Statistical Prediction of Center Negative Bending Capacity of Pretensioned Concrete Crossties

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Abstract: When concrete railway crossties are installed in North American freight track and subjected to flexural loads, center negative bending is one of the most critical demands. However, the ultimate flexural capacity at the crosstie center is often unknown and hard to obtain. Because railroads do not always know what the remaining flexural capacity of concrete crossties is, it becomes difficult to assess whether crossties should be removed from service or if it is safe to increase axle loads as an example. To address this challenge, we present a predictive mathematical model based on laboratory experimentation data of various common pretensioned concrete crosstie designs to estimate their center negative bending strength. The model is developed using least absolute shrinkage and selection operator (LASSO) techniques. The final derived equation uses predictor variables that are easily interpreted and applied, and the results are adequate for approximations when limited information is available about the crossties' characteristics and lengthy structural calculations or additional laboratory testing is not practical. For the investigated crosstie designs, the maximum prediction error was 5.5%. **DOI: 10.1061/JTEPBS.0000313.** *© 2019 American Society of Civil Engineers.*

Introduction

Freight railroad lines in the United States rely on crossties to support rail seat loads and hold proper track gauge. Although these track components have been traditionally made from timber, the use of concrete has increased after prestressed concrete crossties were developed in the late 1950s as a competitive alternative for high-speed or heavy axle load (HAL) corridors (Weber 1969). Currently, most prestressed concrete crossties are made with high-strength concrete and bonded pretensioned steel tendons (wires or strands). Used as beams, the most critical structural demands that concrete crossties must resist are flexural loads.

There are two critical locations on concrete crossties that should be examined during the design process: the crosstie center and the section underneath the rail (rail seat) (Wolf et al. 2014). These sections are often designed to be uncracked when subjected to service loading conditions, which often leads to designs with high stiffness and significant reserve capacity with respect to their ultimate condition (Bastos et al. 2018). However, the ultimate flexural capacity at the crosstie center is often unknown and difficult to obtain by railroads because it requires the use of structural formulae using design characteristics and material properties are not always readily available. Even if the necessary data from the design stage were available, the calculations would still be rough estimates because design equations are conservative and concrete strength and prestress losses change over time, and crosstie geometry changes due to inherent deterioration. Thus, it is difficult for railroads to know the reserve flexural capacity of their concrete crossties, leading to suboptimal maintenance interventions and interruptions to train operations. Moreover, not knowing the actual flexural capacity of crossties can unnecessarily limit axle loads transported over them or result in overspending on crossties that have excessive capacity beyond a reasonable safety factor.

To provide a quick and simple tool to estimate ultimate strength, this paper develops a statistical model based on regression analysis of previous results of laboratory experimentation. Given that center cracking is a critical problem for concrete crossties, and the fact that rail seat cracking is uncommon (Bastos et al. 2015; Van Dyk 2014), this paper focuses on the crosstie center. Center positive bending is a rare case in railroad track. In reality, crossties are often subjected to center binding support conditions, which generate center negative bending moments (Bastos et al. 2015, 2017). Thus, only center negative bending is investigated, which is the most common and critical field condition that governs crosstie designs at this cross section (Bastos et al. 2017).

Previous Experimentation

The study presented in this paper is a regression analysis using data from previous laboratory experimentation reported by Bastos et al. (2018), in which concrete crossties of eight different designs were subjected to four-point bending (Fig. 1) and loaded to ultimate flexural capacity in center negative bending. Most of the tested crosstie designs had pretensioned strands or indented wires that rely on bonding to transfer prestress loads to the concrete matrix. However, one design had anchoring end-plates, and another used posttensioned wires. Because of their different technologies, the results of these two designs are not used in this study.

The key characteristics and replicates used for each crosstie design (Table 1) serve as a proxy for material property and crosstie

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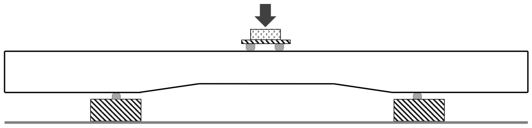

Fig. 1. Four-point bending configuration for center negative flexural testing.

geometry ranges where the regression model is most reliable, as extrapolation is not recommended. The design numbers used in this study is consistent with what has been reported previously (Bastos et al. 2018), which is the reason for omitting Designs 4 and 6 (excluded from this investigation). All crossties were made with high strength concrete, with compressive strengths greater than 7 ksi (48 MPa) (Bastos et al. 2018).

Regression Analysis

To generate a simple mathematical equation to predict the flexural capacity of concrete crossties using simply bonded pretensioned steel tendons, the regression analysis will prioritize prediction of flexural capacity over interpretation of the fundamental science behind it. Thus, the regression model should not be interpreted as a description of causation, but only of correlation among the predictor variables and the response variable. For a more fundamental approach, the field of structural engineering and prestressed concrete provide physical explanation to address this problem through more laborious calculations.

A second-order linear model will be investigated, which has the general form of Eq. (1) (refer to the Notation section for description of terms)

$$y_i = \beta_0 + \sum_{j=1}^{p}(\beta_j x_{ij} + \beta_{jj} x_{ij}^2) + \sum_{j=1}^{p-1}\sum_{k=j+1}^{p}(\beta_{jk} x_{ij} x_{ik}) + \varepsilon_i \quad (1)$$

Response Variable

The response variable (dependent variable) to be modeled is the ultimate center negative bending moment of North American pretensioned concrete crossties, with the restrictions that they rely entirely on bonding of steel tendons to transfer the prestressing force to the concrete. In this paper, the negative sign of center negative bending moments is omitted for simplicity. It is important to clarify that the ultimate bending moment referred to in this paper corresponds to the case of a stress-controlled loading scenario. This would still correspond to the maximum bending moment obtained for cases of strain-controlled loading, but failure would ultimately occur with higher curvature and lower bending moments (postpeak behavior).

Table 2. Notation of independent variables

Description	Statistical notation	Abbreviation
Gross moment of inertia	x_{i1}	I
Number of tendons	x_{i2}	t
Ratio of steel in center cross-sectional area	x_{i3}	ρ
Center cross-sectional area	x_{i4}	A
Cross-sectional height at center	x_{i5}	h
Tendon type	x_{i6}	w
Total tendon bonding area	x_{i7}	A_b

Independent Variables

The independent variables considered for this regression analysis are all readily obtainable properties. These are seven types: tendon type (categorical variable); number of tendons; ratio of steel in center cross-sectional area; center cross-sectional area; cross-sectional height at center; total tendon bonding area; and gross moment of inertia (Table 2). Tendon type is a binary indicator to differentiate indented wire (one) from seven-wire strand (zero), a relevant distinction given that it has been found that beams with strands have different bending performance than those with wires (Momeni 2016). The second variable, number of tendons, is a proxy for the magnitude of prestressing force that is applied to the crosstie. In addition to the number of tendons, the ratio of steel in the center cross-sectional area is also included. This is calculated by dividing the total area of steel in a cross section by the gross cross-sectional area and it is a normalized way of indicating how much steel is used, also commonly referred to as volumetric ratio.

The center cross-sectional area and height represent the geometry of a given design. The height of the section accounts for how distant the cross-sectional forces can be from the neutral axis and it is expected to correlate with the relative position of concrete stress blocks used in ultimate design procedures, such as those developed by Whitney (1937) and Hognestad et al. (1955). The sixth independent variable is the total tendon bonding area, which is the total area of contact between the tendons and the concrete matrix—a function of tendon diameter and geometry, the number of tendons, and the length of the crosstie. The last considered variable is the gross moment of inertia of the cross section about its bending axis, which is both a geometric property and an indication of stiffness. For sake of abbreviation, shorthand notations were created for each independent variable, as indicated in Table 2.

Table 1. Replicates, characteristics, and center cross-sectional properties of investigated concrete crosstie designs

Design	Replicates	Tendon type	Number of tendons	Ratio of steel in cross-sectional area	Cross-sectional area in.^{2}	Cross-sectional area mm^2	Cross-sectional height in.	Cross-sectional height mm	Total tendon bonding area in.^{2}	Total tendon bonding area mm^2	Gross moment of inertia in.^{4}	Gross moment of inertia mm^4 × 10^6
1	10	Wire	20	0.011	60.45	39,000	7.5	190.5	1,342	866,015	283	117.9
2	8	Wire	18	0.009	66.13	42,661	6.75	171.5	1,206	777,743	251	104.3
3	9	Wire	24	0.011	70.36	45,393	7.125	181	1,592	1,027,068	297	123.8
5	7	Strand	8	0.009	71.75	46,290	7	177.8	961	620,210	293	121.9
7	6	Wire	22	0.014	60.37	38,949	6.25	158.8	1,714	1,105,979	196	81.6
8	3	Strand	8	0.009	74.15	47,842	7	177.8	961	620,210	303	126

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Table 3. Correlation coefficients among considered predictor variables

Variable	I	t	ρ	A	h	w	A_b
I	1.00	−0.32	0.41	0.63	0.81	−0.42	−0.45
t	−0.32	1.00	−0.93	−0.54	−0.02	0.94	0.92
ρ	0.41	−0.93	1.00	0.66	0.02	−0.999	−0.76
A	0.63	−0.54	0.66	1.00	0.05	−0.68	−0.50
h	0.81	−0.02	0.02	0.05	1.00	−0.03	−0.23
w	−0.42	0.94	−0.999	−0.68	−0.03	1.00	0.78
A_b	−0.45	0.92	−0.76	−0.50	−0.23	0.78	1.00

There are additional parameters that could have been accounted for but are not included in this investigation, such as: concrete strength, steel grade, amount of prestressing forces, and eccentricity of the centroid of the tendons relative to the centroid of the area. Most of these were left behind because they cannot be easily and precisely determined, defeating the purpose of a simple model. Others were intentionally omitted because their variation across tie designs is minimal, which is the case for most material properties for example.

However, before proceeding to the model development, the correlation among independent variables is investigated to anticipate the effects of multicollinearity on the final model. Correlation coefficients vary from −1 (perfect negative correlation) to 1 (perfect positive correlation), with 0 suggesting no linear correlation (Pearson 1895). The high correlation coefficients among the investigated variables indicate that multicollinearity may further challenge the interpretation of regression coefficients in the final model (Table 3).

Model Development

The model was developed with ordinary least squares (OLS) using 70% of the available data (the reminder 30% is used for model validation, as will be detailed later). The variables were selected by the *least absolute shrinkage and selection operator* (LASSO) method (Tibshirani 1996). To mitigate the effects of correlation among the considered predictor variables (Table 3), only a subset of these variables was included in the model development. The area, A, and the height, h, were excluded at this point, as the moment of inertia, I, is a mathematical function of these two variables. The classification variable tendon type, w, and the total bond area, A_b, were also excluded because of their correlation with the ratio of steel in the cross-sectional area, ρ. However, the number of tendons, t, was not excluded despite its evident correlation with ρ because it seemed better to the authors to allow for the LASSO method to select the most appropriate of these variables. A purist structural approach might argue that ρ is more appropriate than t given that it is normalized variable, but it is also true that the simple number of tendons, which does not account for geometric properties, has a lower probability of being linearly correlated with the moment of inertia (Table 3). The cross-product of these remaining predictors, which correspond to the double-summation term in Eq. (1) (also called interactions) were also accounted for when building this model. In addition, after an exploratory investigation of the data, the response variable was transformed, and two extreme outliers were removed from the dataset. The details of the model development stage are described next.

Variable Transformation

A transformation of the ultimate bending moments was performed to avoid problems of unequal error variance. When first exploring laboratory data, the authors observed that crossties with greater

Table 4. Analysis of variance results

Source of variance	Degrees of freedom	Sum of squares	Mean square	F statistic	p-value
Model	2	42.02	21.008	35.28	<0.0001
Error	24	14.29	0.596	—	—
Corrected total	26	56.31	—	—	—

ultimate capacity also incurred greater residuals than those designs with smaller strength. A square root transformation was used to reduce this disparity, leading to $y_i = \sqrt{M_{ni}}$, where M_{ni} is the maximum bending moment of Trial i. Other transformations were tried, such as raising M_{ni} to a power recommended by the Box-Cox procedure (Box and Cox 1964), but the square root transformation was preferred as it was both satisfactory and simple.

Model Selection

As previously stated, the LASSO procedure was adopted (Tibshirani 1996). The procedure was conducted in a forward manner, meaning that the model starts with no predictors and the variables are added by steps. The selected model is the one with lowest Schwarz Bayesian information criterion (Schwarz 1978). At the end of five steps, the variables selected were the squared values of the moment of inertia and the number of tendons. Therefore, the form of the final model includes only three regression parameters and can be written with the statistical terms of Eq. (2)

$$y_i = \beta_0 + \beta_{11}x_{i1}^2 + \beta_{22}x_{i2}^2 + \varepsilon_i \tag{2}$$

After estimating the regression parameters, the final fitted model is obtained in Eq. (3), where $\hat{y}_i$ is the expected value of y_i (true value)

$$\hat{y}_i = 15.68 + (8.48 \times 10^{-5})x_{i1}^2 + (2.32 \times 10^{-3})x_{i2}^2 \tag{3}$$

An analysis of variance of the recommended model shows a p-value lower than 0.0001 (Table 4), an indication that the model is performing well in explaining the variability of the response variable. The analysis of variance also breaks down the error sum of squares into lack of fit and pure error sum of squares. The lack of fit is not significant (p-value 0.9686). This is a sign that the model is complex enough to predict the ultimate capacity of concrete crossties, and that the error in the model is due to the natural randomness of the response variable (not because of a poor model choice).

The coefficient of multiple determination, R^2, is 0.746, which is a good indication of the adequacy of the fitted model, especially when considered in combination with the lack of fit analysis. The adjusted coefficient of multiple determination, R_a^2, is 0.725, which is also acceptable. Given that the fitted model has been able to explain the variability of the bending strength of various crosstie designs, it is useful to rewrite the fitted model by substituting y_i by $\sqrt{M_n}$ and by using a notation that is more easily interpreted, as shown in Eq. (4) (refer to Notation section for description of terms)

$$M_n = [15.68 + (8.48 \times 10^{-5})I^2 + (2.32 \times 10^{-3})t^2]^2 \tag{4}$$

For metric units, Eq. (5) should be used. M_n is given in kilonewton meters, while I is expressed in mm^4

$$M_n = [5.27 + (1.64 \times 10^{-16})I^2 + (7.80 \times 10^{-4})t^2]^2 \tag{5}$$

A word of caution for using Eqs. (4) or (5). Extrapolation of crosstie properties beyond what was used to develop this equation should be done with care. The range of some relevant properties used are shown in Table 1. In addition, concrete materials strength should be above 7,000 psi (48 MPa) at 28 days. Prestressing tendons should be made of either low-relaxation seven-wire steel strand or low-relaxation indented steel wire of diameter near 5.3 mm. Again, Eqs. (4) and (5) predict the center negative bending strength of pretensioned concrete crossties that rely solely on bonding to transfer the prestressing forces from the tendons to the concrete matrix.

Note that Eqs. (4) and (5) predict a higher ultimate strength for concrete crossties made with a bigger cross section (I) and with more prestress (t), which is consistent with what would be expected based on structural analysis. Moreover, the fact that the maximum bending capacity of various common North American crossties can be estimated using only these simple variables is an evidence that there has not been much diversity in design approach across different manufacturers. For instance, it appears that the eccentricity of the wires centroid relative to the centroid of the cross-sectional area has been consistent for the different designs. Additional evidence of lack of design diversity is that the model seems to be sufficient without considering material properties. Therefore, there appears to be room for innovation of pretensioned concrete crossties if railroads are willing to experiment. Nevertheless, the proposed model seems to be adequate for a good prediction while crosstie design remains consistent.

Diagnostics

The derivation of ordinary least-square estimators assumes that the error terms are normally and independently distributed with constant variance. Given there was no sequential data, the authors did not expect autocorrelation to be a problem, and there was no formal verification of independence of error terms. When this problem is anticipated, the Durbin-Watson (Durbin and Watson 1950, 1951) test is commonly used for detection of serial correlation. The assumption that the error terms are normally distributed was confirmed at a significance level of 5% with the Shapiro-Wilk test (Shapiro and Wilk 1965) yielding a p-value of 0.45, which is illustrated using studentized residuals for dimensionless values (Fig. 2). Finally, the hypothesis of homogenous variance of the error terms (homoscedastic errors) was tested using the Brown-Forsythe test (Brown and Forsythe 1974). To perform this procedure, it was necessary to randomly divide the data in two groups, each containing half of the replicates of all designs used in the model building dataset, as illustrated using studentized residuals (Fig. 3). The homoscedasticity of error terms was then confirmed with a p-value of 0.91, which is greater than most common significance levels.

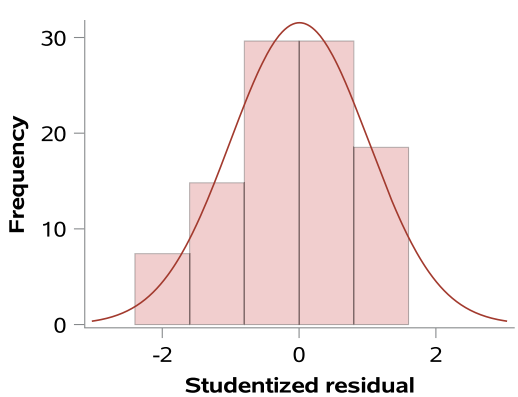

Fig. 2. Histogram of studentized residuals overlaid with nearest normal distribution.

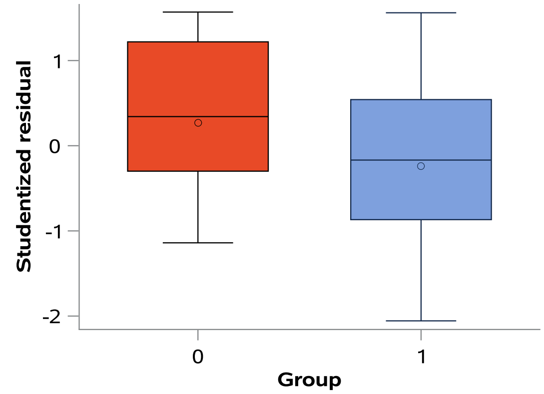

Fig. 3. Box plots of studentized residuals grouped for verification of homogenous variances.

This supports the sufficiency of the transformation of the response variable previously described.

Regarding outliers and influential points, no remedial measure was necessary at this stage, given that two datapoints had been previously removed in the data exploring phase as mentioned before. These outliers had been removed so unreliable observations would not negatively influence the model development. The first outlier was an extremely high observation, and its removal contributed to a more conservative model. The second outlying observation was relative to a used, worn crosstie that did not have its properties properly quantified before the test was run. Only one observation had a Cook's distance statistic (Cook 1979) slightly above the threshold $4/n$, where n is the number of datapoints (i.e., 27), an indication that no point should be classified as abnormally influential. One Y-outlier was detected with an absolute studentized residual greater than two (−2.22), a common threshold (Neter et al. 1996), but this was of little concern because it was not influential. Datapoints of Design 7 had high leverage (i.e., diagonal values of the so-called hat matrix), an indication that the properties of this crosstie were disparate from those of the other designs, which is attributed to the fact they use a high number of tendons (second highest of all designs) for a small moment of inertia (smallest of all designs). Finally, correlation among independent variables (multicollinearity) is not a concern in this investigation, as the variables were selected to mitigate these effects as explained previously.

Model Validation and Application

As previously mentioned, 70% of the available data was used to build the model, leaving 30% for validation. With the validation dataset, a new model was developed using the same predictor variables used in the original model. Then, the new regression coefficients and their standard errors were compared with the ones relative to the first model. The model is validated if the coefficients and standard errors from both models are similar. One may argue that such validation might be accidental if the datapoints are grouped conveniently in the process of splitting the data into the building and validation datasets. To avoid this problem, two actions were taken. First, the datapoints had been intentionally split to ensure that representative datapoints were in both datasets. Second, the entire process was repeated nine times, each time randomly splitting the data into different building and validation datasets, always keeping the ratio 70%:30%. Therefore, a total of 10 data splitting procedures were carried out, one being intentional (Model A) and nine being random (Models B through J) (Table 5). It is apparent that the regression coefficients of

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Table 5. Parameter estimate (regression coefficients) and standard error for all models

Model	Purpose	β_0		β_{11}		β_{22}	
		Estimate	Standard error	Estimate ($\times 10^5$)	Standard error ($\times 10^5$)	Estimate ($\times 10^3$)	Standard error ($\times 10^3$)
Model A	Build	15.68	0.89	8.48	1.03	2.32	0.81
	Validate	16.27	1.11	7.92	1.17	2.45	1.29
Model B	Build	15.47	0.72	8.34	0.80	3.17	0.70
	Validate	18.08	1.17	7.22	1.29	−1.24	1.32
Model C	Build	15.74	0.88	8.55	0.99	2.21	0.78
	Validate	15.89	1.18	7.35	1.27	3.72	1.75
Model D	Build	16.17	0.90	7.96	1.01	2.01	0.81
	Validate	15.96	1.17	8.31	1.19	2.53	1.70
Model E	Build	15.91	0.72	8.32	0.80	2.48	0.71
	Validate	16.82	1.61	6.95	1.81	1.77	1.75
Model F	Build	16.52	0.80	7.19	0.91	2.76	0.82
	Validate	15.01	1.08	9.70	1.17	2.14	1.12
Model G	Build	15.98	0.73	8.24	0.80	2.54	0.71
	Validate	17.05	1.45	6.79	1.65	1.14	1.62
Model H	Build	16.23	0.75	7.43	0.87	2.87	0.75
	Validate	15.46	1.40	9.18	1.50	2.58	1.50
Model I	Build	15.90	0.79	8.53	0.87	1.42	0.78
	Validate	16.17	0.85	7.37	0.96	4.25	0.90
Model J	Build	16.49	0.82	7.53	0.93	2.15	0.81
	Validate	14.91	1.14	9.38	1.24	3.1	1.23

the several models and their standard errors agree well with the coefficients of the proposed model shown in Eq. (3), which corresponds to the first line of Table 5.

For a final comparison of the observed and predicted values of bending strength, the results after back-transforming the response variable were also calculated (Table 6). The greatest final prediction error was −5.5%, which shows that the model can be valuable for approximations.

Having a simple mathematical expression to predict ultimate capacity is useful and considerably easier than performing time consuming and costly laboratory testing, back-calculating bending capacity with structural formulas, or developing a numerical finite-element (FE) model. After estimating the ultimate center negative capacity of new crossties, railroads can have an idea of how much reserve capacity there is in each design beyond the first crack capacity recommended by the American Railway Engineering and Maintenance-of-Way Association (AREMA), as discussed by Bastos et al. (2018). The approximate results provided by using this model can also serve as a maintenance metric for bottom-abraded concrete crossties, which is a common defective condition for these components (Bastos et al. 2015; Riding et al. 2018; Vemuganti and Moreu 2017; Yu 2016). As concrete crossties abrade away, their cross-sectional geometry changes and their ultimate capacity is reduced. If this reduced capacity is estimated, regulators and railroads may be able to decide whether it is acceptable or not for a given railroad environment and wheel loads.

As an example of reduced capacity estimation, Fig. 4 shows the effect of section loss on the ultimate center negative capacity of crossties of Design 1, where the statistical model curve is relative to Eq. (4). The structural estimation curve is provided for comparison an it is based on the methodology proposed by the American Concrete Institute [ACI Committee 318 (ACI 2014)] when using Whitey's stress block (Whitney 1937). There is reasonable agreement among the two methodologies. The sharp change around 1.4 in. (35.6 mm) is an indication that prestress tendons are uncovered and infective due to excessive section loss.

Table 6. Observed and predicted bending strength of studied concrete crosstie designs

Crosstie design	Average laboratory result (kip-in.)	Predicted value (kip-in.)	Average laboratory result (kN·m)	Predicted value (kN·m)	Prediction error (%)
1	538.1	547.8	60.8	61.9	1.8
2	496.0	473.2	56.0	53.5	−4.6
3	619.2	600.8	70.0	67.9	−3.0
5	525.2	533.5	59.3	60.3	1.6
7	408.6	402.5	46.2	45.5	−1.5
8	589.0	556.4	66.5	62.9	**−5.5**

Note: Bold value corresponds to the greatest error.

Conclusions

This paper focused on developing a mathematical equation to predict the center negative bending strength of the most common type of concrete railroad crossties in North America. To obtain such an equation, a regression analysis was conducted with previous laboratory experimentation results, which was accomplished using ordinary least square estimators and analysis of variance through LASSO methodology. Eqs. (4) [or Eq. (5] for metric units) is the result of this process. This equation was validated with experimental data from crosstie designs that had not been used in its formulation and the maximum error obtained for the investigated designs was −5.5%, which can be better than many structural engineering estimation procedures.

The proposed model is of easy application and can provide a quick estimation of center negative flexural capacity of common concrete crossties. This can assist railroads in predicting the reserve capacity of their pretensioned concrete crossties to better inform their maintenance and operational decisions. Only two design characteristics are needed, namely the number of prestressed tendons and the gross area moment of inertia of the center cross section of a crosstie. Having an elementary model typifies the uniformity across several concrete crosstie designs in the US market with regards to aspects such as material properties and prestress eccentricity. There seems to be an opportunity for design innovation and product differentiation.

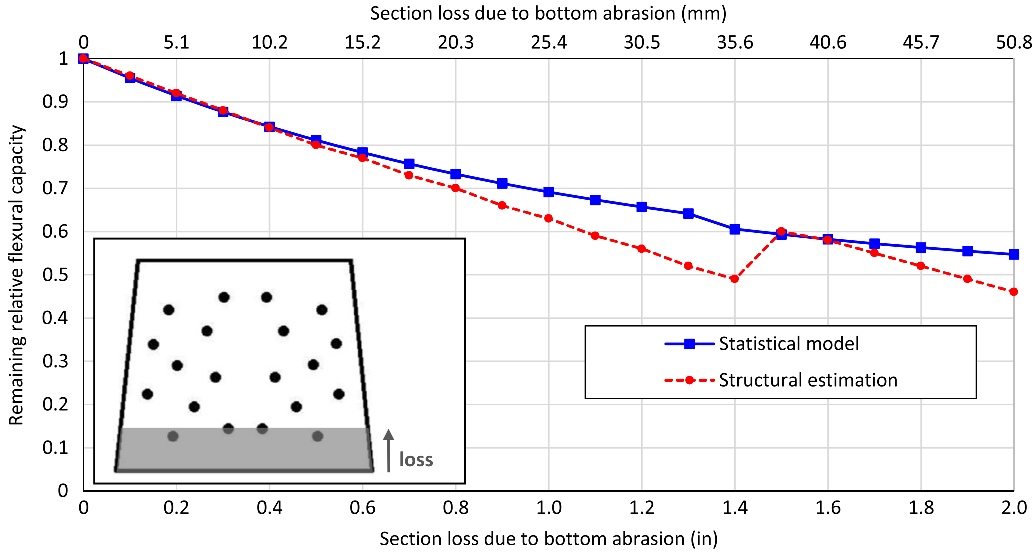

Fig. 4. Remaining flexural capacity of bottom abraded crosstie (Design 1) normalized by its initial capacity estimated by each approach.

There are three primary applications for the derived statistical model. First, it is useful in the estimation of reserve capacity of new crossties. Optimized designs could have lower reserve capacity than premium, more onerous products that need extra strength for extreme railroad environments. Second, the model serves as a maintenance tool to prevent accidents caused by concrete crossties that fail due to bottom abrasion. Railroads can stipulate a minimum flexural capacity for abraded crossties, and those for which the prediction is lower than the requirement should be monitored or replaced. Third, the model is useful to estimate how safe it is to increase axle loads without causing concrete crosstie failure in a given rail line.

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Notation

The following symbols are used in this paper:

I = gross area moment of inertia about flexural axis at crosstie center (in.^{4});
M_n = center-negative bending strength of pretensioned concrete crosstie (kip-in.);
p = total number of independent variables (predictors) in the model;
t = total number of tendons of a crosstie;
$x_{i1}, x_{i2}, \ldots, x_{ij}, x_{ik}, \ldots, x_{i(p-1)}, x_{ip}$ = values of independent variables included in the model for Trial i;
y_i = value of response variable for Trial i;
β_j = regression parameter associated with x_{ij};
β_{jj} = regression parameter associated with x_{ij}^2;
β_{jk} = regression parameter associated with the interaction (cross product) term $x_{ij}x_{ik}$;
β_0 = regression parameter for the intercept; and
ε_i = random error term for Trial i.

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