger corridor

Maximum versus Actual Train Speeds

As suggested by Figure 3, even within a given train type, a range of actual train speeds below the maximum allowable speed for that train type may be observed. When this range of train speeds below maximum becomes wide, a design optimized for the maximum train speed may not adequately support the slowest trains. Depending on the exact frequency distribution of train speeds, a lower design speed may be more appropriate even if it means that certain trains operate with additional cant deficiency.

Reasons for the existence of a range of actual train speeds below the maximum include:

- Grades may substantially reduce train speeds in the ascending direction
- Certain trains may have insufficient tractive effort or horsepower to sustain maximum speed
- Individual railcars or commodities in the train consist may be subject to speed restrictions
- Train speed may be limited by signal indications
- Weather conditions may dictate slower speeds
- Operating crews may have different train-handling styles
- Overall reductions in train speed to save fuel

Site-Specific Factors

In addition to the above factors, there are certain locations where it is likely that a range of different operating speeds will be observed. Particular care should be taken when designing curve superelevation within typical train acceleration and braking distances of the following locations where trains frequently operate well below the posted maximum train speed:

- Terminals, passing sidings or interlockings where trains stop or slow down to negotiate turnouts
- Stations where express and/or local passenger trains stop
- Track sections adjacent to civil speed restrictions (including other curves)
- Spur connections to rail customers or areas where local switching takes place
- Crew change points
- Segments subject to congestion and train delay

Site-specific factors can have a more substantial effect on mixed-use corridors due to the different acceleration and braking distances of freight and passenger trains. While passenger trains may quickly accelerate after a civil speed restriction and negotiate an adjacent curve at maximum track speed, freight

trains may still be travelling at much lower speeds when they reach the same curve. Such conditions broaden the range of speeds that must be accommodated by the superelevation design for the curve.

IMPLICATIONS OF CURVING AT NON-EQUILIBRIUM SPEEDS

Distribution of Normal and Lateral Wheel Loads

The distribution of normal and lateral wheel loads changes when a train operates above or below the equilibrium speed on a superelevated curve (Figure 4). In this figure, the red arrow acting on the vehicle mass center is the net resultant centripetal force that is accelerating the vehicle around the curve. The black arrow in each scenario is the force due to gravity acting on the vehicle mass center. The dashed black lines illustrate track frame vertical and lateral components of the gravitational force. The blue arrows represent the vertical and lateral forces acting on the rail vehicle at the wheel/rail interface.

At equilibrium speed, the high and low rail vertical forces are equal in magnitude and there is no lateral force at the wheel rail interface. In an overbalance condition, where cant excess exists, the low rail vertical force is greater in magnitude than high rail vertical force. In this condition, there is also a lateral force acting outward on the rail vehicle from the low rail. In an underbalance condition, where cant deficiency exists, the high rail vertical force is greater in magnitude than that of the low rail. In addition, there is a lateral force that acts inward on the rail vehicle from the high rail. The corresponding sets of force vector additions demonstrate how the different forces combine to produce a resultant force that guides the rail vehicle in a circular path around the curve at different speeds.

Summing forces perpendicular and parallel to the track in Figure 4 and taking moments about the rail vehicle center of gravity can yield equations for the vertical (normal) wheel load on each rail and the lateral wheel/rail force. To illustrate the magnitude of the increased or decreased wheel loads under typical overbalance and underbalance conditions, these equations were solved for a 3-degree curve over a range of train speeds and corresponding values of cant deficiency (negative cant deficiency is cant excess). The wheel loads were calculated for a loaded 286,000-pound railcar with a static wheel load of 35.75 kips.

The results of the calculation (Figure 5) indicate that both the vertical and lateral wheel loads increase linearly as cant deficiency is increased. This is equivalent to a parabolic relationship to train speed. At higher-speeds, small increases in train speed result in a greater change in wheel loads than at lower speeds. At the balancing speed of 43.6 mph, the forces on the high and low rail are equivalent to the static wheel load of 35.75 kips and the lateral force is negligible. When the train speed increases to a cant deficiency of 4 inches, the wheel load on the high rail increases by over 20-percent. Similarly, at a cant excess of 4 inches, the wheel load on the low rail increases by over 20-percent.

Maintenance Considerations

Increasing vertical wheel loads above the static value observed on tangent track (ignoring other dynamic effects) and introducing additional lateral forces at the wheel/rail interface has maintenance implications on curves. Increased forces due to the overbalance condition increase vertical wear on the low rail (12). The overbalance condition has also been shown to increase the frequency of track maintenance on curves and promote development of rolling contact fatigue (4-6). Union Pacific Railroad concluded that correcting improperly superelevated track to eliminate overbalance conditions is the most significant factor in reducing excessive flange wear on the high and low rail on curves (13). Sadhegi (14) examined the deterioration of crossties due to cant deficiency and concluded that the underbalance condition showed less wear compared to overbalance. The overbalance condition may also result in increased derailment risk due to "string-lining" or rail roll-over.

Tournay (4-6) has suggested that the overbalance condition should be avoided if possible. Since in-train forces on grades and variation in train speeds can produce overbalance conditions under current railroad, some margin is needed between the design speed and the equilibrium speed to avoid cant excess. For this reason, cant deficiency is preferred to excess cant (4-6) and current design practice sets maximum track speed above the equilibrium speed by specifying a design value of cant deficiency. Cant deficiency allows for higher curving speeds for a given amount of superelevation, facilitating reduced running time on existing track geometry. However, the underbalance condition may result in wear on the high rail, vehicle overturning derailments and poor ride comfort.

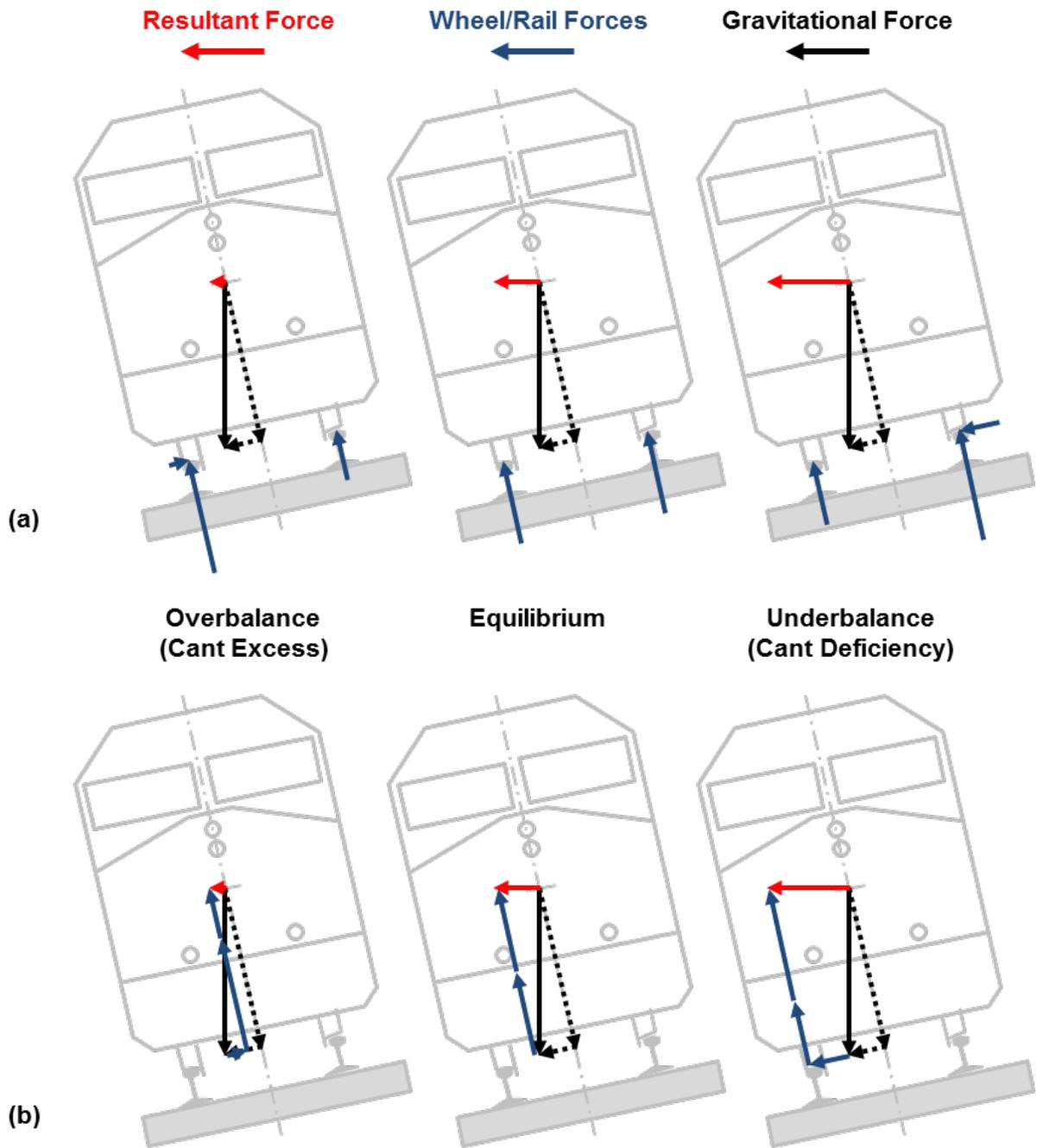


Figure 4: Illustration of (a) forces acting on a rail vehicle while curving at different speeds and (b) vector force addition to produce centripetal force required for circular motion at speed

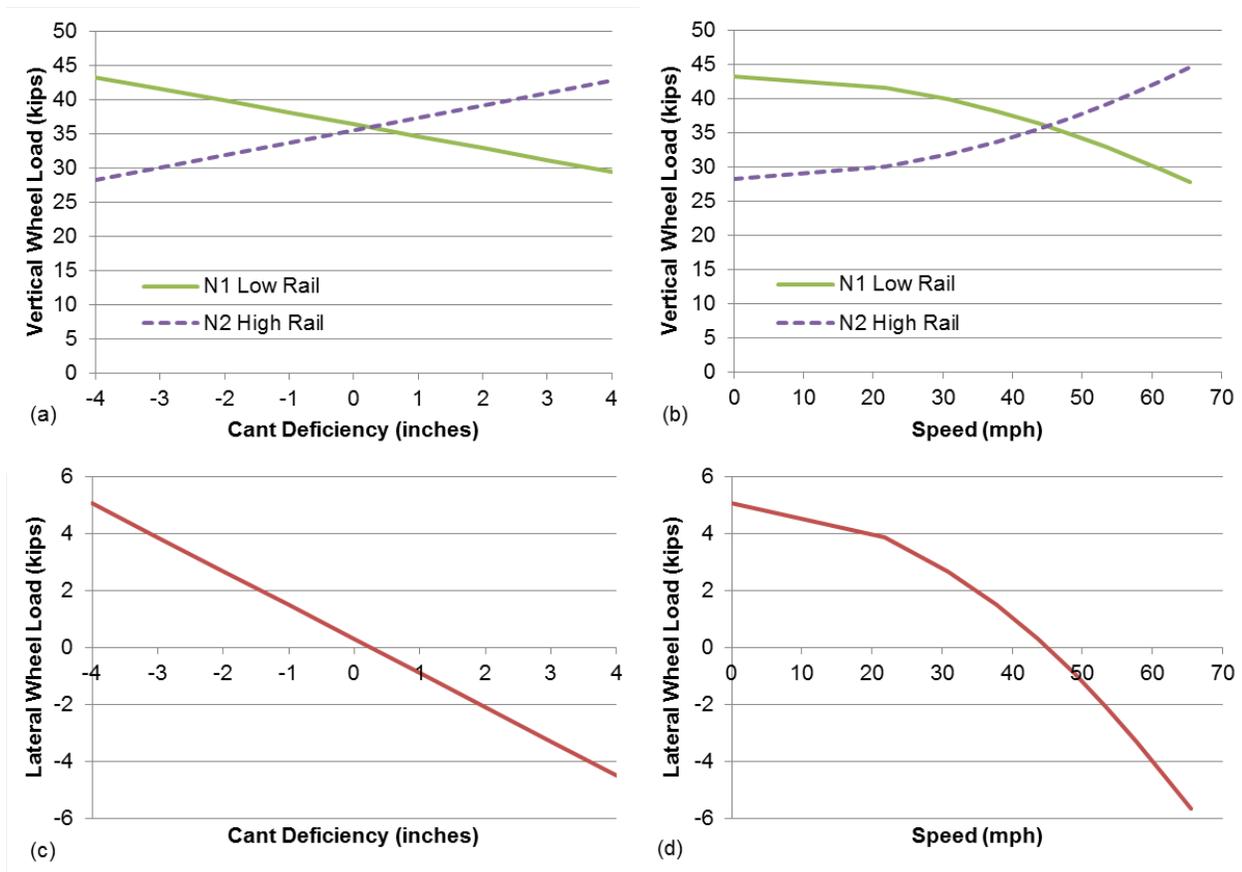


Figure 5: Variation in vertical and lateral wheel loads over a range of cant deficiency (a & c) and speed (b & d) for a 286,000 pound railcar on a 3-degree curve

SELECTION OF SUPERELEVATION FOR MULTIPLE TRAIN SPEEDS

Plotkin (15) analysed mixed-use rail corridors and concluded that setting a common superelevation is quite complex and may create overbalance condition for freight trains. Tournay (4-6) concluded that superelevation for the speed of prevailing tonnage is the most desired condition. An empirical approach to handling these speed ranges is to set the superelevation on the curve to the minimum required to achieve the desired passenger speed and observing rail wear over a period of time. The superelevation is then gradually increased until a desirable even wear pattern is observed on both the high and low rails. This paper applies the concept of superelevation bandwidth to maximize coverage of observed train operating speeds.

Superelevation Bandwidth

For a given degree of curvature and actual superelevation (E_a), Equation 2 indicates that the maximum train speed is dictated by the allowable cant deficiency. Increasing cant deficiency facilitates faster train speeds. As discussed in the earlier maintenance section, a consensus of most papers is that the overbalance condition is undesirable due to wear on the low rail. Thus the balanced speed (as calculated with Equation 2 and $E_u=0$) sets a desirable minimum train speed for the curve. The conditions of $E_u=0$ and $E_u=\text{Maximum } E_u$ set the upper and lower bounds on a “bandwidth” of desirable speeds to negotiate a curve with given actual superelevation (4-6, 7).

Since the required superelevation and cant deficiency increases with the square of the speed, the width of the superelevation bandwidth decreases as desired maximum speed increases (Figure 6). This makes finding an optimal combination of superelevation and cant deficiency with sufficient bandwidth to cover a range of operating speeds more difficult as maximum train speed increases.

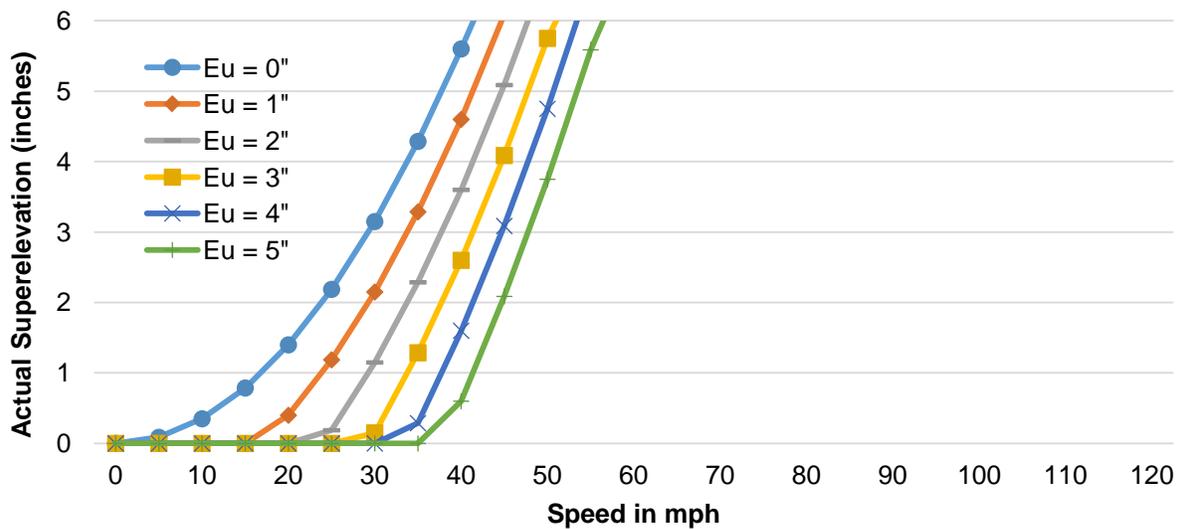
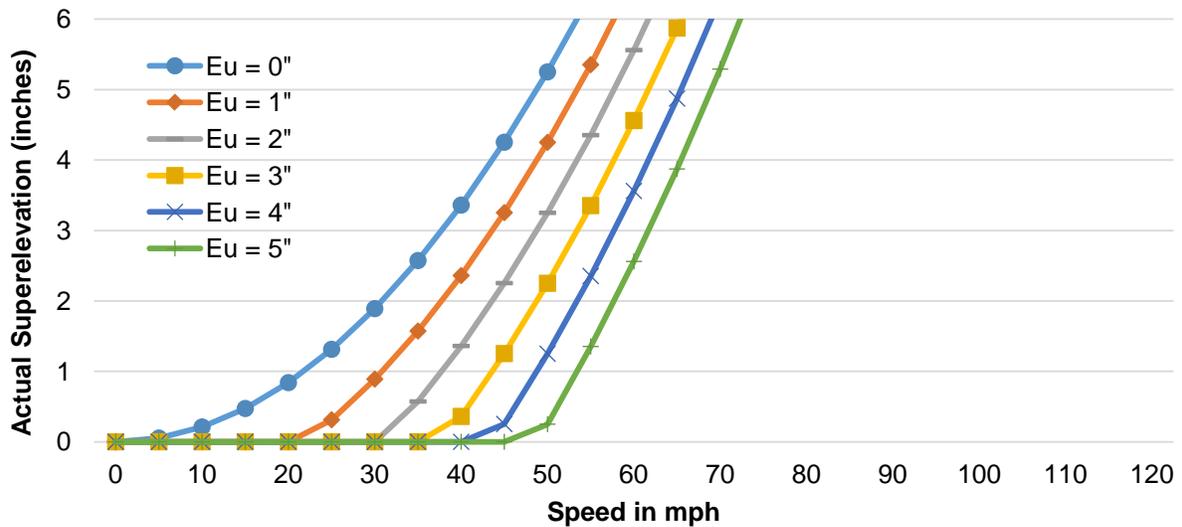
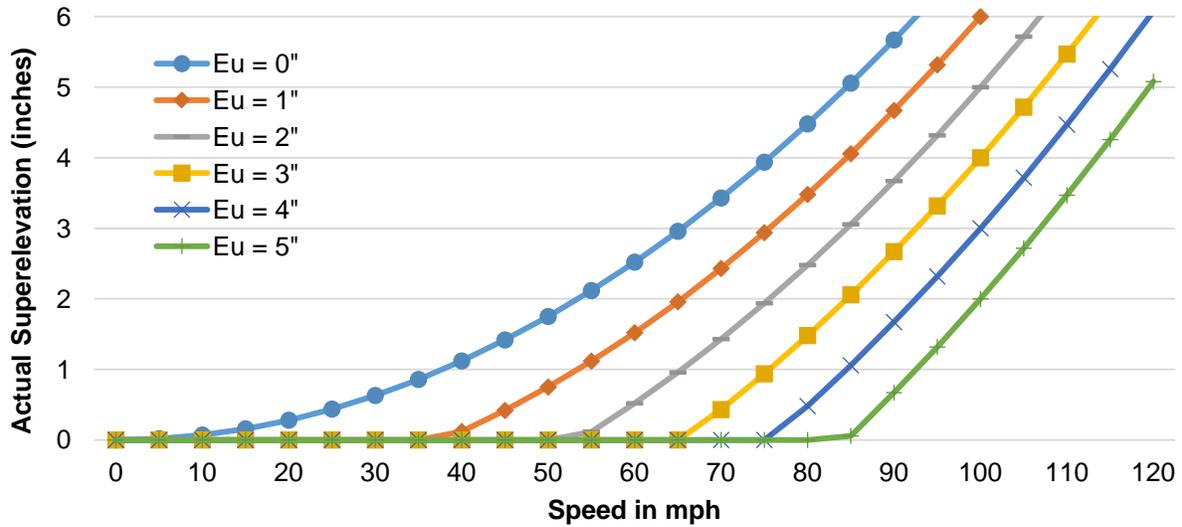


Figure 6: Range of speeds between minimum balanced speed for freight and passenger speed at maximum cant deficiency (a) 1-degree curve (b) 3-degree curve and (c) 5-degree curve

For a given degree of curvature, although maximum speed increases, superelevation bandwidth decreases as actual superelevation is increased. Finally, as degree of curvature increases, superelevation bandwidth decreases rapidly. On sharper curves, operating with high cant deficiency is less effective at increasing maximum train speeds compared to broader curves.

Bandwidth and Distribution of Train Speeds

Since superelevation bandwidth is defined over a range of train speed, it can be directly overlaid on a train speed frequency distribution (Figure 7). Combining these two figures provides a visual framework for selecting the optimal combination of actual superelevation and cant deficiency to cover the majority of trains in the train speed distribution.

For the example of a 1-degree curve with one inch of actual superelevation and five inches of cant deficiency, the corresponding superelevation bandwidth "A" can be directly compared to the train speed frequency distribution (Figure 7a). Trains falling below the lower bound are freight trains that will operate on the curve in the undesirable overbalance condition. Trains falling above the upper bound will be subject to civil speed restrictions that increase running time (and effectively create built-in train delay). To avoid maintenance issues and delaying trains, the number of trains in each "tail" outside the bandwidth "A" should be minimized.

Using this graphical approach, the effectiveness of different superelevation solutions can be compared. Continuing with the example, increasing actual superelevation to two inches while maintaining five inches of cant deficiency increases maximum passenger train speed, resulting in fewer delayed trains (Figure 7b). However, the decrease in overall superelevation bandwidth "B" and increased maximum speed result in the lower bound speed increasing from 40 to 55 mph. At this equilibrium speed, the majority of freight trains will operate in the undesirable overbalance condition.

A third superelevation solution with actual superelevation of 4 inches and cant deficiency of three inches (Figure 7c) maintains the same maximum speed as bandwidth "B". However, with lower cant deficiency and greater actual superelevation, bandwidth "C" is much narrower and all trains operating below 75 mph will experience overbalance conditions.

Later in the paper, this graphical technique is developed into a more formal mathematical model to optimize superelevation.

Industry Standards

For existing mixed-use corridors, superelevation and the ability of the bandwidth concept to cover a range of operating speeds may be constrained by current industry standards. Comparison of superelevation design practices for twelve freight, passenger and commuter railroads reveals that no two operators are using the exact same criteria (16-25). Nearly all operators calculate superelevation for passenger operations with at least 3 inches of cant deficiency and several allow higher levels of cant deficiency where equipment meets FRA requirements. Low values of freight cant deficiency result in narrow speed ranges for freight train operation (26).

Several freight operators indicate that the larger of actual superelevation required for freight or passenger train speeds shall be installed. This approach may create a situation where slower freight trains operate in the overbalance condition. CSX is the only freight operator that explicitly states that superelevation is designed for freight train speeds and then passenger train speeds are set according to the actual superelevation and allowable cant deficiency.

Design practices that involve large values of passenger cant deficiency and low values of freight cant deficiency maximize superelevation bandwidth. The Amtrak design standard offers the best potential in this regard but CN and CSX also facilitate wider superelevation bandwidth on certain corridors subject to operation of higher-speed passenger trains.

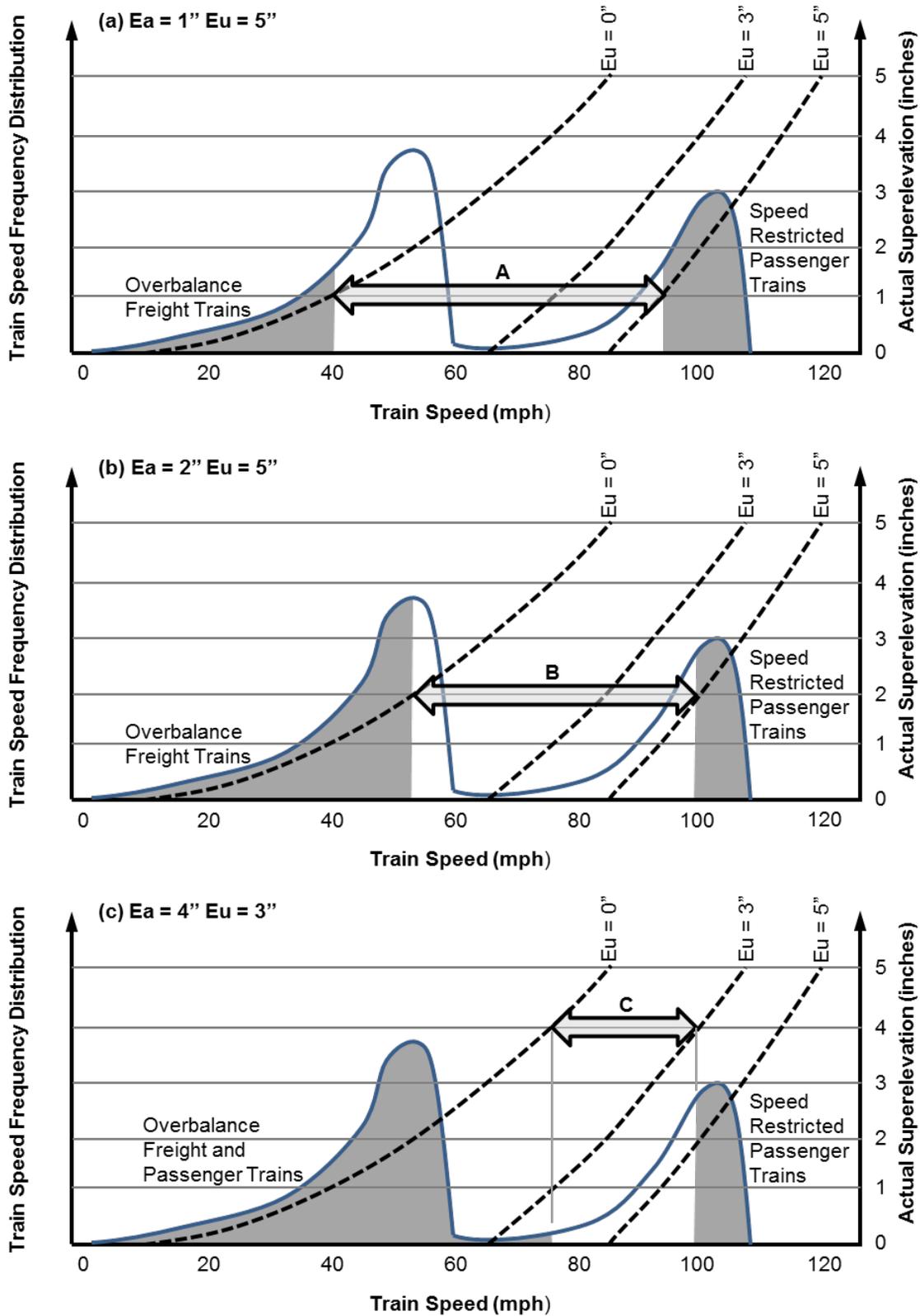


Figure 7: Portion of train speed frequency distribution covered by the speed bandwidth corresponding to different combinations of E_a and E_u on a 1-degree curve

Table 1: Summary of current US railway superelevation design criteria

Railway or Operator	Freight Cant Deficiency (in.)	Passenger Cant Deficiency (in.)	Maximum Actual Superelevation (in.)	Type of Train Governing Actual Superelevation Installed
BNSF	2	3	5	Largest E_a of freight or passenger
Canadian Pacific	2	3	N/A	Selected with software tool
CN Railway	2	3 or 6	5	Minimum E_a in range of operating speeds
CSX	1-1/2 or 2-1/2	3 to 4	5	Freight
Norfolk Southern	2	3	5	Largest E_a of freight or passenger
Union Pacific	1	3	5	Not indicated
Amtrak	1-1/2	4 to 7	5-1/2	Passenger
California High-Speed Rail	1	3 to 4	3 to 6	Proportional to support passenger speeds
Caltrain	N/A	3	5	Passenger
Mass. Bay Transport Auth.	N/A	2-3/4	6	Passenger
Metrolink (So. California)	2	3-1/2	5	Passenger but check freight cant deficiency
Utah Transit Authority	N/A	3	5	Passenger

N/A = information not available

SUPERELEVATION OPTIMIZATION FRAMEWORK

Since the possible combinations of actual superelevation and cant deficiency is limited, the graphical approach of matching superelevation bandwidth to a train speed frequency distribution is effective at suggesting a superelevation design for a single curve or small number of curves. However, it could become cumbersome for a route with many curves or complex distributions of train speeds. To facilitate rapid optimization over many curves, a formal mathematical model for optimizing superelevation has been formulated.

Mathematical Model

As depicted graphically in Figure 7, the bandwidth concept seeks to maximize the number of trains (or equivalent tonnage or number of passengers) that fall between the equilibrium speed and speed at maximum cant deficiency. This is also equivalent to minimizing the number of trains falling outside the bandwidth. By dividing the train speed distribution into a series of discrete train speed groups, optimizing superelevation bandwidth can be formulated as a “set-covering problem”. Such problems are often solved with a Mixed-Integer Program (MIP). However, since superelevation and train speed are related quadratically, the problem is formulated as a Mixed Integer Quadratically Constrained Program (MIQCP).

Equations 4 through 12 describe the form of the model:

$$\text{Minimize } Z = \beta \sum_i S_{fi} G_i + \gamma \sum_i S_{pi} T_i \quad (4)$$

Subject to

$$E_a + E_u = 0.0007 D V_{max}^2 \quad (5)$$

$$E_a = 0.0007 D V_{min}^2 \quad (6)$$

$$V_{min} - V_i \leq S_{fi} V_{top} \quad \forall i \quad (7)$$

$$V_i - V_{max} \leq S_{pi} V_{top} \quad \forall i \quad (8)$$

$$E_{amin} \leq E_a \leq E_{amax} \quad (9)$$

$$0 \leq E_u \leq E_{umax} \quad (10)$$

$$S_{fi} = 0,1 \quad \forall i \quad (11)$$

$$S_{pi} = 0,1 \quad \forall i \quad (12)$$

Where:

D = degree of curvature

E_a = actual superelevation (design)

E_u = cant deficiency (design)

E_{amin} = minimum actual superelevation allowed by design criteria

E_{amax} = maximum actual superelevation allowed by design criteria

E_{umax} = maximum cant deficiency allowed by design criteria

V_{min} = minimum bandwidth speed

V_{max} = maximum bandwidth speed

V_{top} = fastest observed train speed

V_i = median speed of trains in speed group *i*

G_i = annual million gross tons of freight traffic in speed group *i*

T_i = annual passenger traffic in speed group *i*

β = coefficient quantifying cost of 1 MGT of freight train operation in overbalance condition

γ = coefficient quantifying revenue loss of increased passenger running time per passenger

S_{fi} = freight binary variable for speed group *i* (0 if freight traffic in bandwidth, 1 if outside bandwidth)

S_{pi} = passenger binary variable for speed group *i* (0 if passenger traffic in bandwidth, 1 if outside)

When solved, the model will provide the design values of E_a and E_u that, given the factors considered by the model, minimize the negative effects of trains operating outside the superelevation bandwidth. Equation 4, the objective function, minimizes the weighted sum of MGT of freight traffic falling below the minimum bandwidth speed and passenger traffic falling above the maximum bandwidth speed. Equation 5 defines the maximum bandwidth speed according to the actual superelevation and cant deficiency design. Equation 6 defines the minimum bandwidth speed as the equilibrium speed for the actual superelevation design. Equations 7 and 8 determine if speed group *i* falls within the superelevation bandwidth and set the freight and passenger binary S_{fi} and S_{pi} variables accordingly. Equations 9 and 10 ensure the design actual superelevation and cant deficiency are positive and do not exceed the allowable

values specified in the design criteria. Equations 11 and 12 define the freight and passenger binary variables for all speed groups.

One compromise of this model formulation is that actual superelevation can assume any real value between the minimum and maximum superelevation allowable under the railway design criteria. In practice, actual superelevation is installed in even increments of 0.25 or 0.125 inches. The solution provided by the model must be rounded to the nearest feasible value of actual superelevation. Reformulating the model to only consider even increments of actual superelevation is possible but would greatly increase the number of integer variables and make it more difficult to solve.

Of practical concern in implementing this mathematical model as a superelevation optimization framework is selection of the coefficients β and γ . Although many research papers document the negative maintenance implications of operating with cant excess in the over balance condition below equilibrium speed, none of the papers quantify the incremental maintenance expense as a function of freight traffic. Zarembski (27) presents a detailed model of the cost of additional track maintenance for passenger trains on freight corridors but it does not include unbalance or superelevation as specific parameters. Additional research is needed to select values of β that directly relate to maintenance costs or the maintenance agreements in place on a specific mixed-use corridor. Passenger operators may have a good feel for the revenue implications of imposing civil curve speed restrictions on passenger trains but in the absence of data, setting γ also requires engineering judgement. To simplify implementation of the model, the coefficients can also be set to reflect the relative importance of passenger and freight trains to the track designer and weight their influence on the superelevation design accordingly.

CONCLUSIONS

Many factors influence the design of curve superelevation on mixed-use railway lines where trains operate at different speeds. Disparity in train speeds may be due to the different business objectives of certain types of freight and passenger service, the curving capability of different types of rail vehicles and local site conditions that may cause certain trains to negotiate nearby curves at less than normal timetable operating speeds. Grades and the resulting in-train forces experienced by long freight trains present additional challenges for setting superelevation to avoid excessive lateral wheel/rail forces. Since the actual superelevation in track is fixed, under conditions of varying train speeds and in-train forces, different trains will experience cant deficiency or cant excess. Under these conditions, quasi-static vertical wheel loads can be increased by over 20 percent, increasing maintenance. In particular, the overbalance condition should be avoided as it promotes rolling contact fatigue.

On a given curve, a design superelevation bandwidth can be defined and compared to the frequency distribution of train speeds operating on the route. Trains falling below the lower bound set by the equilibrium speed will operate in the undesirable overbalance condition. Trains falling above the upper bound speed set by maximum cant deficiency will be subject to civil curve speed restrictions. A combination of actual superelevation and allowable cant deficiency that satisfies railway design criteria while maximizing the number of trains falling within the superelevation bandwidth will provide the best solution for a mixed-use corridor. Using the bandwidth approach, trains can be weighted by traffic in terms of freight tonnage or number passengers to better reflect the maintenance and revenue implications of operating outside the design superelevation bandwidth.

The paper presents both a graphical framework and mathematical approach to identifying the optimal superelevation design parameters with the bandwidth approach. The optimization framework could be improved by additional research to quantify the specific maintenance cost of operating freight trains with overbalance. Additional knowledge of this parameter would allow for more effective practitioner decisions regarding the trade-off between increasing maximum passenger train speeds to eliminate civil speed restrictions while simultaneously increasing the fraction of freight trains operating with overbalance.

ACKNOWLEDGEMENTS

This research was sponsored by the Federal Railroad Administration under BAA-2014-2: Research Initiatives In Support of Track Research, Project FRA-TR-008A. The authors were also supported by the National University Rail (NURail) Center, a US DOT-OST Tier 1 University Transportation Center

REFERENCES

1. American Railway Engineering and Maintenance-of-Way Association, 2014. *Manual for Railway Engineering*, Chapter 5, Section 3. The American Railway Engineering and Maintenance of Way Association, Landover, MD, USA.
2. Elkins, J.A. and Gostling, R.J., 1977. A general quasi-static curving theory for railway vehicles. *Vehicle System Dynamics*. 6 (2-3), pp.100-106.
3. Moody, J.C., 2014. *Critical Speed Analysis of Railcars and Wheelsets on Curved and Straight Track*. Honors Thesis, Bates College, Spring 2014.
4. Tournay, H., S. Cakdi, S. Trevithick and K. Morrison, 2014. The effect of track cant on vehicle curving: theory and single car test results, Part I of III. *Technology Digest*, TD-14-013. Transportation Technology Center, July 2014.
5. Tournay, H., M. Akhtar, S. Cakdi, and K. Morrison, 2014. The effect of track cant on vehicle curving: in-service site selection and analysis, Part II of III. *Technology Digest*, TD-14-014. Transportation Technology Center, July 2014.
6. Tournay, H., M. Akhtar, S. Cakdi, and K. Morrison, 2014. The effect of track cant on vehicle curving: in-service test results, Part III of III. *Technology Digest*, TD-14-015. Transportation Technology Center, July 2014.
7. Igwemezie, J.O., 2006. Re-engineering superelevation: Part 2 of 2. *Railway Track and Structures*, 102 (8).
8. Roney, M., 2009. Reducing the stress state on Canadian Pacific's Western Corridor. *Interface: The Journal of Wheel/rail Interaction*.
9. Roney, M., Bell, S., Paradise, S., Oldknow, K. and Igwemezie, J., 2010. Implementation of distributed power and friction control to minimize the stress state and maximize velocity in Canadian Pacific's heavy haul/heavy grade operations. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*. 224 (5), pp.465-471.
10. Oldknow, K.D. and D.T. Eadie, 2010. Top of rail friction control as a means to mitigate damaging lateral loads due to overbalanced operation of heavy axle load freight traffic in shared high speed rail corridors. *Proceedings of the 2010 Joint Rail Conference*, American Society of Mechanical Engineers.
11. Federal Railroad Administration, 2015. *Track Safety Standards*. US Code of Federal Regulations Title 49 Part 213 Subpart C Section 213.57.
12. Sadeghi, J. and Akbari, B., 2006. Field investigation on effects of railway track geometric parameters on rail wear. *Journal of Zhejiang University Science A*, 7 (11), pp.1846-1855.
13. Igwemezie, J., D. Marcks, W.E. Van Trump and D.W. Clark, 2006. High rail wear on the Tehachapi: problem identification, analysis and implementation of solutions. *Proceedings of the 2006 AREMA Annual Conference & Exposition*.
14. Sadeghi, J. and Shoja, H., 2013. Impact of superelevation deficiencies on the loading pattern of railway sleepers. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*. 227 (3), pp.286-295.
15. Plotkin, D., 1997. Carrying freight on high-speed rail lines. *Journal of Transportation Engineering*. 123 (3), pp.199-201.
16. BNSF Railway. 2000. *BNSF Engineering Instructions*, Section 5: Track Geometry. Fort Worth, TX, USA.
17. California High-Speed Rail Authority. 2008. *Alignment Standards for Shared Use Corridors*, TM 1.1.6.
18. CN Railway, 2002. *CN Engineering Recommended Method for Determining Superelevation, Spiral Length and Maximum Train Speed on Curves*, RM 1305-0.
19. CSX Transportation. 2002. *Standard Plan 2511: Superelevation of Curves*. Jacksonville, FL, USA.
20. Massachusetts Bay Transportation Authority. 1996. *Commuter Rail Design Standards Manual*, Volume 1, Section 1: Track and Roadway. Boston, MA, USA.
21. National Railroad Passenger Corporation. 2015. *Amtrak Engineering Track Design Specification No. 63*.

22. Norfolk Southern Railway. 2013. *Public Projects Manual*, Track Design Guide, Curve Superelevation. Atlanta, GA, USA.
23. Southern California Regional Rail Authority. 2014. *SCRRA Design Criteria Manual*. Los Angeles, CA. USA.
24. Union Pacific Railroad. 2011. *Track Standard Drawings*, Standard Drawings 0019-0023. Omaha, NE, USA.
25. Utah Transit Authority. 2010. *Utah Transit Authority Commuter Rail Design Criteria*, Chapter 3: Alignment. Salt Lake City, UT, USA.
26. Kollmar, R.A., 2006. The Balancing Act: *The Fine Art of Using Unbalance to Maximize the Efficiency of Railroad Infrastructure*. W.W. Hay railroad Engineering Seminar Series presentation, University of Illinois at Urbana-Champaign, April 28, 2006.
27. Zarembski, A.M. and Patel, P., 2010. Estimating maintenance costs for mixed higher speed passenger and freight rail corridors. *Proceedings of the 2015 Joint Rail Conference*, American Society of Mechanical Engineers.

LIST OF FIGURES

Figure 1: Influence in-train forces on required superelevation for (a) upgrade and (b) downgrade conditions at the same train speed

Figure 2: Equivalent superelevation due to in-train forces at different positions in a 100-car train on a 1-percent downgrade or upgrade

Figure 3: Example distribution of train speeds on mixed-use freight and passenger corridor

Figure 4: Illustration of (a) forces acting on a rail vehicle while curving at different speeds and (b) vector force addition to produce centripetal force required for circular motion at speed

Figure 5: Variation in vertical and lateral wheel loads over a range of cant deficiency (a & c) and speed (b & d) for a 286,000 pound railcar on a 3-degree curve

Figure 6: Range of speeds between minimum balanced speed for freight and passenger speed at maximum cant deficiency (a) 1-degree curve (b) 3-degree curve and (c) 5-degree curve

Figure 7: Portion of train speed frequency distribution covered by the speed bandwidth corresponding to different combinations of E_a and E_u on a 1-degree curve

LIST OF TABLES

Table 1: Summary of current US railway superelevation design criteria

Superelevation Optimization for Mixed Freight and Higher-Speed Passenger Trains

C. Tyler Dick, P.E.,
Conrad J. Ruppert, Jr.
Luv Sehgal
Shashad Gujuran

Rail Transportation and Engineering Center (RailTEC)
University of Illinois at Urbana-Champaign



AREMA 2016 Annual Conference & Exposition

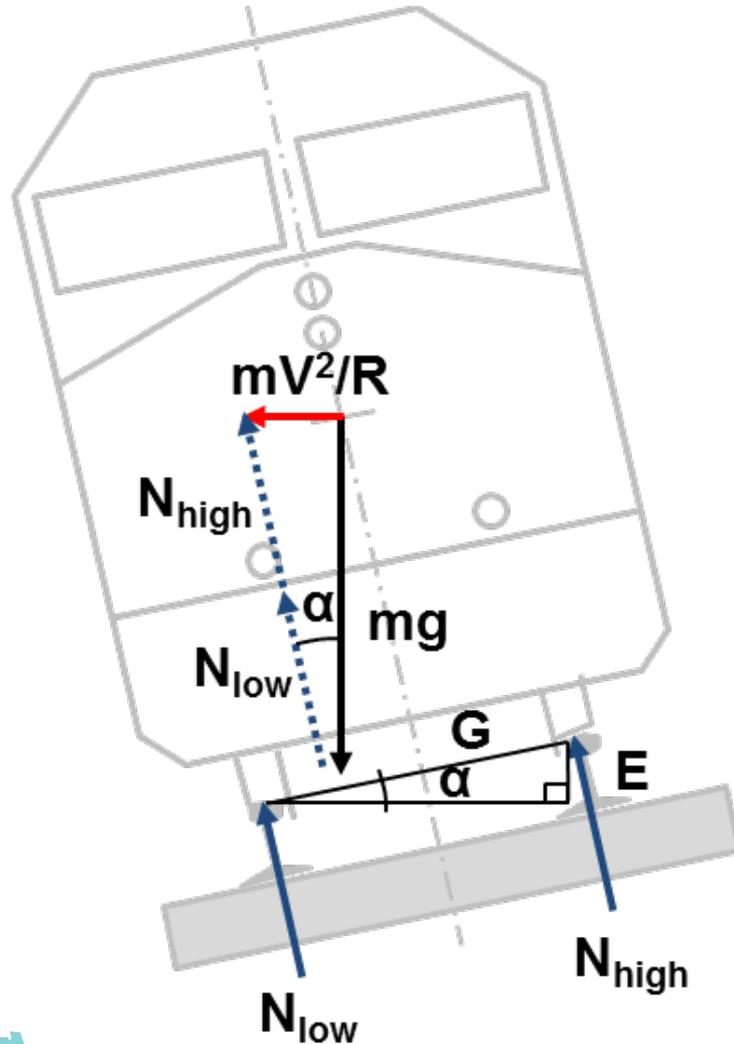


Outline

- Physics of curve superelevation
- Curving at non-equilibrium speed
- Selection of design speed
- Design for multiple train speeds
 - Graphical technique
 - Mathematical optimization



Physics of Superelevation



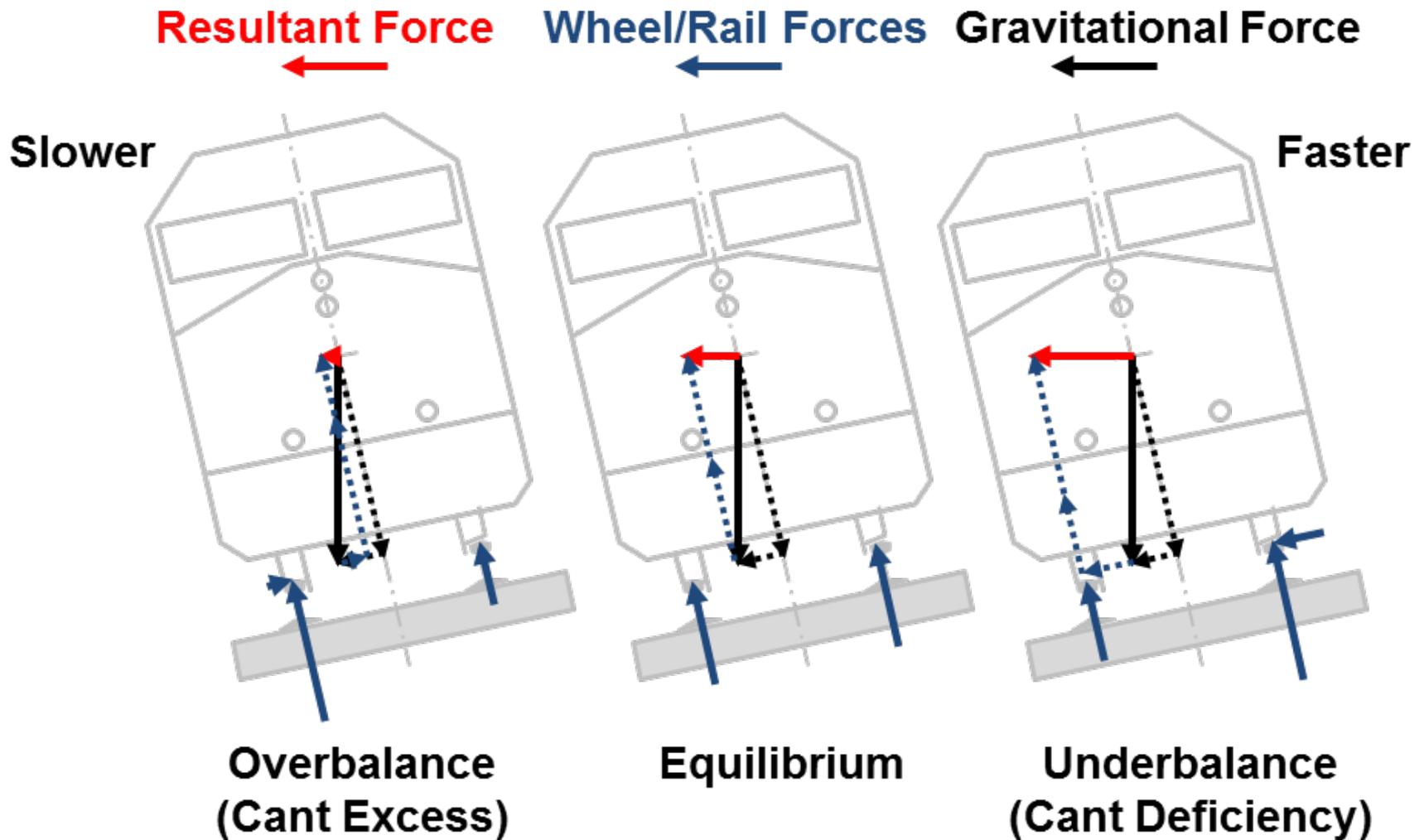
- Equilibrium superelevation

$$\frac{E}{G} = \frac{mV^2}{mgR}$$

$$E = 0.0007DV^2$$

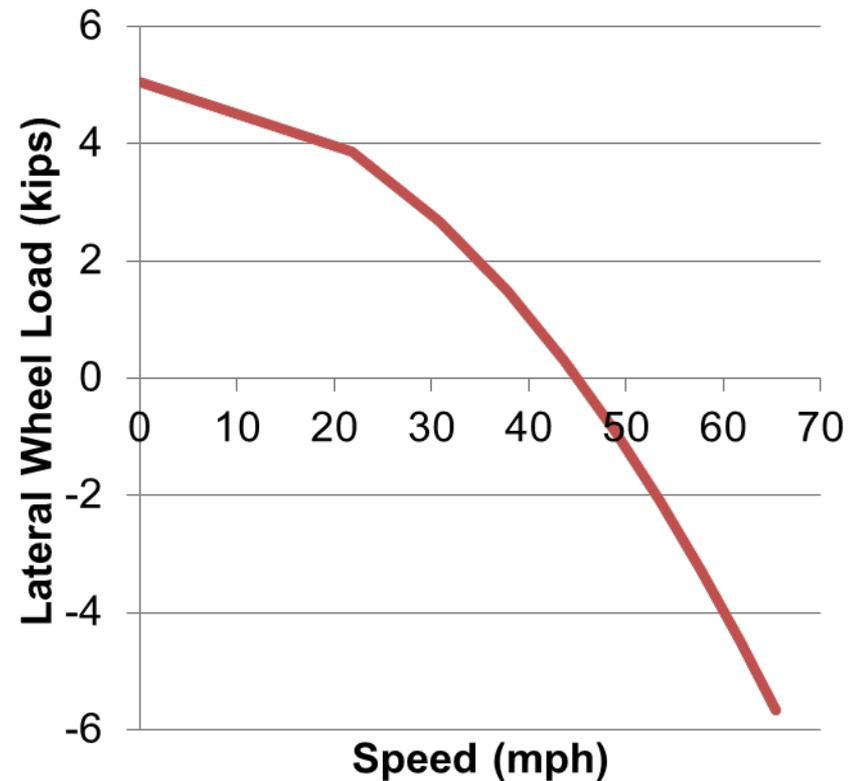
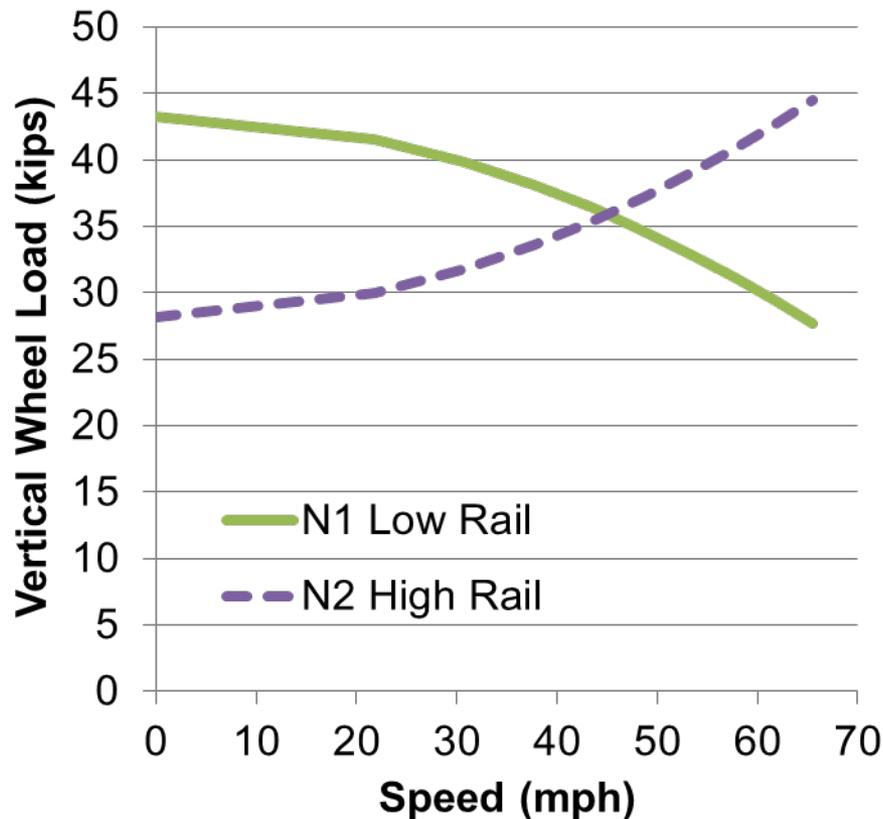


Non-equilibrium Speeds



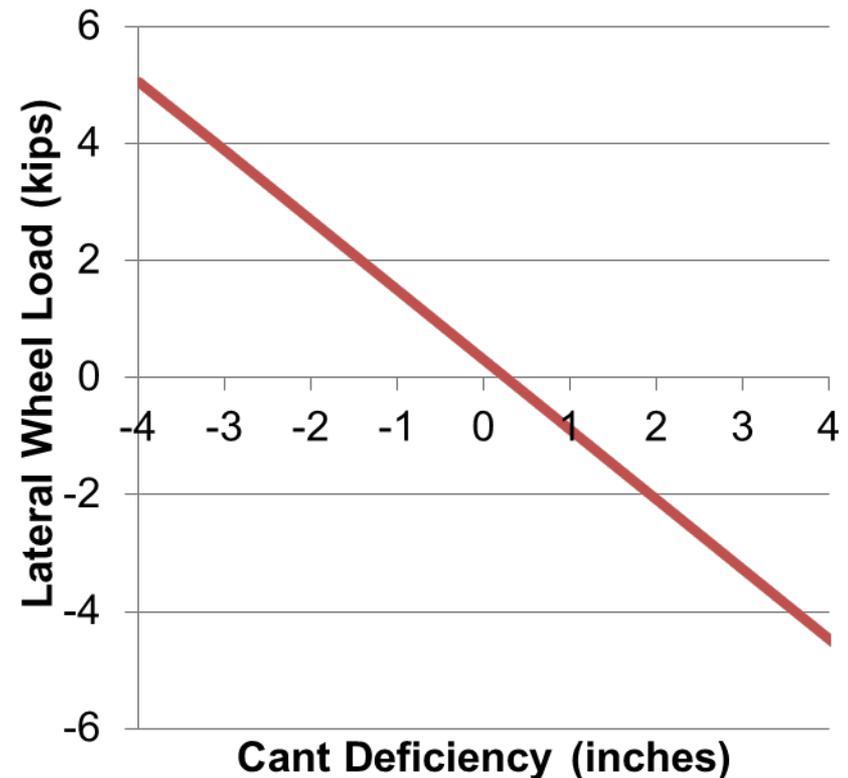
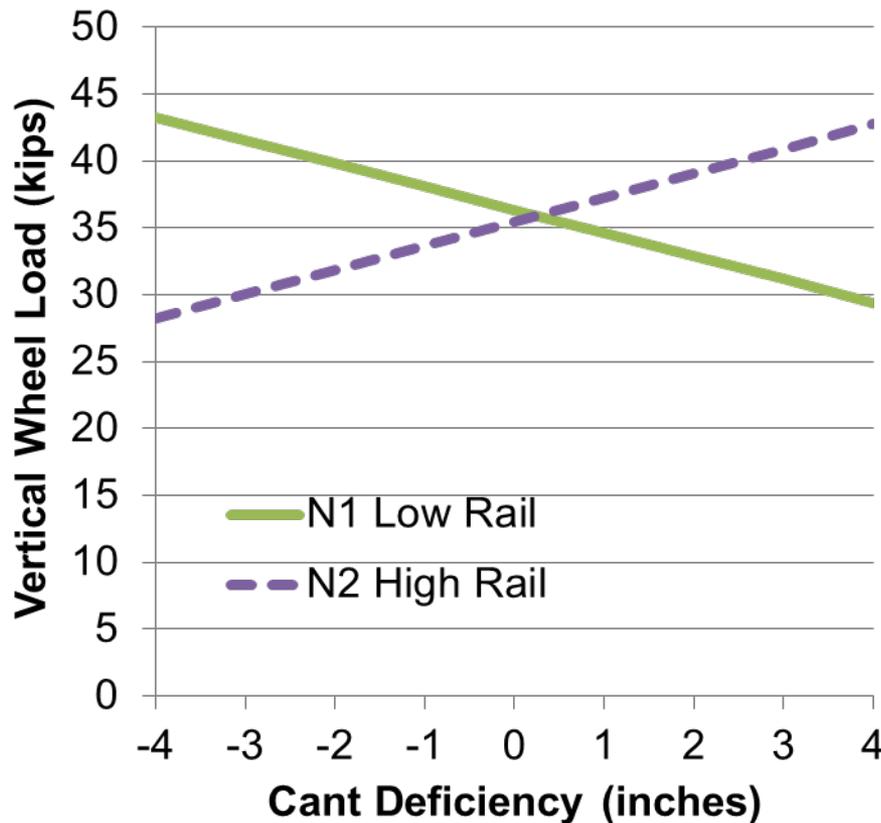
Distribution of Wheel Loads

- Loaded 286k railcar, 3-degree curve



Distribution of Wheel Loads

- Loaded 286k railcar, 3-degree curve



Implications

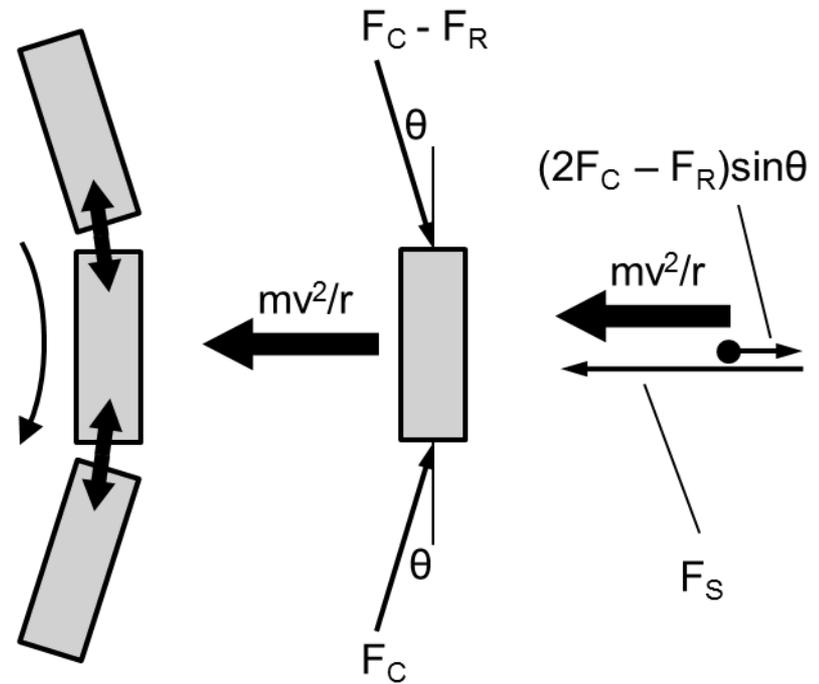
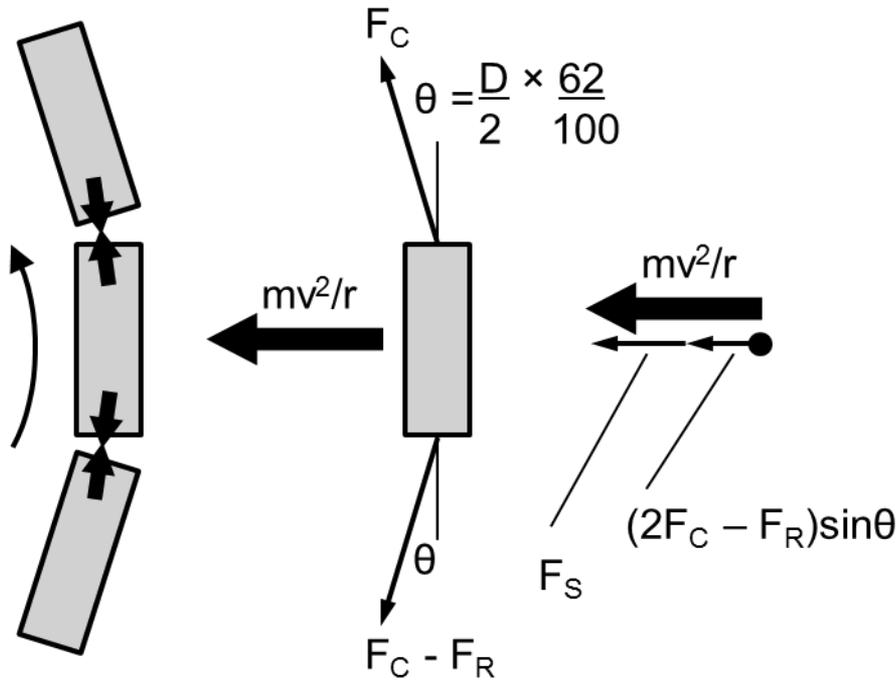
- Overbalance (Cant Excess)
 - Increased wear on low rail
 - Promote rolling contact fatigue
 - String-line or rail roll-over derailment
- Underbalance (Cant Deficiency)
 - Increased wear on high rail
 - Overturning or wheel-climb derailment
 - More desirable than overbalance



Superelevated Curves on Grades

Upgrade (Draft)

Downgrade (Buff)



Required superelevation force

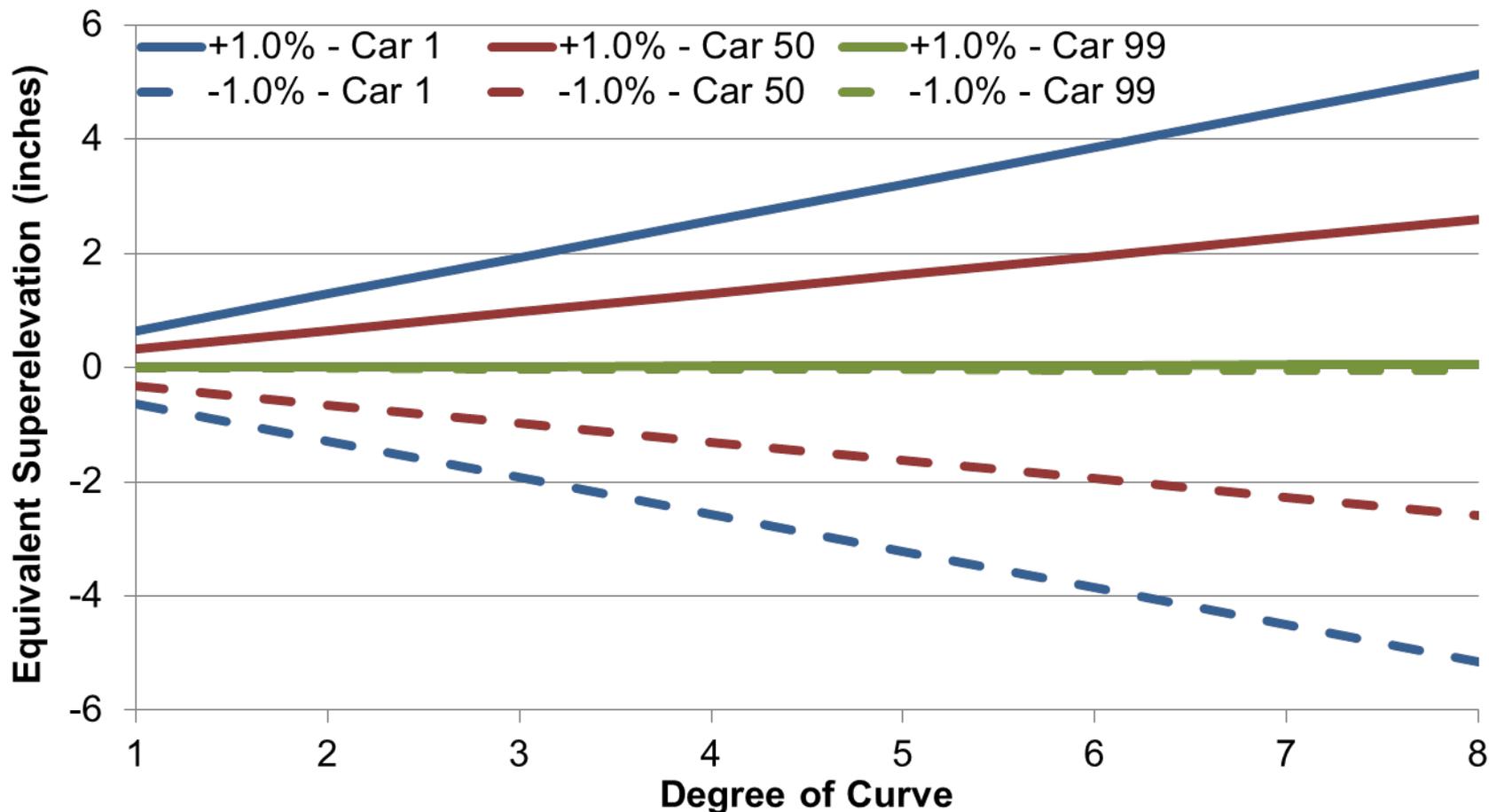
$$F_S = mv^2/r - (2F_C - F_R)\sin\theta$$

$$F_S = mv^2/r + (2F_C - F_R)\sin\theta$$



Equivalent Superelevation of In-Train Forces

- 100-car train, loaded 286k GRL railcars

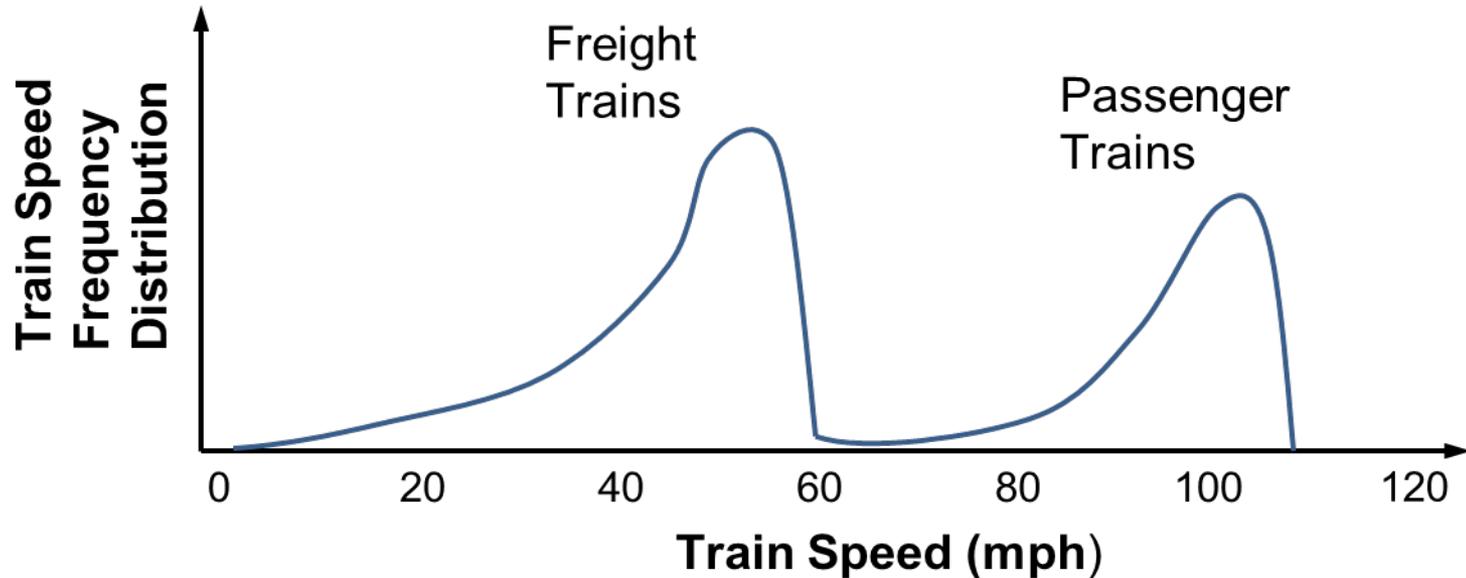


Design vs. Actual Train Speeds

- Type/priority of train
- Tractive effort and horsepower-per-ton
- Grade and direction of travel
- Commodity/railcar speed restrictions
- Signal indication or weather
- Train handling or fuel saving
- Curves adjacent to
 - Passing sidings or turnouts
 - Stations, crew change points
 - Spurs and local switching areas



Train Speed Distribution

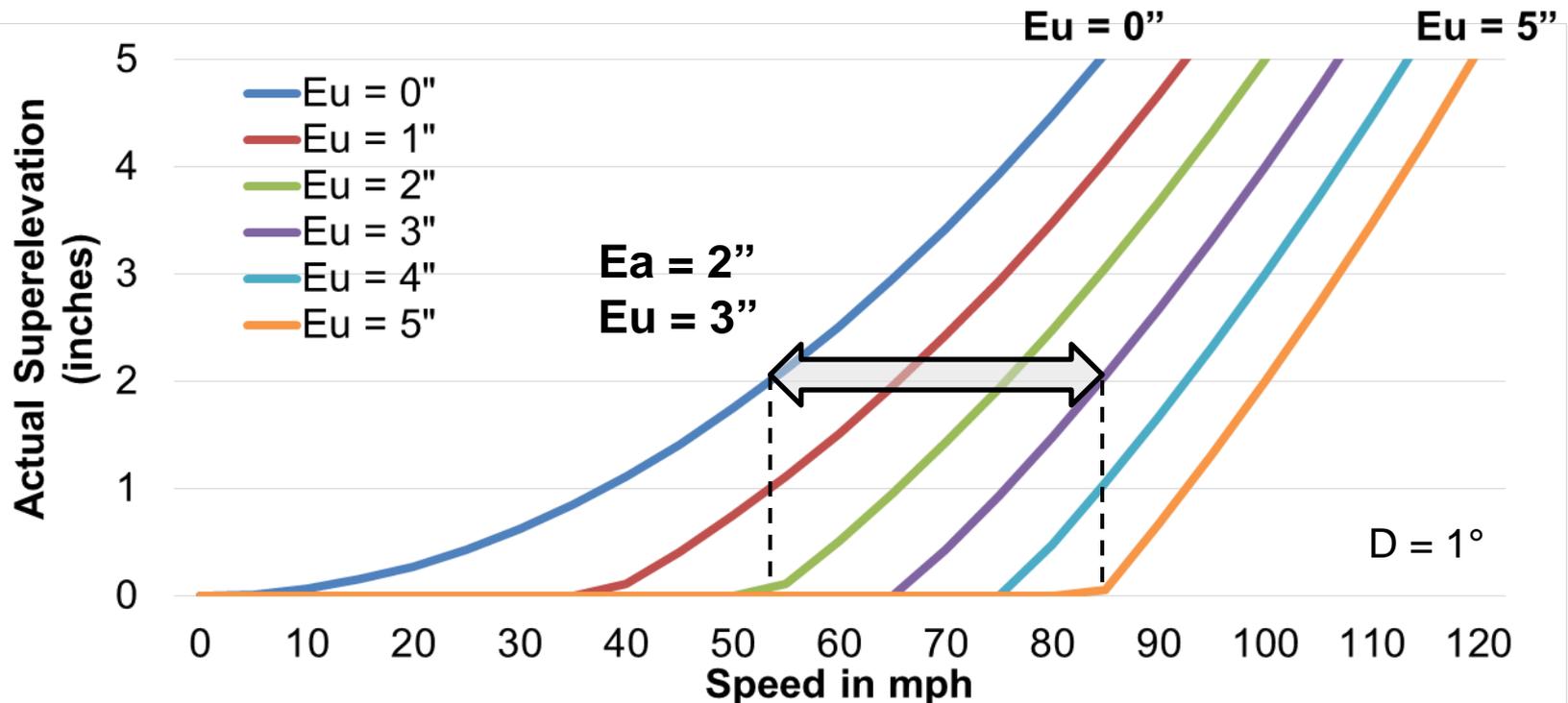


- Which speed governs superelevation?
 - Avoid freight overbalance
 - Avoid passenger civil speed restrictions



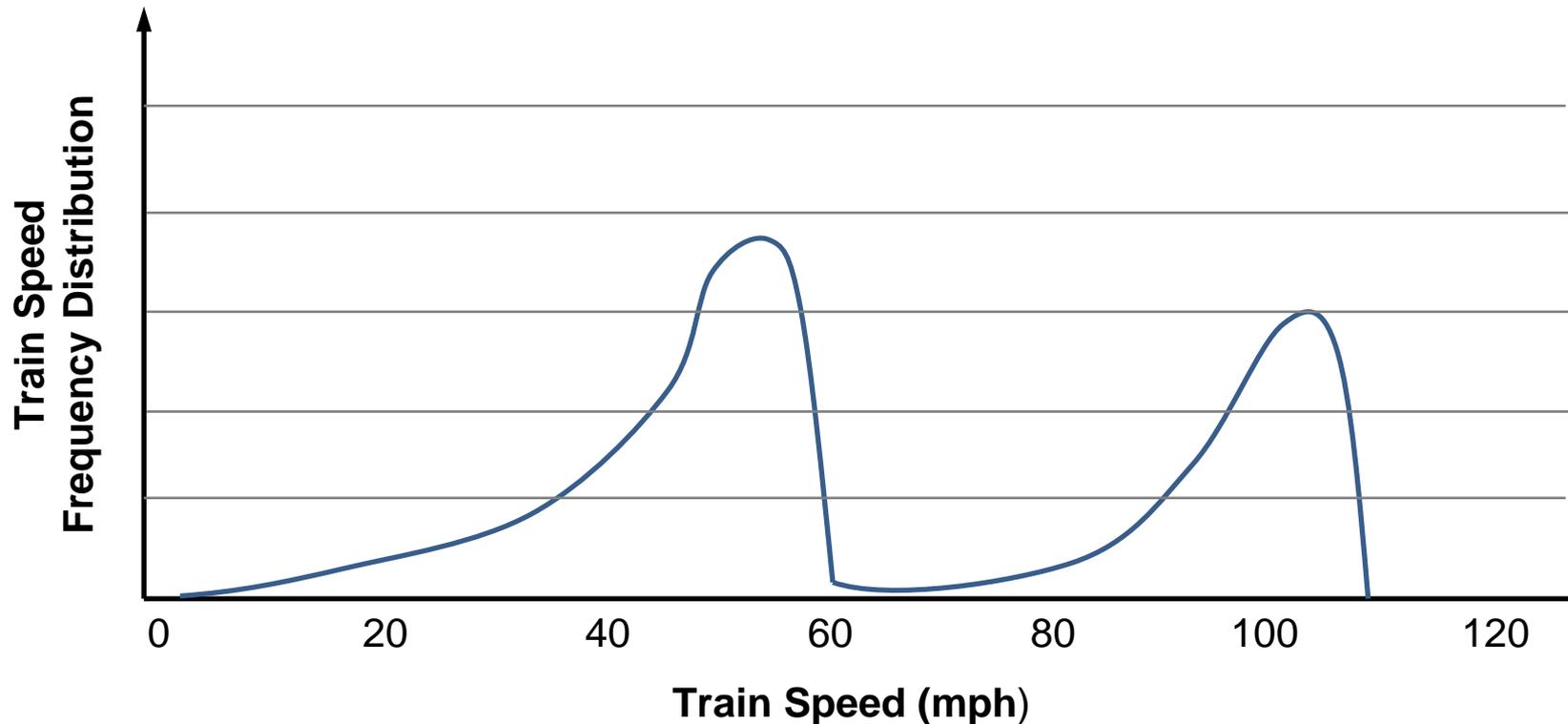
Superelevation Bandwidth

- Maximum unbalance superelevation limited by FRA regulations for different types of equipment
- Range of curve speeds between equilibrium and maximum unbalance for actual superelevation



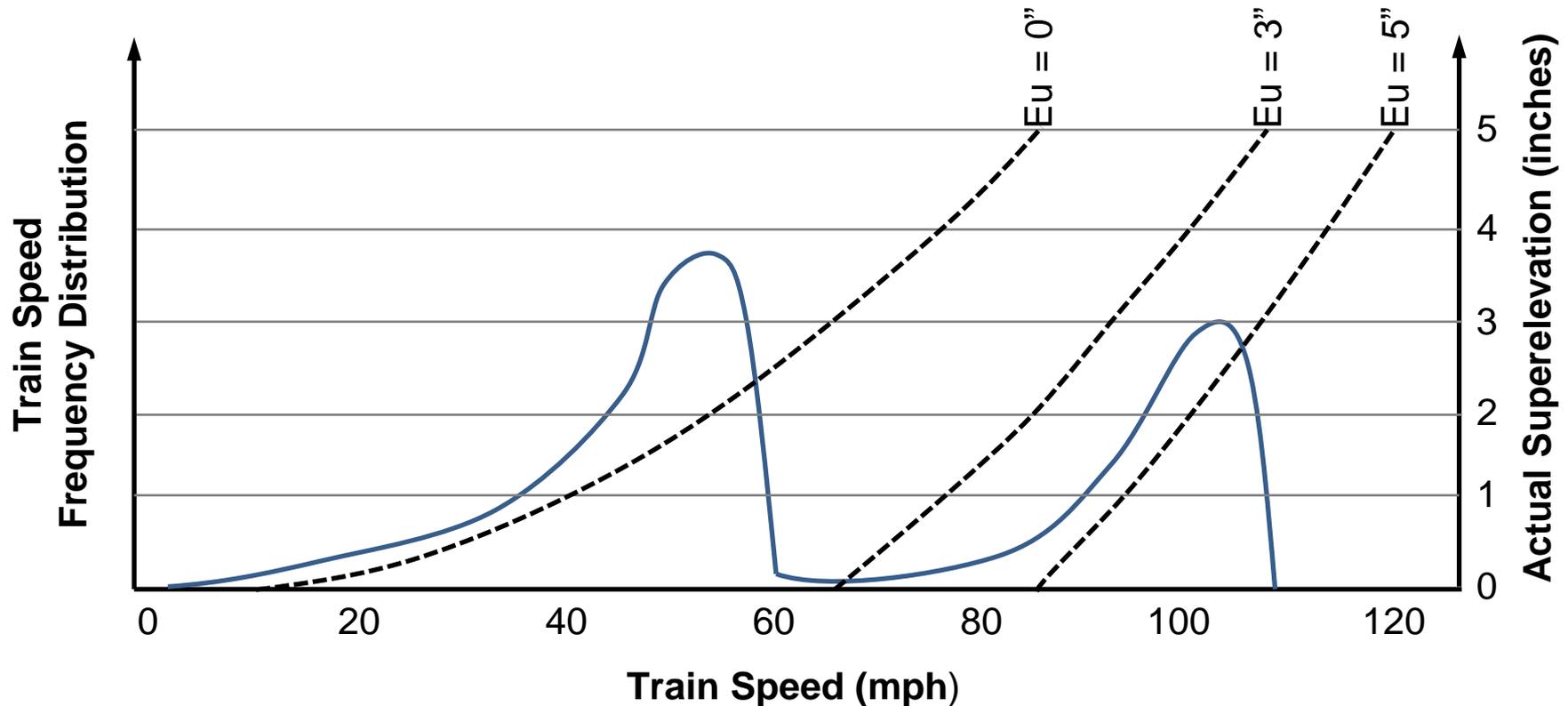
Graphical Technique

- Set superelevation for range of design train speeds on a given curve



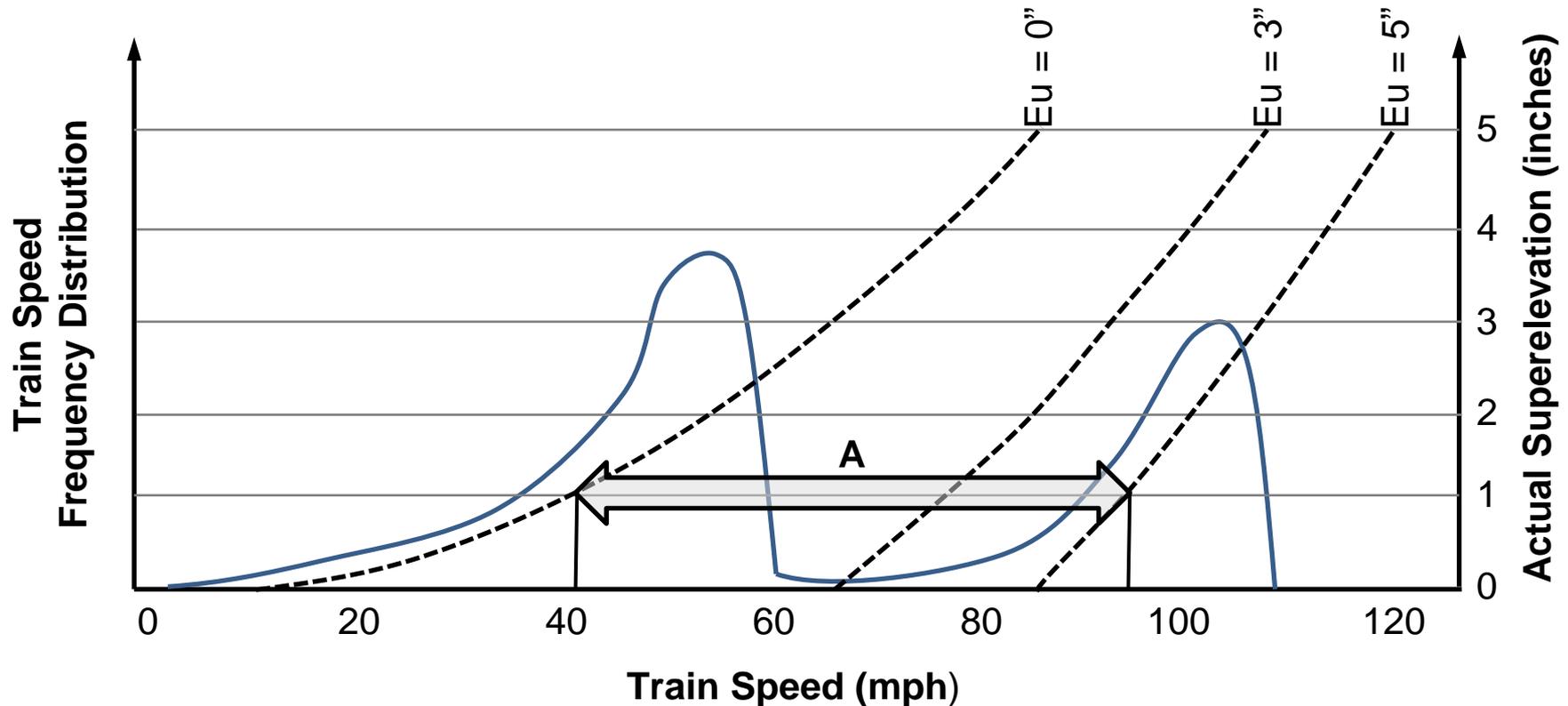
Superelevation Bandwidth

- 1-degree curve



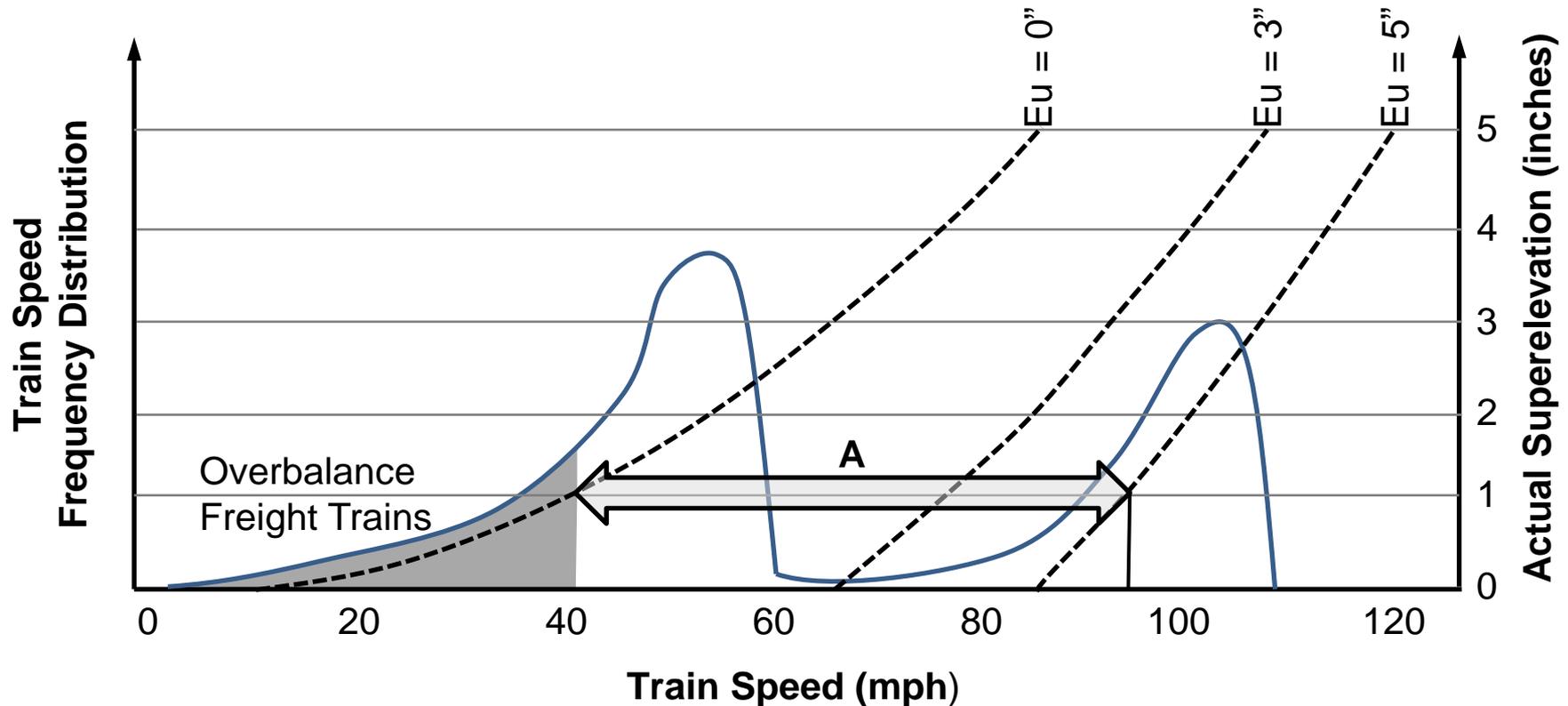
Trial Superelevation Design

- Actual Super. = 1" and Unbalance = 5"



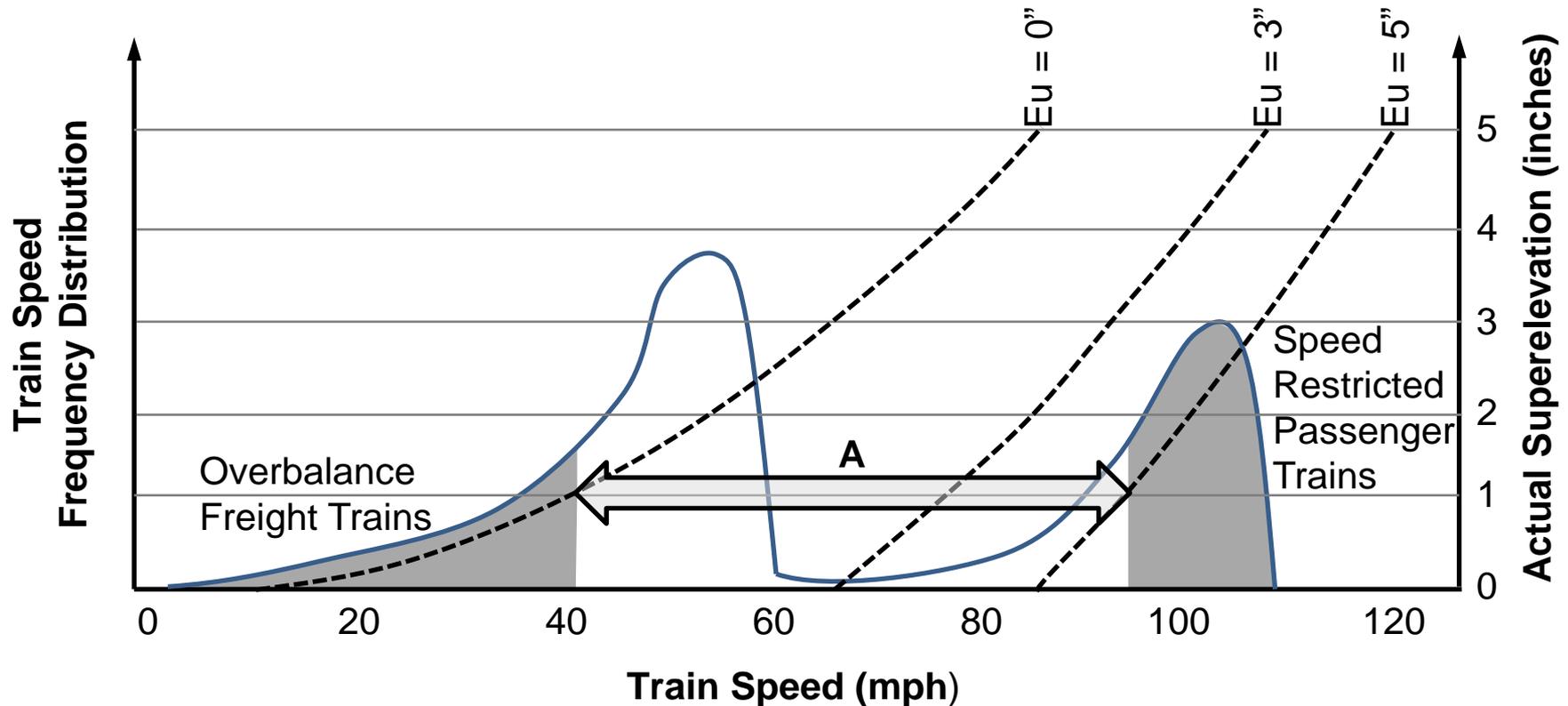
Overbalance Trains

- Freight trains below equilibrium speed



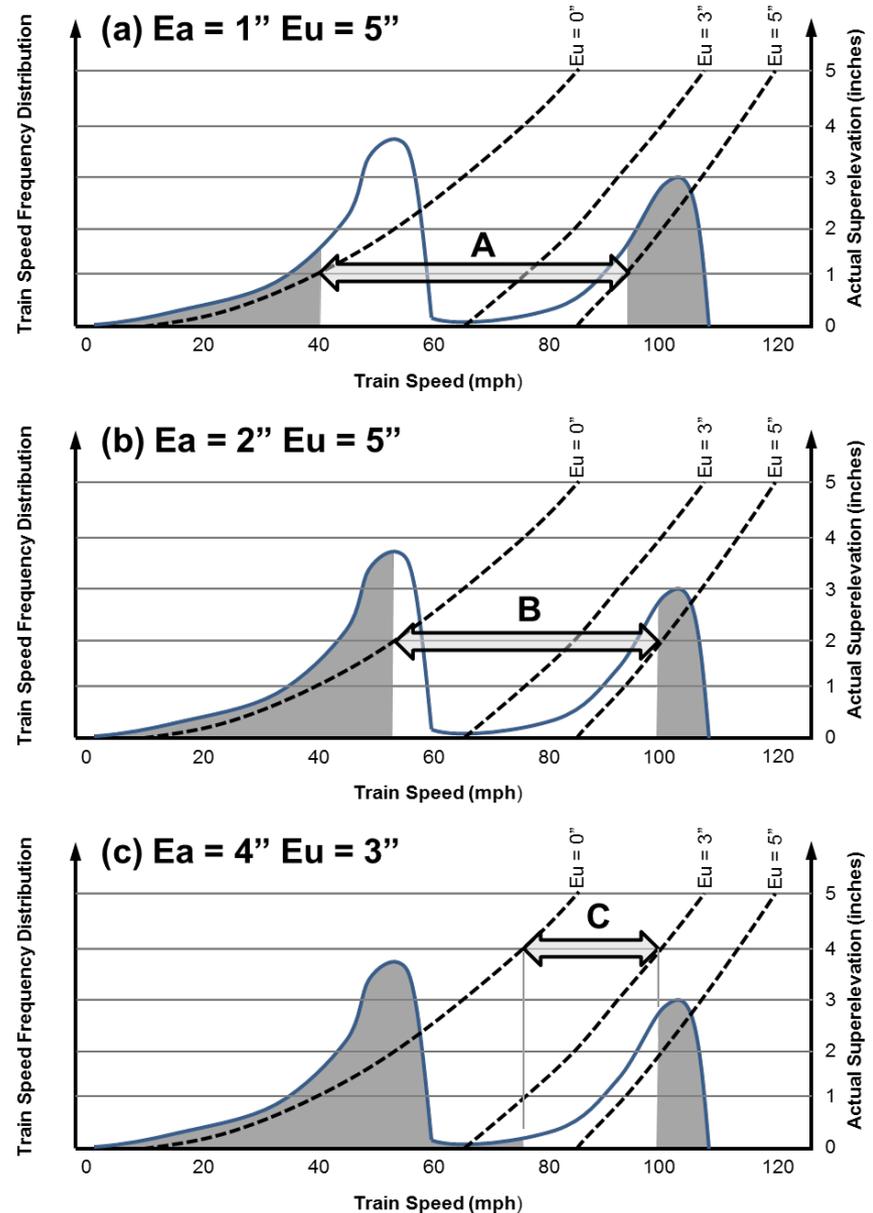
Speed Restrictions

- Passenger trains above max. unbalance



Comparing Different Designs

- Evaluate different combinations of E_a and E_u
- Select design to minimize rail traffic (MGT or passengers) falling outside bandwidth



Observations

- Bandwidth decreases as actual superelevation increases and unbalance decreases
- Bandwidth decreases for tighter curves (larger degree)
- May want to place different emphasis on maintenance (overbalance) vs. performance (speed restrictions)



Mathematical Optimization

- Limited combinations of E_a and E_u make graphical technique effective for single curve
- Developed a mathematical model to facilitate batch processing of multiple curves
- Divide freight and passenger train traffic into i groups by train speed
- Select optimal E_a and E_u to minimize MGT and passengers in speed groups outside bandwidth



Mixed Integer Quadratically Constrained Program (MIQCP) for Optimal Superelevation Design

$$\text{Minimize } Z = \beta \sum_i S_{fi} G_i + \gamma \sum_i S_{pi} T_i$$

(Minimize traffic outside superelevation bandwidth)

$$E_a + E_u = 0.0007 D V_{max}^2$$

(Max and min curve speed for selected E_a and E_u)

$$E_a = 0.0007 D V_{min}^2$$

$$V_{min} - V_i \leq S_{fi} V_{top} \quad \forall i$$

(indicate if outside bandwidth)

$$V_i - V_{max} \leq S_{pi} V_{top} \quad \forall i$$

$$E_{amin} \leq E_a \leq E_{amax}$$

(E_a and E_u design criteria)

$$0 \leq E_u \leq E_{umax}$$

$$S_{fi} = 0,1 \quad \forall i$$

(integer variables for freight and passenger speed groups)

$$S_{pi} = 0,1 \quad \forall i$$



Implementation

- Mathematical model requires selection of weighting parameters: β, γ
- Need to quantify:
 - Maintenance implications of overbalance MGT on given curve
 - Implications of increased running time due to speed restrictions on faster trains
- Former requires new research



Summary

- Superelevation on mixed-use corridors is complicated by range of train speeds and in-train forces on grades
- Bandwidth concept provides graphical framework to visualize trade-offs
- Optimization model has been formulated but requires research to better quantify effects of over and underbalance traffic



Thank you for your attention!

C. Tyler Dick, P.E.

Senior Research Engineer

Rail Transportation and Engineering Center (RailTEC)

University of Illinois at Urbana-Champaign

E-mail: ctdick@illinois.edu

This project sponsored by:



U.S. Department of Transportation

Federal Railroad Administration

Additional support from:

National University Rail Center
(NURail)

a USDOT-OST Tier 1

University Transportation Center



AREMA 2016 Annual Conference & Exposition

