Field Validated Discrete Element Model for Railroad Ballast

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Abstract:

This paper presents field validation results of a realistic railroad ballast model, developed recently at the University of Illinois based on the Discrete Element Method (DEM), with ballast settlement data collected from the Facility for Accelerated Service Testing (FAST) for Heavy Axle Load (HAL) applications at Transportation Technology Center (TTC) in Pueblo, Colorado. By addressing adequately the particulate nature of different sized and shaped ballast aggregate particles and their interactions with each other at contact points, the ballast DEM model provides a quantitative track performance simulation capability to conduct field applications and investigate various aspects of railroad ballast designs and behavior. For the field validation, four 100-ft test sections were constructed in early 2010 with four different aggregate materials used as the new ballast layer installations on a curve at the TTC FAST test track. During the ballast layer construction, settlement plates were also installed on top of subgrade in the middle and outside rail locations in order to measure deformations within the ballast. In particular, ballast layer settlements due to train loadings were measured to assess track surface degradation, track stiffness, and ballast breakdown. Each aggregate material, donated by one partnering US railroad company, was processed for gradation and imaging based shape, angularity and texture properties for generating the corresponding ballast DEM particles and performing the full-scale track DEM model simulations. Both field measured ballast settlements and the DEM model predictions are presented and compared in this paper. The settlement predictions due to the repeated train loading patterns applied indicate that the ballast DEM model could predict magnitudes of the field ballast settlements from both early loading cycles and over 90 MGT performance trends reasonably accurately. The ballast settlement predictions were sensitive to both aggregate shape and gradation. In addition, a proper assessment of the initial compaction condition (void ratio) of the constructed ballast layer was essential for accurately predicting field settlements. The DEM model can be used as a validated tool for engineering ballasted track designs and addressing critical substructure concerns such as those related to variable track stiffness and track transition zones.

Key words: Railroad ballast, gradation, aggregate shape, void ratio, settlement, Discrete Element Modeling, field validation

Introduction

A large portion of the annual budget to sustain railway track system goes into maintenance and renewal of track ballast. Railroad ballast is uniformly-graded coarse aggregate placed between and immediately underneath the crossties. The purpose of ballast is to provide drainage and structural support for the dynamic loading applied by trains. Aggregate type, size distribution (gradation) and particle shape, texture and angularity are among the major properties that impact the mechanical behavior of ballasted railroad track designs. Superior ballast aggregate shape properties, e.g., provided by angular crushed stone, have proven to be critical for ballast strength and stability. Ballast layers with large air voids that are more uniformly-graded have also produced higher permanent deformations under repeated train loading often due to a lower density (Tutumluer et al. 2009). Hence, for a better evaluation of the serviceability and proper functioning of the existing ballast layer, ballast strength, modulus and deformation behavior needs to be characterized in the laboratory and then linked to field performance by means of a realistic and robust modeling capability that would establish the basis of a quantitative track performance simulation tool.

With the objective to provide better engineering insight into the design of ballasted track for improving railroad safety and network reliability, recent Association American Railroads (AAR) Technology Scanning Program research at the University of Illinois has developed a ballast performance model based on the Discrete Element Method (DEM) which uses rigid but random shaped 3-dimensional (3D) “polyhedrons or blocks” as the basic elements to realistically simulate interactions such as interlock/contact of actual aggregate particles (Tutumluer et al. 2006, 2007, 2009, and Huang et al. 2009, 2010). The ballast DEM model requires as input imaging based aggregate size and shape quantifications. Among the various particle shape/ morphological indices, the flat and elongated (F&E) ratio, the angularity index (AI), and the surface texture (ST) index, all developed using University of Illinois Aggregate Image Analyzer (UIAIA), are key indices (Rao et al. 2002, Pan et al. 2006). The UIAIA system features taking images of an individual aggregate particle from three orthogonal views to quantify imaging based F&E ratio, AI, and ST morphological indices. The image-aided DEM approach then recreates the 3D aggregate shapes as individual discrete elements based on the UIAIA scanned images (see Figure 1).

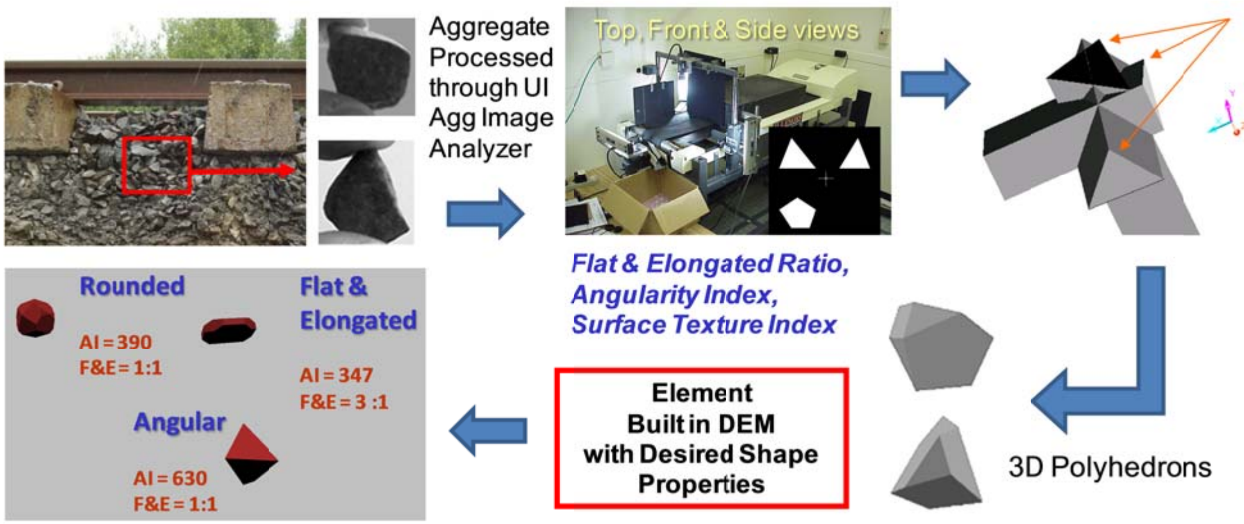


Figure 1 Aggregate Imaging based Railroad Ballast Discrete Element Modeling

The ballast DEM model was calibrated in our early research efforts with laboratory direct shear (shear box) ballast strength test results and then utilized to model strength and settlement behavior of railroad ballast for the effects of multi-scale aggregate morphological properties and ballast fouling influencing track performance (Tutumluer et al., 2006, 2007, and Huang et al., 2009, 2010). Tutumluer et al. (2009) recently investigated the effect of gradation, including those of AREMA gradations currently in use, on ballast settlement and drainage using the ballast DEM model. Following the concept of 0.45-power maximum density gradation charts, “characteristic gradation curves” were generated for different minimum aggregate sizes and the commonly used AREMA gradations such as No. 4 and No. 24. According to the DEM methodology employed, there was certainly room to further engineer current specifications by optimizing the gradation for a minimum allowable particle size of 1.4 in. in a new gradation, which still accommodated large enough air voids for drainage and minimized the overall settlement potential of the ballast layer.

This paper describes field evaluation of the ballast DEM model with ballast settlement data collected from the Facility for Accelerated Service Testing (FAST) for Heavy Axle Load (HAL) applications at Transportation Technology Center (TTC) in Pueblo, Colorado. Four different ballast materials donated by AAR member railroads for the field performance study are presented first for the gradation and imaging based aggregate shape properties. Next, field test section designs, compaction/construction, and testing conditions are discussed in detail to emphasize the most pertinent inputs needed for establishing the full-scale DEM model simulations considering the typical track geometry and dynamic train loadings. Finally, DEM model settlement predictions are compared with field measured results for four ballast test sections in an effort to demonstrate the prediction ability of the aggregate imaging based ballast DEM modeling approach and establish field validation.

Field Ballast Test Sections

The field ballast performance study was conducted in Section 3 of the TTC FAST High Tonnage Loop (HTL) under heavy axle loading (39 tons per axle) conditions, as shown in Figure 2. There were four test zones constructed in early 2010 with different ballast materials donated by AAR member railroads, BNSF, CSX, Norfolk Southern and Union Pacific. The ballasts were randomly designated as Railroad 1 to Railroad 4 (RR1 to RR4) in this paper, installed as new ballast layers on a 5-degree curve. Each test section had 80-ft long test zone and 20-ft long transition zone between two test zones to eliminate the influence of different test zones as configured in Figure 3. Since Section 3 of the FAST track was curved track, an average of 4.24

in. superelevation was achieved during construction. The average thickness of ballast layer constructed was around 14 in.

The following performance measures in particular were of interest: permanent deformation of the ballast layer; track surface degradation; and ballast breakdown. The field tests allowed measurement of ballast vertical settlement in each test section over time at two locations using subgrade settlement plates. To more accurately measure the settlements of the ballast layer and the subgrade, respectively, three settlement plates (see Figures 4 and 5) were installed in the middle of the rails, at field side, and at gauge side of the track for every location field measurements were taken.

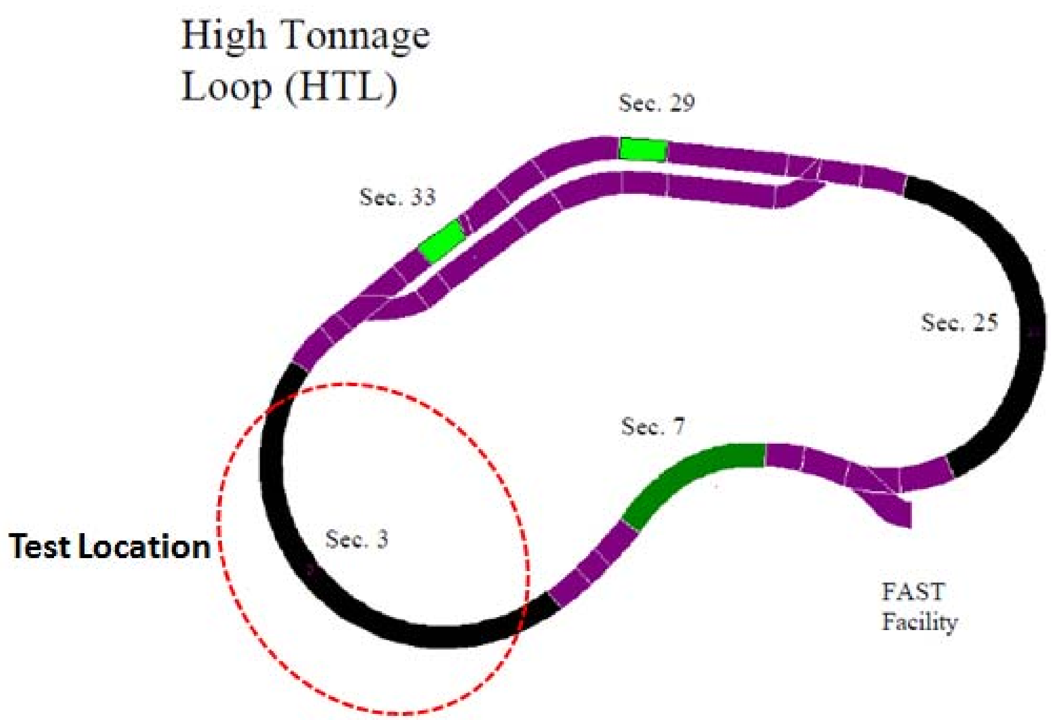


Figure 2 Field Test Locations at the TTC FAST Track in Pueblo, Colorado

Figure 3 Ballast Test Sections Constructed in Section 3 of the High Tonnage Loop

Figure 4 Photo Showing Settlement Plate Installations

Figure 5 Photo Showing the Final Constructed Ballast Test Section

Ballast Material Properties

The ballast materials donated by all four AAR member railroads were clean granite type 100% crushed aggregates. Figure 6 shows size distributions of all the ballast materials studied. The three ballast materials RR 1, RR 2, and RR 3 had gradations that complied with the AREMA No. 24 requirements whereas the RR 4 ballast had a large proportion of the total sample as 1-1/2 inch size particles. As a result, RR 4 had fewer particles smaller than 1-1/2 inch size than are required for an AREMA No. 24 gradation. This ballast gradation was closest to being a single size. On the other hand, the ballast material donated by railroad 2 (RR 2) had the smallest proportion of 1-1/2 inch particles and a wider distribution of particle sizes.

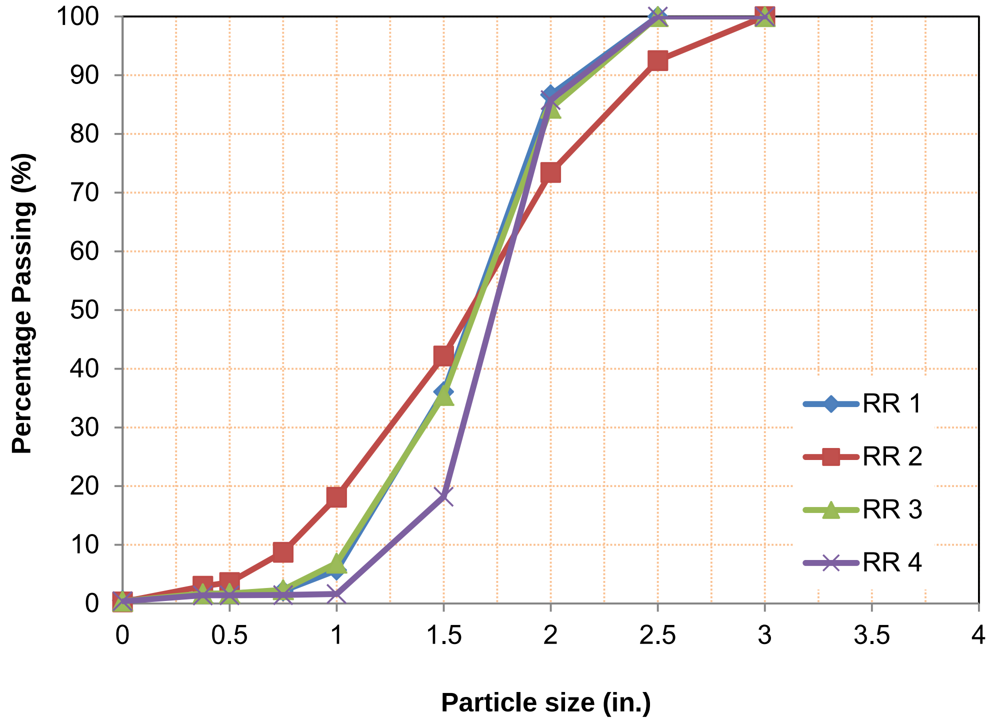


Figure 6 Gradations of the Four Ballast Materials Studied

Besides gradation, aggregate shape properties, especially the flat and elongated (F&E) ratio, the angularity index (AI), and the surface texture (ST) index, are key indices quantified by the University of Illinois Aggregate Image Analyzer (UIAIA). One full bucket of each ballast material was scanned and analyzed using the UIAIA to determine the average values of the F&E ratio, AI, and the ST index. These shape indices were then used to create the aggregate particles as discrete elements in the ballast DEM model (see Figure 1). Table 1 lists the details of shape properties of each granite type ballast material used in this field test study. The ballast materials donated by RR 1 and RR 4 had both high angularity (AI) and high surface texture (ST). Ballast material from RR 3 had more rounded particles (lower AI) and high surface texture (ST). Ballast material donated by RR 2, had both high AI and ST, however, this material had the largest flat and elongated (F&E) ratio.

Table 1 Ballast Material Characteristics

Test Section	Angularity Index	Surface Texture Index	Flat & Elongated Ratio	AREMA Gradation
RR 1	584	2.3	2.2	No. 24
RR 2	590	2.5	3.5	No. 24
RR 3	461	2.2	2.6	No. 24
RR 4	509	1.8	2.3	No. 24*

* Does not meet all gradation requirements

Ballast Compaction (Initial) Condition

The level of field compaction or achieved density influences ballast deformation behavior significantly. Similarly, the density (or void ratio) of the simulated ballast layer in the numerical model dictates the predicted settlement results. It is a required input and establishes initial conditions for the ballast DEM model used in any full-scale track loading simulations. However, an appropriate and convenient method to quantify the ballast compaction level or density in the field is not readily available.

To study the appropriate ballast compaction conditions in the field, preliminary settlement data were obtained from a ballast test section constructed and tested in Section 40

Tangent Line at the TTC FAST track. The ballast material used in the Section 40 test section was the same RR 1 aggregate. Accordingly, a half-track simulation was established using the ballast DEM model for the known ballast material properties and track geometry data (see Figure 7). Since the tangent section was symmetric in both geometry and loading conditions with no superelevation, the half-track model saved time and computational resources. The ballast layer in the DEM simulation was compacted and prepared to study how different initial conditions, i.e., initial void ratios, influenced settlement predictions. By comparing the predicted settlements obtained from the DEM simulations for up to 1,000 train loadings, it was found that an initial void ratio of 37% yielded the closest results to the field ballast settlements measured by both the settlement plates and top of the rail measurements.

Figure 8 presents the predicted results and the field measurements of the ballast settlement in Section 40. The ballast in Simulation I was compacted to an initial void ratio of 37% and allowed slight rebound of the ballast layer, which caused an aggregate rearrangement after the compaction force was eliminated. The ballast in Simulation II was compacted to a void ratio slightly less than 37%, which compensated for the rebound after the compaction force was eliminated. Figure 8 shows both predictions from Simulations I and II to be close to the field measurements, especially the initial 200 passes of car loading. When the field test and the numerical simulation had identical ballast compaction (initial) condition and material (gradation and shape) properties, the settlement predictions were quite similar. However, even slight differences in the initial conditions, i.e., Simulations I and II, were found to significantly influence the particle rearrangement and settlement trends with increasing load passes. Since the DEM simulations did not consider particle breakage or particle size degradation with load passes, the settlement predictions from Simulations I and II, shown in Figure 8, either indicate increasing-decreasing-increasing trends (particle reorientation with no edge breakage) or gradually increasing trends when compared with the field measured values, respectively.

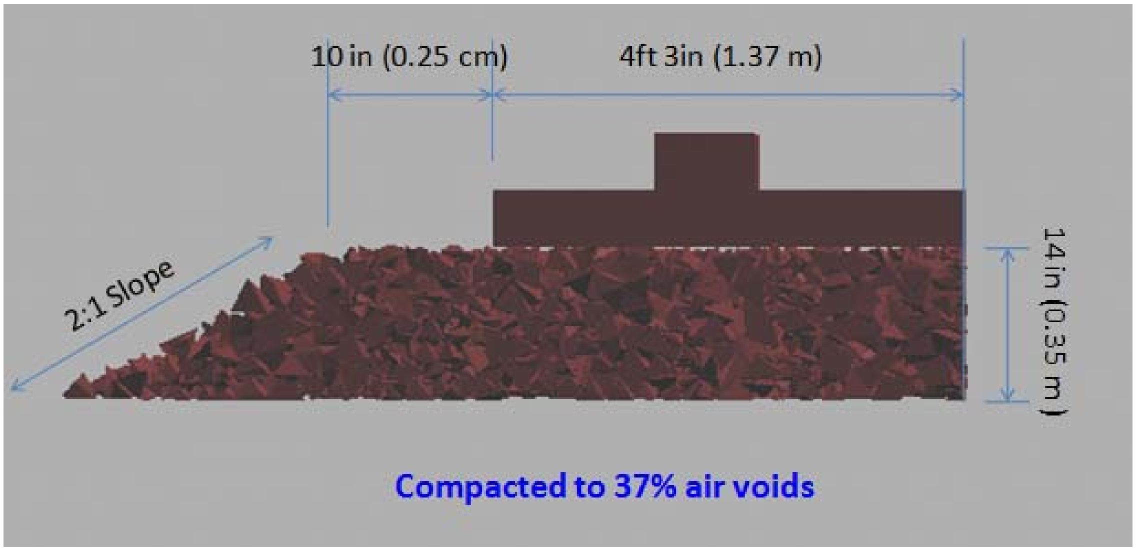


Figure 7 Half-track Simulations for the Section 40 Tangent Line (RR 1 Ballast)

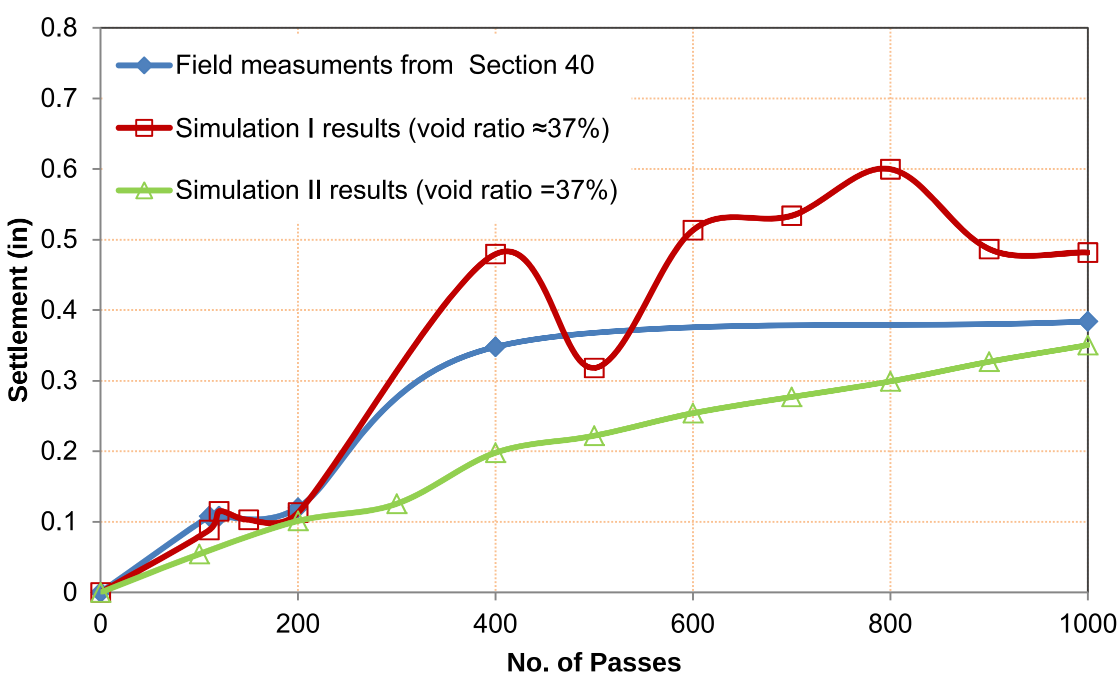


Figure 8 Measured and Predicted Settlements for Section 40 Tangent Line (RR 1 Ballast)

As discussed above, ballast compaction (initial) condition is quite important to know in the DEM simulations for predicting accurately the field settlement magnitudes. However, an appropriate and convenient method to quantify the ballast compaction level or density in the field is not readily available. To overcome this difficulty and evaluate the compaction levels of the ballast layers constructed in the different test zones, an open metal box, 12 in. by 14 in. and 6-in. deep, was placed on the subgrade during the construction of the RR 1 ballast layer according to the standard field practice, i.e., compactive effort (see Figure 9). The box was then recovered and the total weights of the ballast materials inside the box before and after compaction were measured. Using this approach, the compacted density (void ratio) of the ballast layer in RR 1 test zone could be computed.

Figure 9 Metal Box Used to Measure Field-Compacted Aggregate Weight to Determine Field Compaction Condition (density or void ratio)

To adequately determine the initial conditions of the ballast layers in other test zones, laboratory compaction tests were conducted. First, the same field-compacted weight amount of RR 1 ballast material in the metal box was again compacted to fully fit in the box using a vibratory compactor in the laboratory. The time it took the vibratory compactor to accomplish this task was recorded. A similar level and duration of compaction (same compactive effort) was then applied by the same operator to compact other ballast materials in the box using the same vibratory compactor. By analyzing the aggregate weights packed in the box, the void ratios were computed for all the ballast materials to account for any discrepancies in the initial compaction field conditions.

Although the above described approach worked well in general, the calculated void ratios still could not be used directly as the input initial conditions for the ballast DEM model. This is because in the DEM model all the particles created are solid particles without any fractures or permeable (external) voids. However, many aggregate particles found among the four ballast materials used in the test zones were observed to have fractures and porous surfaces on the outside; especially this was the case with the RR 1 ballast material (see Figure 10). Accordingly, all the calculated void ratios from the laboratory compaction tests had to be adjusted using the measured aggregate specific gravities to account for the porous surfaces. Table 2 lists the void ratios used as initial condition input for the ballast DEM model. Note that the field void ratio for the RR 1 compacted ballast is given as 44% in Table 1. However, for the half-track DEM model of Section 40 study, when the RR 1 ballast material was compacted to a void ratio of 37%, the numerical simulation yielded the closest settlement predictions to the field measurements. Accordingly, both 44% and 37% void ratios were selected to prepare the RR 1 ballast layer in the Section 3 curved track DEM simulations.

Figure 10 RR 1 Ballast Aggregates with Porous Surfaces

Table 2 Initial Compaction Conditions used in Section 3 Curved Track DEM Simulations

Ballast Material Source	Void Ratio in DEM
RR 1	44%
RR 1 (from Section 40 study)	37%
RR 2	32%
RR 3	37%
RR 4	45%

Full-Track DEM Simulations

Five full-track DEM simulations models were established according to the track geometry data of the field ballast structures built and tested in Section 3 curved line at the TTC FAST track. A front view of the DEM model is given in Figure 11. Each full-track DEM simulation had approximately 13,000 individual particles that established the 14-in thick ballast layer with around 4-in. superelevation in the field side of the track structure. The aggregate particles used in the DEM models were created according to the sieve analysis results and the imaging based shape indices of the ballast materials from different test zones. The crosstie used in the simulations was a typical tie size used in North America, 8 ft 6 in. (2.591 m) long, 8 in. (0.203 m) wide, and 7 in. (0.178 m) deep. The ballast layers in the DEM simulations were compacted to the void ratios listed in Table 2 with a 2:1 slope used for shoulders on both sides. After the DEM simulations were prepared for the initial compaction conditions (void ratios), a dynamic train loading pattern, derived recently from a realistic "Sandwich Model" by Huang et al. (2009), was applied to simulate the dynamic loading caused by the 315-kip rail car traveling at a speed of 45 mph. Figure 12 shows the 4-peak moving wheel pulse loading applied with a rest period which was considered as one load pass in the repeated train loading DEM simulations.

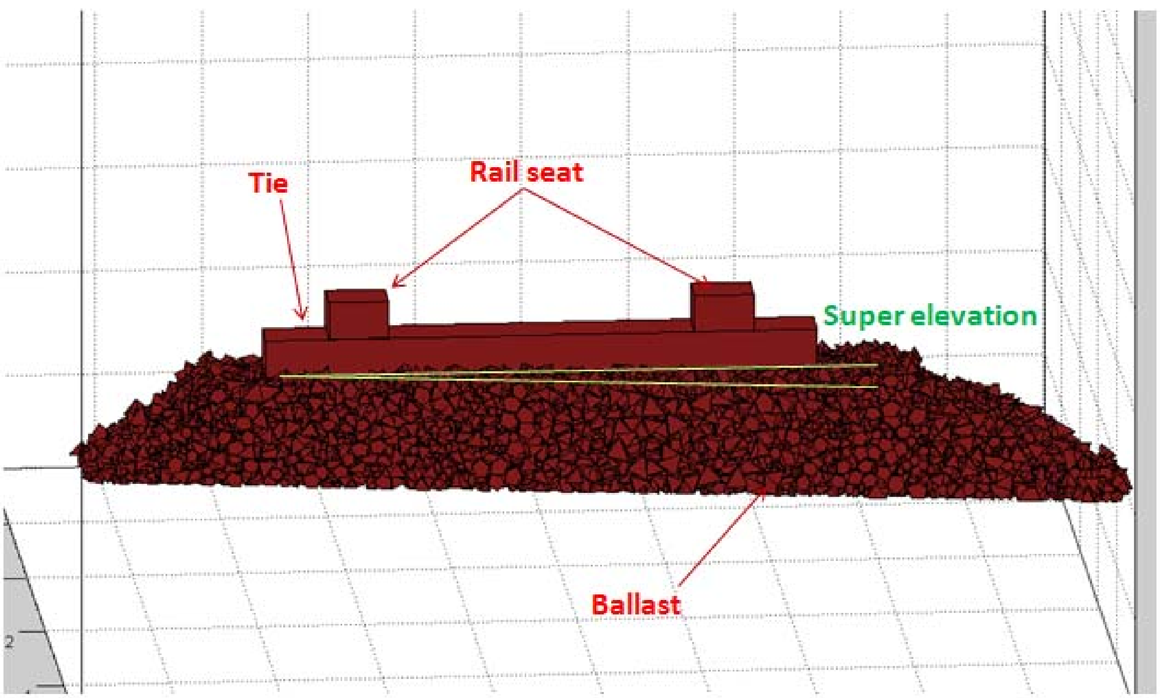


Figure 11 Front View of Full-Track DEM Simulations

Field Test and DEM Simulation Results

The field tests were conducted by Transportation Technology Center, Inc. (TTCI). Figure 13 shows the average total settlements accumulated with increased tonnage for each ballast section for up to 90 MGTs. Note that the ballast material donated by RR 2, shown with the highest settlements in Figure 13, also had the most flat and elongated particles prone to particle breakage. The ballast material donated by RR 3 is shown with the lowest settlement in Figure 13, which may be primarily attributed to the more rounded (low AI) nature of the RR 3 ballast material having the least tendency to crush particles. The field test results agreed with earlier studies on the influence of aggregate shape properties on ballast performance (Tutumluer et al. 2007, Huang 2010).

Settlement plates installed on top of the subgrade were used to determine how much settlement was occurring in the foundation below the ballast layer and accordingly, the settlement within the ballast could be computed from the top of rail measurements. Figure 14 indicates that the major contribution of the track settlement was in fact from the ballast layer. In this field test, the subgrade accounted for about 10% of the total settlement as presented in Figure 14.

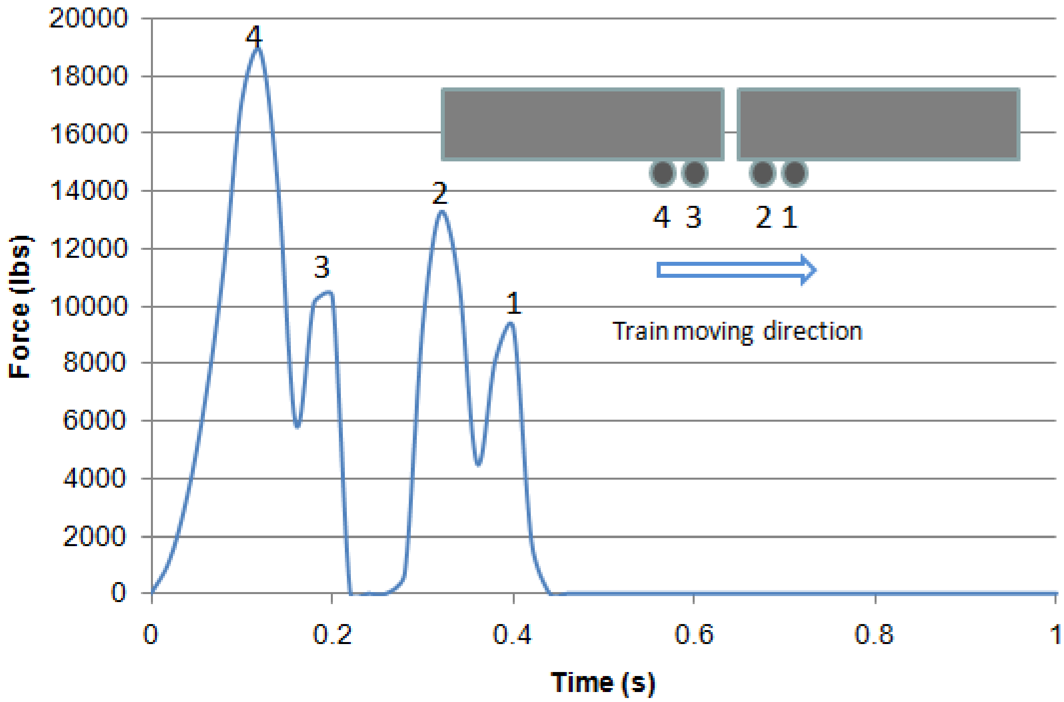


Figure 12 Dynamic Loading Pattern for a 315-kip Car Traveling at 45 mph

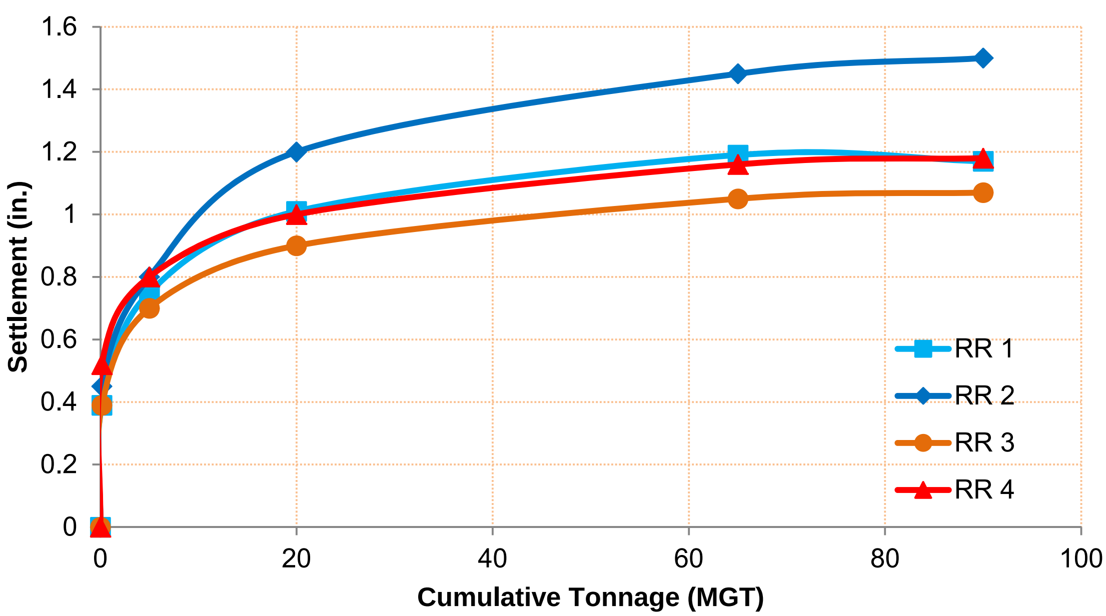


Figure 13 Field Test Results of Ballast Settlement graphed with Cumulative Tonnage

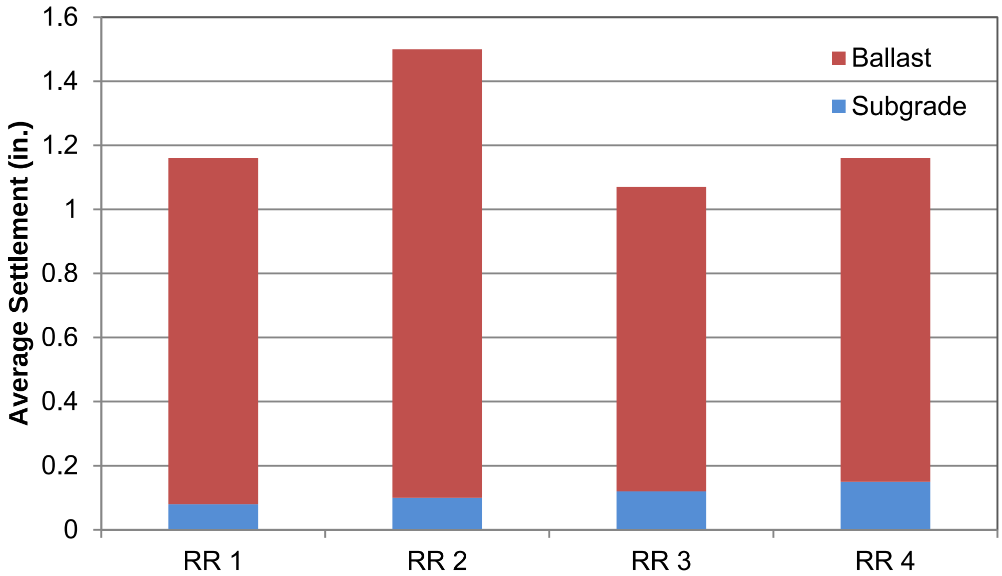


Figure 14 Field Test Results for Ballast Layer and Subgrade Settlements

Additionally, the lateral stability performance of each test zone was assessed using a single tie push test. This test gives a measure of the lateral stiffness of the track panel. It measures the lateral force needed to move a crosstie through the ballast. The average of two ties is reported for each test section. The test results are summarized in Figure 15. The zone having the RR 2 donated ballast had the largest lateral strength despite the largest settlement (see Figure 13).

Note that due to the significantly large amount of aggregate particle contact forces computed and checked for global granular assembly equilibrium at each iterative time step, the full-track DEM model could not simulate the same amount of loadings as the field tests did with limited time and computational resources available. Although the field tests applied over 90 MGTs, the DEM simulations could only be finished for up to 2,000 car passes, which took approximately 5 months and equaled to around 0.32 MGTs. Figure 16 shows the ballast DEM model settlements predicted in each test zone with the number of car passes. The DEM simulations predicted the track with the RR 3 ballast material to have the lowest settlement, which is in agreement with the field observed trends. This can be primarily attributed to the more rounded (low AI) nature of the RR 3 ballast material having the least tendency to crush particles. A similar, more compact ballast layer packing by rounded particles was also observed to yield low settlements in an earlier modeling effort by Tutumluer et al. (2007). The RR 1 ballast with a void ratio of 37% performed better than the 44% one in the DEM simulations. Note that the ballast DEM model so far cannot accommodate particle breakage in simulations and hence could not predict the much higher settlements observed for the RR 2 ballast which had more flat and elongated particles.

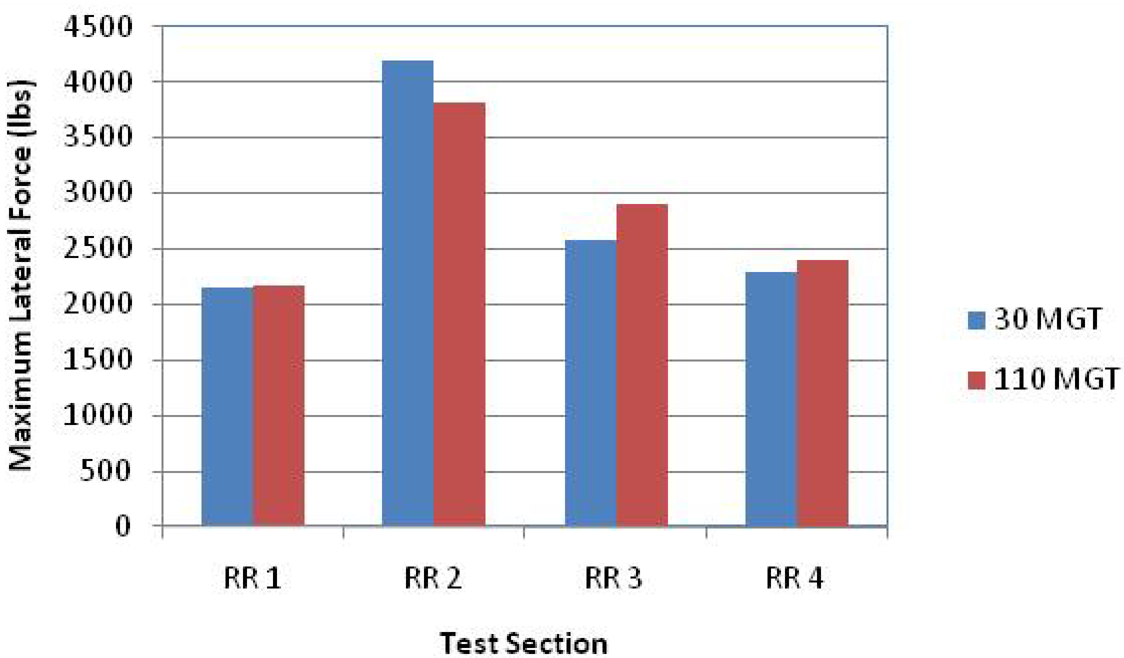


Figure 15 Lateral Strength Test Results

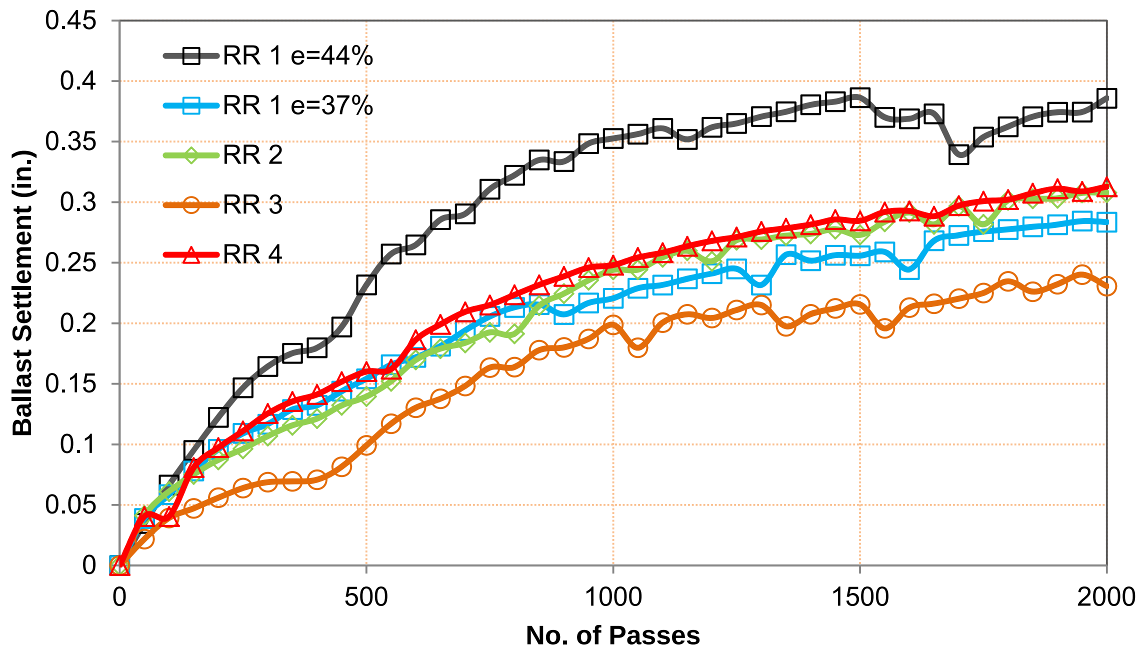


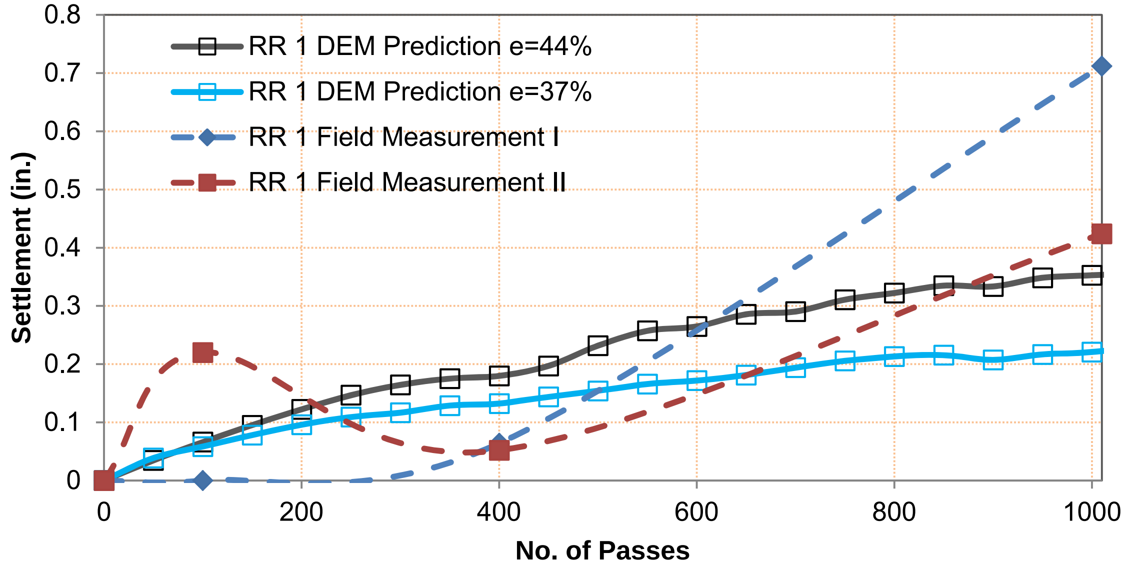
Figure 16 DEM Ballast Settlement Predictions for up to 2,000 Car Passes

Utilizing the predicted settlement data for only up to 2,000-car passes (around 0.32 MGTs), DEM settlement prediction models were developed based on regression analyses to extrapolate the settlement trends and predict the long-term performance of the ballast test zones. Table 3 lists the developed settlement prediction models and the DEM predicted long-term ballast settlements, which in general compare favorably to the field measurements at 90 MGTs.

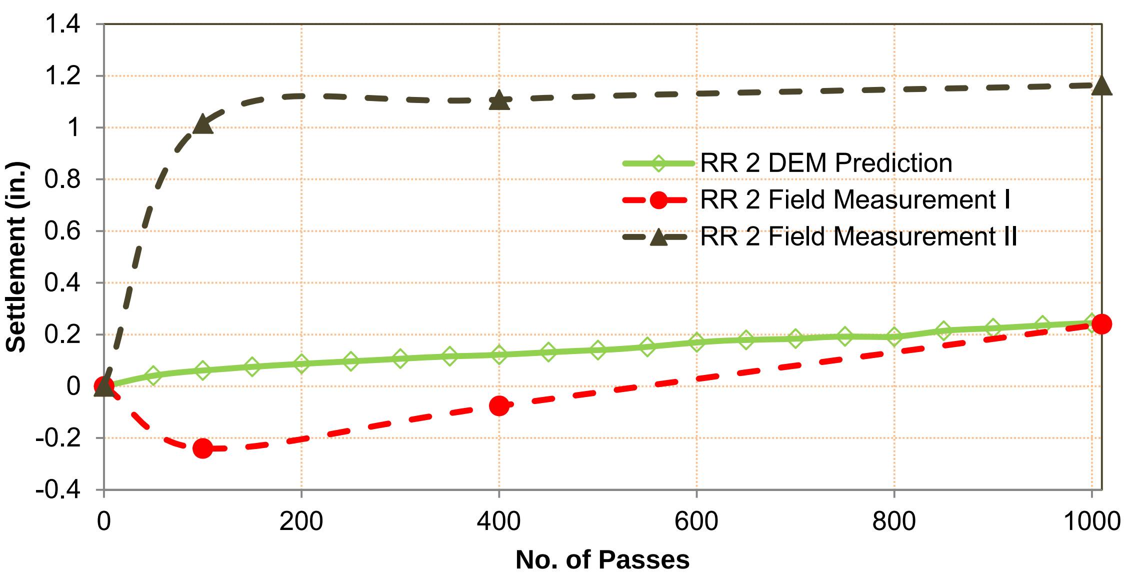
Figure 17 compares the DEM predicted settlements with the field measurements in two locations for only up to the first 1,000 car passes. The predicted settlements increase always gradually in the DEM simulations due to the better control of compaction and loading when compared to the field measurements, which show sudden increases and often heaves due to unstable field shakedown conditions. Note that the field measurements are sometimes in agreement with the DEM predictions, e.g., Figures 17 (a) and (c), but also, the field measurements can be quite surprising when compared to the DEM predictions in other cases, such as in Figures 17 (b) and (d). For example, Figure 17 (d) indicates heave measured in both rail locations; this is somewhat unexpected even on a 5-degree curved track. In addition to the difficulties in maintaining uniform compaction/construction for ensuring proper test zone track geometries, the existing superelevation of the curved track would definitely influence the settlement characteristics on both sides of the track due to uneven loading and lateral forces applied in the rails. Meanwhile, in the DEM simulations, the loading was applied evenly onto the two rail seats and there was no lateral force applied to track substructure, which eliminated some random factors and yielded gradual settlement accumulations predicted in the ballast.

Table 3 Field and DEM Settlement Prediction Results

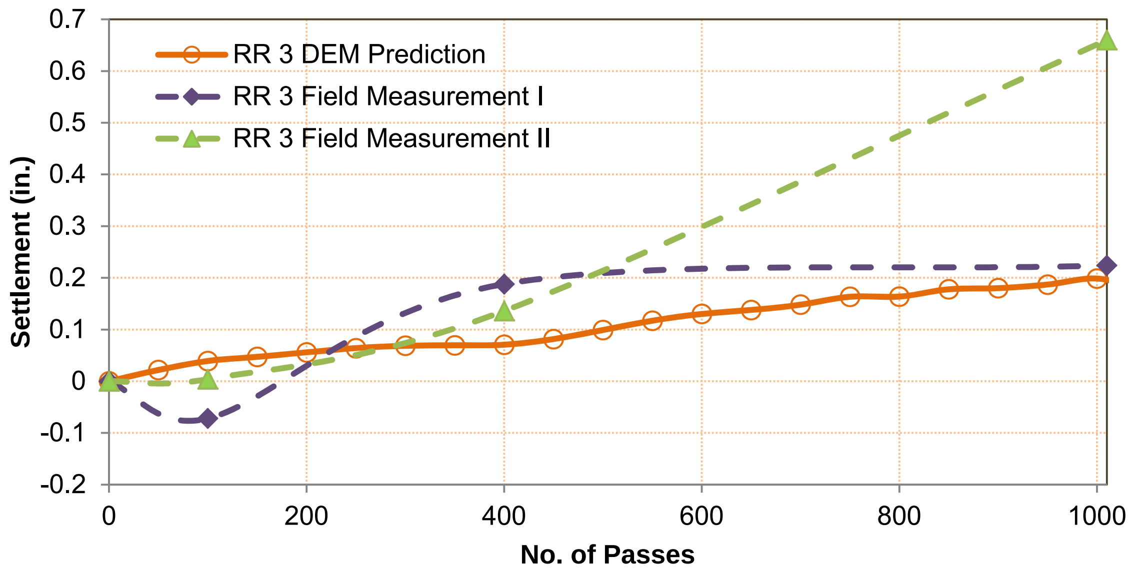
	DEM Settlement Prediction Models (N = No. of Passes)	**DEM Predicted at 90 MGTs (in.)**	**Field Measured at 90 MGTs (in.)**
RR 1	$S = 0.74N^{0.29}$ $R^2 = 0.93$ $e = 37\%$	1.395	1.446
RR 1	$S = 1.44N^{0.24}$ $R^2 = 0.85$ $e = 44\%$	1.438	1.446
RR 2	$S = 0.64N^{0.32}$ $R^2 = 0.91$ $e = 32\%$	1.566	1.768
RR 3	$S = 0.62N^{0.30}$ $R^2 = 0.92$ $e = 37\%$	1.430	1.251
RR 4	$S = 0.84N^{0.29}$ $R^2 = 0.88$ $e = 45\%$	1.505	1.466



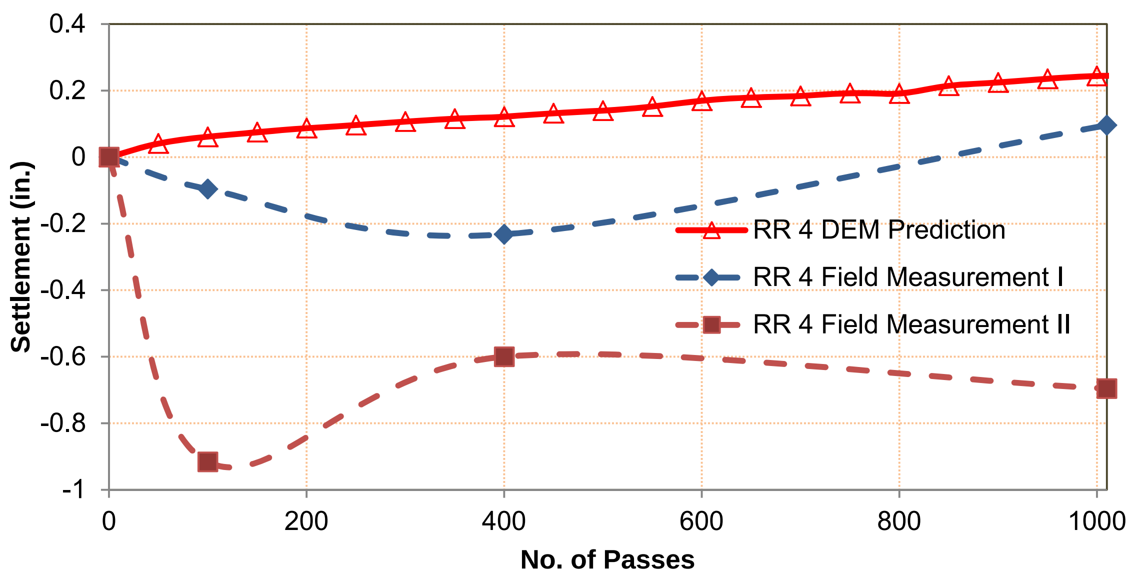
(a) Test Section with RR1 Ballast Material



(b) Test Section with RR2 Ballast Material



(c) Test Section with RR3 Ballast Material



(d) Test Section with RR4 Ballast Material

Figure 17 Detailed Ballast Settlement Results for the Initial 1,000 Car Passes

Summary and Conclusions

Numerical simulations of railroad track ballast settlements were conducted in this study utilizing a ballast performance model developed at the University of Illinois based on gradation and image analyses of individual aggregate particles for shape, texture and angularity indices and the Discrete Element Method (DEM). To validate the ballast DEM model with the field settlement data, four ballast materials donated by Association of American Railroads (AAR) member railroads were used to construct ballast test zones at the Facility for Accelerated Service Testing (FAST) for Heavy Axle Load (HAL) applications at Transportation Technology Center (TTC) in Pueblo, Colorado. The four ballast materials had different imaging quantified aggregate shape indices and accordingly, accumulated settlements differently. Further, the superelevation in the curved track caused uneven measured settlements of the two rails and made it hard to predict ballast settlement behavior in the initial loading stages.

The ballast DEM model generated the corresponding ballast particles as discrete element based on the four different ballast materials with varying aggregate shape, texture and angularity properties and performed numerical simulations of the full-scale curved track test zones under realistic heavy axle train loadings. By properly accounting for the initial compaction conditions, the ballast DEM simulations closely predicted the lowest settlement performance of one of the ballast materials with only 2,000 car passes investigated. The test section with more flat and elongated particles had the most particle breakage and degradation which contributed to the highest field settlements. The ballast DEM model currently does not consider particle breakage. This is a future research area to enhance and fully develop the ballast DEM model as a performance prediction tool.

Results from the dynamic, repeated train loading simulations indicate that the ballast DEM model could predict magnitudes of the field ballast settlements over 90 MGT performance trends reasonably accurately. The ballast settlement predictions were sensitive to both aggregate shape and gradation. In addition, ballast initial compaction condition (density or void ratio) played a very important role in ballast performance predictions and it is a key input for DEM simulations. The ballast DEM model has been successfully validated using the field settlement data for predicting ballast deformation behavior under realistic train loading. The ballast DEM model has the potential use as a tool for engineering ballasted track designs and addressing critical substructure concerns such as those related to variable track stiffness and track transition zones.

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