

Probabilistic framework for the assessment of the flexural design of concrete sleepers

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Abstract

An extensive study of the flexural performance of monoblock prestressed concrete sleepers in a light rail system was conducted as part of a research program funded by the Federal Transit Administration. Five consecutive sleepers deployed on the track were instrumented with strain gauges at their critical design cross-sections (center and rail seats) to obtain relevant flexural information during an uninterrupted period of 14 months. Results were compared with the projected design capacities obtained from the application of current design standards, resulting in glaring differences. The current design methodologies were deemed insufficient for the development of optimal design solutions for light rail applications. Furthermore, structural reliability analysis is employed to study the flexural capacity of the sleeper design. A capacity model based on the material and geometric properties of the sleeper design was developed. The demand model was derived from the field flexural data of over 27,000 train passes, fitting this information to predefined probability distributions. Four limit-state functions were defined to represent the typical flexural failure modes. The probability of failure was calculated using first-order reliability method, second-order reliability method, and Monte Carlo simulation. Ultimately, the analysis yielded consistent results for the three methods, showing largely low probability of failure at both design cross-sections under the studied demand level. In conclusion, the sleeper's capacity was higher than the existing field demands, indicating an overly conservative design approach.

Keywords

Structural reliability, railroad engineering, rail track design, concrete sleeper, design standards

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Introduction

Rail transit is a common mode of passenger transport used in medium to large cities throughout the world. Within the classification of rail transit, there are different modes serving different purposes based on qualifiers that include the type of infrastructure, the type of service (urban or suburban), and operating speed, which result in the use of different types of railcars (rolling stock). Because of this, variable loading environments can be found within the different rail transit modes in North America¹ and worldwide. Due to the varied loading environments, both among rail transit systems and with the freight railroad and high-speed rail domains, differences in the design of railroad infrastructure may be necessary.

Presently, the recommended design practices provided within the AREMA Manual for Railway Engineering² (hereafter referred to as the “AREMA Manual”) fail to capture some of the intricacies among the aforementioned differences in operations

and loading environments. Similarities that do exist throughout different railroad systems have led to the use of very common, and oftentimes the same, design procedures for varied modes and rail transit systems. Ballasted track is the most common type of railroad infrastructure in service today, across various rail transit modes. One of the primary elements in a ballasted track system are the sleepers (also known as crossties in North America), which are embedded in

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the ballast and to which the rails are restrained.^{3,4} Sleepers provide rigidity to the track and behave as beams, resisting demands mainly through flexure. Sleepers are required to transfer train loads from the rail to the ballast, restrain the track longitudinally, laterally and vertically, and maintain the track gauge within admissible limits.³ Timber is the most commonly used sleeper material throughout North America (about 90–95%), followed by concrete, adding an additional 5–10%.⁵ Concrete sleepers are traditionally used in more demanding loading scenarios due to the additional resiliency imparted to the system, as they are designed for longer life cycles when compared to other sleeper materials. Additionally, they provide the system with adaptiveness and robustness to extreme demands caused by external events. Traditionally, the design of concrete sleepers has relied on previous experience and empirical results.⁵ The current design practices propose load-based methodologies that fail to address the variability of other key factors in design, including the support conditions or the input loads. Through the use of field instrumentation and a probabilistic approach, a prominent monoblock prestressed concrete sleeper designed for light rail transit applications is evaluated.

Field experimentation

As part of a large rail transit infrastructure research program funded by the Federal Transit Administration (FTA), researchers at the University of Illinois at Urbana-Champaign (UIUC) partnered with St Louis MetroLink (hereafter referred to as “MetroLink”), a light rail transit system, to deploy field instrumentation. This FTA-funded project strives to characterize the desired performance and resiliency requirements for concrete sleepers and fastening systems through the quantification of their field behavior, with the aim of developing more resilient concrete sleepers and fastening systems.

The instrumented track on MetroLink consists of five consecutive concrete sleepers spaced 0.76 m (30 in) apart on center within a tangent segment of track with a track speed of 89 km/h (55 mile/h). An automated data collection system was deployed at the site, continuously capturing and storing large data sets of field data over a 14-month time period. Figure 1(a) shows the instrumentation layout and Figure 1(b) presents a picture of the site after installation of instrumentation.

Figure 1(a) shows the instrumented sleepers capable of providing flexural response data. Specifically, concrete surface strain gauges were installed at the top chamfer of the critical design cross-sections, which are the center and the rail seats. Through laboratory calibration, the bending moment imposed by revenue service loads can be calculated using the measured strain.^{6,7} Based on the application of Euler–Bernoulli

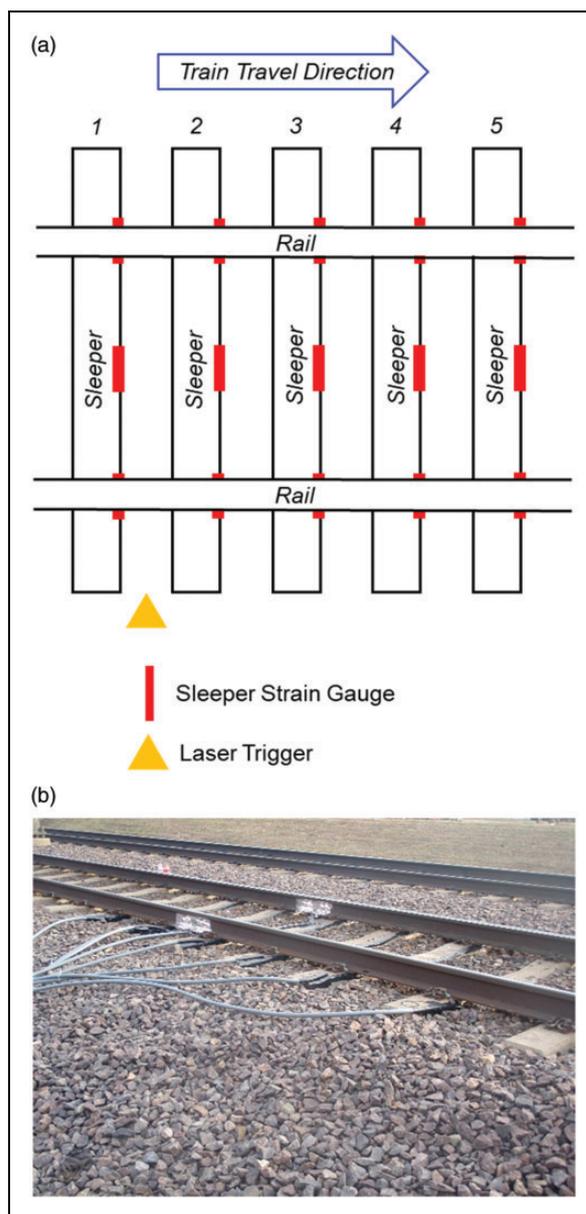


Figure 1. Field instrumentation setup deployed on a light rail system. (a) Field instrumentation plan. (b) Picture of the instrumented site.

beam theory for small deformations, the stress distribution within the critical cross-sections of the sleepers is known. Previous research and laboratory experimentation conducted at RailTEC at UIUC have demonstrated that this is an acceptable assumption, validating the proposed field instrumentation of concrete sleepers used in this study.^{8–11} The laboratory calibration methodology followed, which was developed and described by Edwards et al.,⁶ consists of generating load deformation curves under known loading and support conditions for the exact same sleeper design as the one studied in field. Following the rail seat positive and center negative bending moment test protocols for concrete sleepers provided by AREMA,² a strain-bending moment diagram is developed for each of the design sections, center, and rail

seat. Using these lab-generated data, a calibration factor, mathematically defined in equation (1), is derived for each of the design sections.

$$M_s = \frac{\varepsilon_s E_c I_s}{d_s} = k \varepsilon_s \quad (1)$$

where

- M_s is the sleeper bending moment at section “s” (kip-in (kN-m));
- ε_s is the strain at the surface strain gauge at section “s” (in/in (m/m));
- E_c is the elastic modulus of the concrete (psi (kPa));
- I_s is the moment of inertia at section “s” (in⁴ (m⁴));
- d_s is the distance from the surface strain gauge to the neutral axis of bending of the sleeper at section “s” (in (m));
- k is the calibration factor at section “s” (kip-in (kN-m)). The calibration factors derived for this sleeper are:
 - Center: $k_C = -398,851.31$ kip-in (-45,064.15 kN-m);
 - Rail seat: $k_{RS} = -731,491.44$ kip-in (-82,647.44 kN-m).

The strain gauges used in this study are PFL-30-22-3LT, manufactured by Tokyo Sokki Kenkyujo Co., Ltd, designed for concrete structures applications (TML). National Instruments 9135 Automated Compact Data Acquisition System was used to collect data automatically in the field with a laser trigger manufactured by Micro-Epsilon whenever a train passed the field site.

Design evaluation using field data

Using the field instrumentation setup described above, a large data set was generated for the five instrumented sleepers over the period of 14 months. After developing an automated data processing system using MATLAB, flexural data were processed for a total of 27,092 light rail vehicle passes (12 axles each) during the period of time between 18 March 2016 and 19 May 2017.

These results have been previously discussed and analyzed.⁷ Through extensive monitoring of the track, data representative of a variety of operational conditions were obtained. While deterioration over time is not captured, due to the relatively light loading environment and the monitoring period spanning just over a year, other effects that can increase the flexural demand, as environmental and temperature conditions or high dynamic loading, are collected in terms of bending moments. It is worth highlighting the minor variability in support conditions from sleeper to sleeper found at MetroLink.⁷ Given that the sleeper support conditions are one of the key aspects that affect its flexural performance,⁹ bounding the

variability at this site allows for a more meaningful study and applicable conclusions.

Light rail sleeper design

The design investigated in this paper consists of a prestressed monoblock concrete sleeper developed for light rail transit applications. This sleeper model uses high-strength concrete (HSC) with compressive strength ranging from 48.3 to 75.8 MPa (7 to 11 ksi) and Grade 270 prestressing steel (i.e. ultimate strength of 1861.6 MPa (270 ksi)). At the cross-sectional level, the sleeper has trapezoidal shape. Previous field research found the center cross-section to be more prone to undergo negative bending moments. On the other hand, the rail seat cross-sections are generally subjected to positive bending moments.⁷⁻⁹ For this reason, design standards identify the center subjected to negative bending moment and the rail seat subjected to positive bending moment as the key design cases.^{2,12-14} Hence, to optimize the use of straight prestressing steel wires, typical designs adopt varying cross-sections throughout the length of the member, being higher at the rail seats. Figure 2(a) shows a sketch of the elevation of a typical concrete sleeper.

The idea behind this is to define the overall sleeper geometry and steel wires arrangement in a way such that the eccentricity, being this the distance between the center of gravity of the steel (c.g.s.) and the center of gravity of the concrete (c.g.c.), generates a prestressing-based moment that opposes the main flexural demand on the sleeper. Figure 2(b) graphically represents this concept, where having a constant height of the c.g.s. along the sleeper, the c.g.c. at the center cross-section can be found below the c.g.s. and above it at the rail seats. This specific design uses 12 prestressing wires, each of 5.32 mm (0.21 in) in diameter. The initial prestressing force is 31.1 kN (7 kips). The geometric properties of the design cross-sections are summarized in Table 1.

Review of current design practices (AREMA) and path forward in design

The current design methodologies for prestressed monoblock concrete sleepers are based on the practices of allowable stress design (ASD) and focus on limiting stresses at the critical cross-sections.² When it comes to prestressed concrete sleepers, this approach focuses on preventing crack initiation on the aforementioned design sections when subjected to a design load and support conditions defined by the recommended design practice. The design input load is calculated as function of the static wheel load, the speed, and the sleeper spacing. Assumptions on the rail seat load distribution and support conditions vary among the different design standards followed in the United States (AREMA),² Europe (EN),^{12,13}

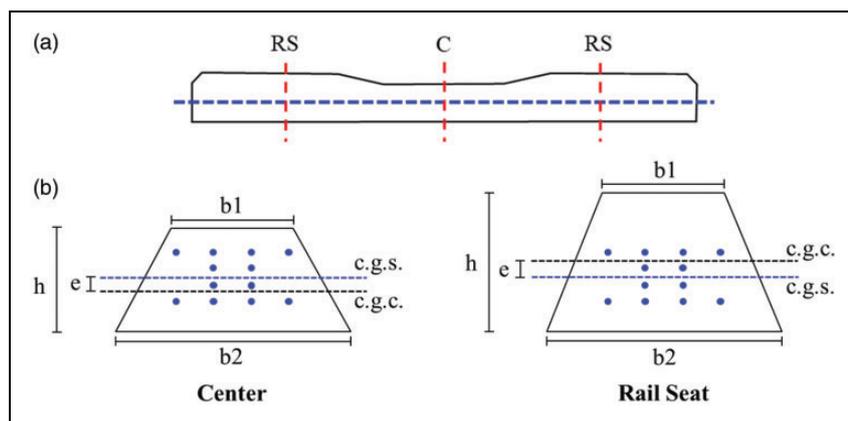


Figure 2. Elevation and design cross-sections of typical concrete sleepers. (a) Elevation sketch of the prestressed monoblock concrete sleeper. (b) Sketch of the design cross-sections of a prestressed monoblock concrete sleeper.

Table 1. Geometric properties of the design cross-sections in a light rail concrete sleeper.

| | Height cm (in) | Top width cm (in) | Bottom width cm (in) | Height of c.g.s. cm (in) |
|------------|-------------------|-------------------------|----------------------------|--------------------------------|
| Center | 15.88 (6.25) | 19.69 (7.75) | 26.37 (10.38) | 8.57 (3.38) |
| Rail seats | 20.32 (8.00) | 18.75 (7.38) | 26.36 (10.38) | 8.57 (3.38) |

c.g.s.: center of gravity of the steel.

and Australia (AS).¹⁴ Figure 3 represents the different support conditions and rail seat load distributions used in the typical design standards just mentioned, including the newly adopted design assumptions included in the 2017 AREMA Manual.^{2,8} These elements are combined to calculate design bending moments that the sleeper shall be able to withstand without reaching the cracking bending moment of the critical cross-sections. Using the assumptions mentioned above and graphically depicted in Figure 3, the two principal design bending moments, which are center negative (C–) and rail seat positive (RS+), can be obtained. Center positive (C+) and rail seat negative (RS–) are calculated as the function of the principal design factors, as are less likely to happen under real loading and support conditions. When they do occur, they are highly unlikely to generate moments that exceed the cracking moment of the sleeper. Previous research has compared these methodologies using heavy axle load (HAL) freight train data, concluding that the current support conditions employed in the different standards do not match actual field conditions.¹⁵

These standards aim to provide design guidelines for all types of rail concrete sleepers, spanning the loading spectrum from HAL freight and high-speed rail applications to the less-demanding rail transit modes, the focus of this article. Using field data collected on MetroLink, both dynamic rail loads and bending moments at the critical cross-sections, different support assumptions are evaluated against field

data. The collection and analysis of dynamic loads on MetroLink was part of the parent project to this study funded by the FTA.¹⁶ Figure 4 shows a comparison between the flexural field data and the predicted moments using the aforementioned design standards in conjunction with the collected load field data.

Observing the results shown in Figure 4(a) for C– and Figure 4(b) for RS+, the challenge in predicting support conditions and developing a uniform set of guidelines for the flexural design of sleepers is further visualized. When comparing the field bending moment data to the predicted moments stemming from the use of commonly accepted design practices, it is found that the applicability and accuracy of support condition assumptions varies for the different design standards. Additionally, it is inferred from these data and other studies that the sleepers have good support under the rail seats, representative of a freshly tamped support condition. Given the case of a track at the end of its maintenance cycle, where the support conditions are primarily center-bound, these graphs would shift, indicating overdesign in the rail seats and a lack of capacity in the center.

Thus, it can be concluded that current standards do not accurately capture the different variables affecting the flexural behavior of concrete sleepers, and ultimately rely on large safety factors to account for the uncertainty. In the case of less demanding loading environments, such as those associated with light rail transit, the lack of precision on the design leads to inefficient sleepers, resulting into larger infrastructure costs for the owner.

Application of structural reliability analysis to concrete sleeper design

Traditional design methodologies propose a deterministic approach where the uncertainty associated with the capacity and the demand is mitigated with safety factors. Most, if not all, of the design standards used in structural engineering propose similar

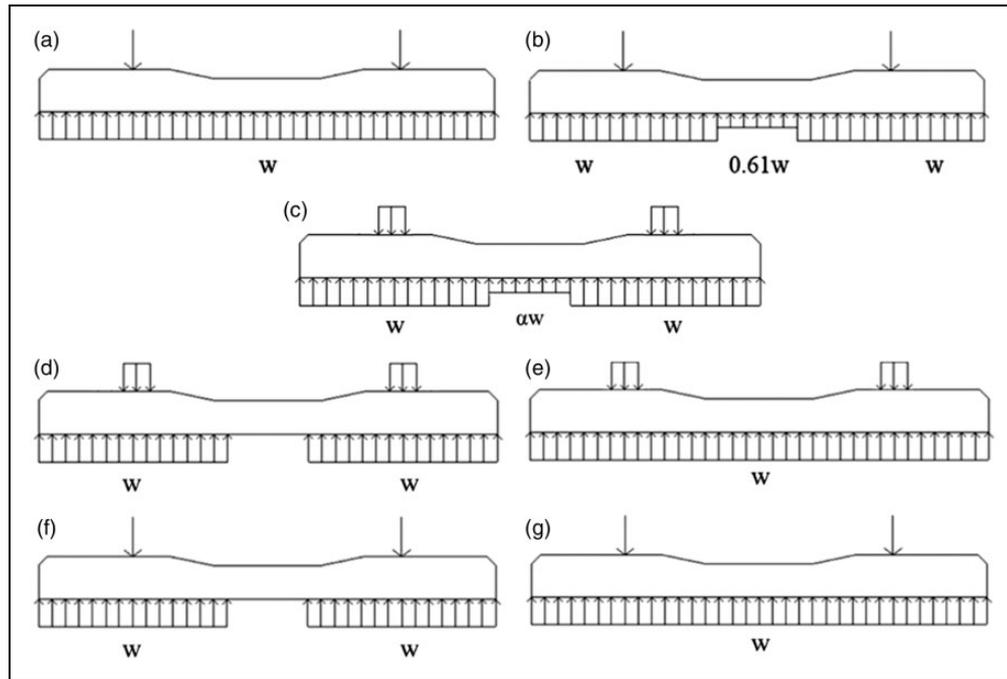


Figure 3. Support conditions and rail seat load distribution assumptions provided in different worldwide used standards for concrete sleeper design: AREMA (pre-2017) RS + (a), AREMA (pre-2017) C– (b), AREMA 2017 (c), UIC/EN RS + (d), UIC/EN C– (e), AS RS + (f), and AS C– (g).

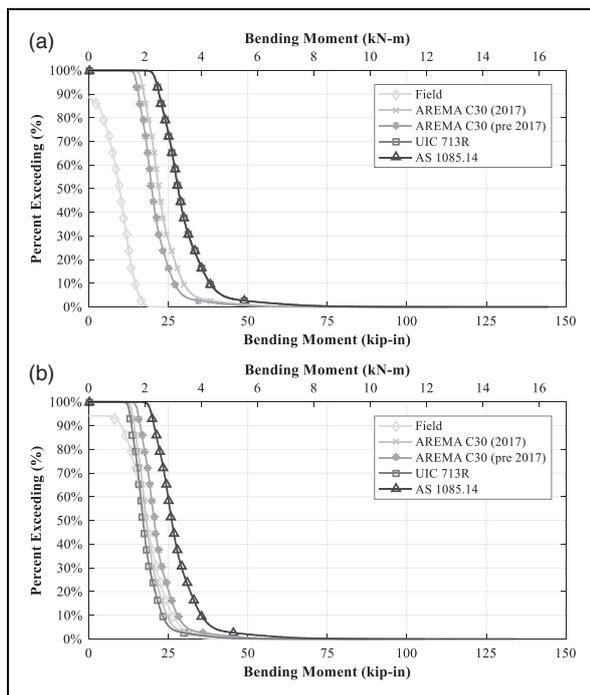


Figure 4. MetroLink light rail transit field data comparing bending moments to moments predicted by widely accepted flexural design criteria. (a) Center negative bending moment. (b) Rail seat positive bending moment.

approaches, which would not be incorrect if the geometric properties, material properties, and actions were known.¹⁷ However, this is never the case, as all the factors influencing a design problem are variable. To account for this variability that can lead to the

failure of the design, defining failure as the demand (function of the actions) being larger than the capacity (function of the geometric and material properties of the designed element), the concept of safety factor is introduced. These safety factors, which are coefficients that multiply some of the previously mentioned components in order to account for less likely situations, are associated with the expected life cycle of the structure, acting as a “black box” which the end user applies, given the type of structure and action for which it is being designed.

Employing a deterministic design approach, as has been widely used in the past, it can yield inaccurate results. Another way to account for the variability in the demand and the capacity is through a probabilistic approach as is used in structural reliability analysis (SRA). SRA is a broadly studied topic for which extensive literature can be found.^{17–21} Uncertainty in the actions, geometric properties, and material properties of the element is considered using probability distributions. However, deterministic parameters are also used when the uncertainty represented by the described metric is minimum or irrelevant to the calculation. This mathematical approach revolves around the concept of probability of failure, which can be defined as the likelihood of the demand being larger than the capacity, hence resulting in failure.

The probability of failure is expressed in SRA through the concept of reliability index β^{17} that can be written as in equation (2)

$$\beta = \Phi^{-1}(P_f) \tag{2}$$

where

- Φ represents the standard normal cumulative distribution function (CDF).
- P_f represents the probability of failure.

When focusing on the concrete sleeper design, to compensate for the use of imprecise support conditions, dynamic load effects, and rail seat load distribution, a conservative design load, in addition to other safety factors including tonnage and speed, are employed. However, this is not an efficient approach, as overdesigned elements can have a negative economic impact. Deploying sub-optimal designs can lead to larger material costs, requiring higher initial investments and replacement costs. Moreover, additional track stiffness can require additional performance from a premium elastic fastening system, resulting in additional expenditures.

Given these reasons, a probabilistic approach for the design and assessment of concrete sleepers is proposed. AREMA² defines a failed concrete sleeper as one in which a crack has initiated from the tensile surface to the first level of prestress. Even though the need for introduction of limit state design concepts in the design of concrete sleepers has been discussed in previous literature,²² this paper aims to present a reliability-based framework built from the current design practices proposed by the AREMA Manual. Previous research has studied the application of these concepts into the design of concrete sleepers, focusing on freight and as an assessment of the Australian Standard.^{23,24} Through this paper, the authors intend to continue to expand the applicability of SRA to the design and assessment of concrete sleepers, focusing on the validation of current industry designs through the use of field data.

In the following sections, a model is developed and presented for the assessment of a light rail concrete sleeper design using field data. The capacity and the demand model are described and discussed, as well as the limit-state functions used to define this study.

Development of demand model based on field experimentation

Demand models are typically more challenging to generate than capacity models, as the uncertainty in loads is inherently higher than which is associated with materials or geometric properties. In this study, the demand model is based on field data and is used to assess an existing design used in rail transit infrastructure. Center and rail seat bending moment data discussed above were used. Both rail seats presented similar support conditions at all locations tested, thus symmetry was assumed. Two random variables were developed from these data: center bending moment and rail seat bending moment. Hence, a distribution fitting exercise was carried to obtain the

probability distributions that best describe the collected data. The MATLAB Distribution Fitter toolbox was employed to perform this analysis both for the center and the rail seat bending moment data. Additionally, the data were truncated using only the positive rail seat bending moment and the negative center bending moments. This decision introduced a small amount of inaccuracy in the model, but it was deemed to be a conservative measure and not significant enough to disturb the final output of the model. The discarded data represent 13% of the center bending moment and 6% of the rail seat bending moment data.

For the analysis, common probability distributions were chosen to obtain simple models that can be extrapolated for future investigations. The data were fitted and compared against Normal, Lognormal, Gamma, Generalized Extreme Value (GEV), and Weibull distributions. Due to the large size of both center and rail seat moment data sets (1,411,052 and 3,060,144 data points, respectively), none of the conventional distributions passed the Chi-squared goodness-of-fit test. Thus, using graphic tools and comparing the different fits among themselves applying the Kullback–Leibler divergence test, the best fits were obtained for each variable. Regardless, it is remarkable how the collected data behaved similar to most of the tested distributions. Figure 5 shows the histogram of the center and rail seat bending moments, as well as the probability density function (PDF) of the different chosen distributions. To ensure a good fit of the proposed distributions to the existing data, the right tails, which correspond to the higher values (negative bending moment in center, positive in rail seat) and more critical for the design of sleepers, are shown. These distributions are also graphically compared using the CDF (Figure 5).

Table 2 shows the numerical results of the Kullback–Leibler divergence test, where a lower value represents a closer fit to the field data. Hence, for the purpose of this analysis, a Weibull distribution was chosen to represent the center bending moment data and a Gamma distribution was used for the rail seat bending moment data. Nevertheless, as stated before, the field data fit quite closely most of the proposed distributions; hence, other distribution options could be selected without adversely affecting the results. Therefore, different distributions were chosen to characterize the demands at the center and rail seat cross-sections, respectively, due to the differing sectional properties and loading behavior among center and rail seat. Loading at the rail seat is known to present particularities not seen at the center cross-section as the rail seat load distribution, usually idealized by design guidelines as a point load or a distributed load, can vary with the rail seat load magnitude and the existing support conditions, presenting a variable deep beam behavior.²⁵

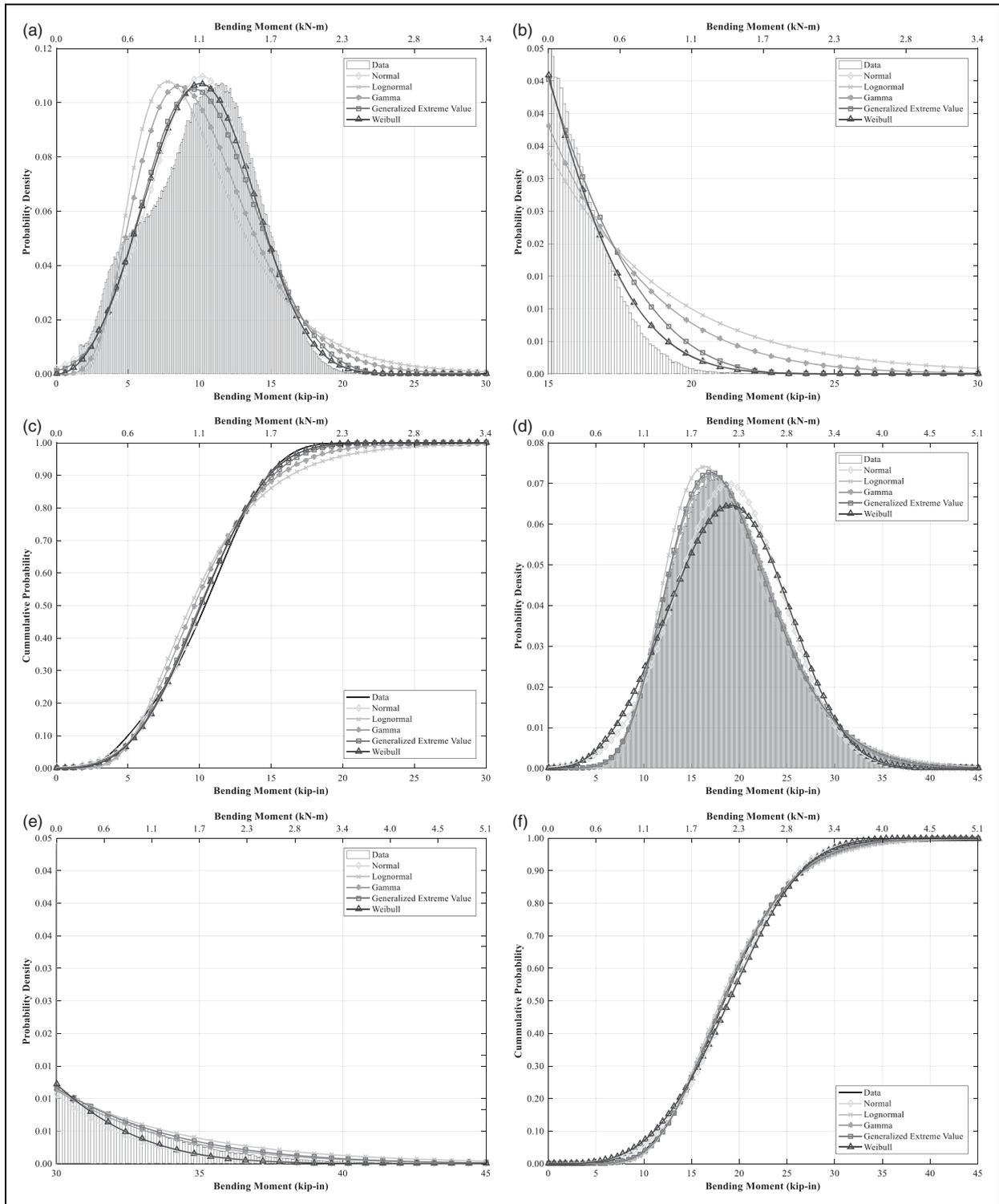


Figure 5. PDF and CDF of proposed demand random variables. (a) Center field bending moment histogram and probability density functions of the proposed probabilities for the fitting. (b) Right tail corresponding to highest values of the center bending moment and proposed probabilities for the fitting. (c) Cumulative density function of the center bending moment data and the proposed distributions. (d) Rail seat field bending moment histogram and probability density functions of the proposed probabilities for the fitting. (e) Right tail corresponding to highest values of the rail seat bending moment and the proposed probabilities for the fitting. (f) Cumulative density function of the rail seat bending moment data and the proposed distributions.

Capacity model and random variables

The capacity model is defined as a function of geometric and material properties. Given that the design

assessment is based on calculations at the critical cross-sections, the dimensions (height, bottom base width, and top width) as well as the reinforcement arrangement are parameters in the model.

The eccentricity of the reinforcement is derived from these. Concrete sleepers are precast elements, and their manufacture requires an intensive quality control during the manufacturing process, resulting in a largely homogenous product. For this reason, unlike cast in place concrete structures, the geometric properties of concrete sleepers present lower tolerances and the resulting product achieves higher accuracy. This has led the author to consider the aforementioned geometric characteristics as constant. Therefore, the values presented in Table 1 are considered the deterministic parameters of the limit-state functions.

The material properties of the concrete and prestressing steel are also required to develop the capacity model. Concrete's actual capacity has long been a topic of discussion and research. Its heterogeneous nature dictates that the primary design material property, which is the compressive strength (f'_c), has a very demanding acceptance criteria as specified by ACI 318.²⁶ With reinforcing steel, although the manufacturing process is more controlled, a similar issue surfaces. The concrete sleeper addressed in this article is prestressed; hence, the steel material properties are defined through the jacking force. This is defined by ACI 318 as 75% of the ultimate capacity of the steel wire.²⁶ The prestressing losses, which vary as a function of time, the manufacturing process, material properties of both concrete and steel, prestress level, element dimensions, or loading, among others,²⁷ are estimated and input into the model. The employed design approach limits cracking of the cross-section, bounding the problem to the linear elastic range. The jacking force (as all the wires have the same area) and losses are assumed to be the same for all prestressing

wires. Consequently, the concrete compressive strength and jacking force are defined as random variables of the capacity model. Thus, these variables shall be defined using probabilistic distributions that can closely capture their variability. Literature reveals extensive research on material properties, using similar studies to define the capacity model.^{24,28} Table 3 summarizes the different random variables used to define the capacity and demand models used in this study.

Limit-state functions

A limit-state function represents, for a failure mode, the boundary between failure and functionality of a component, being the location in which the capacity and demand model cancel each other. Mathematically, it is represented as shown in equation (3)

$$g(x) = C(x_1) - D(x_2) \quad (3)$$

where

- x_1 denotes the vector of random variables that define capacity;
- x_2 denotes the vector of random variables that define the demand;
- x denotes the vector of random variables' combination of x_1 and x_2 ;
- $g(x)$ denotes the limit-state function;
- $C(x_1)$ denotes the capacity model;
- $D(x_2)$ denotes the demand model.

Thus, when the limit-state function has a negative result, failure is achieved, as the demand is higher than the capacity. This analysis aims to evaluate the current design methodologies from the SRA point of view, meaning that the derived limit-state functions depict the analysis of the sleeper capacity at the sectional level. Two equations for each of the design cross-sections were derived, stress level at top and bottom fibers of the sleeper. As a linear elastic analysis is carried due to the fact that most sleepers are designed to not crack, the highest stresses are found at these locations. Following AREMA,² the compressive stress on concrete shall be limited to 60% of its

Table 2. Kullback–Leibler divergence test results.

| Distribution | Center BM | Rail seat BM |
|--------------|-----------|-----------------------|
| Normal | 0.0239 | 0.0232 |
| Lognormal | 0.1378 | 0.0136 |
| Gamma | 0.0735 | 0.0013 |
| GEV | 0.0277 | 0.0023 |
| Weibull | 0.0162 | Ruled out graphically |

BM: bending moment; GEV: Generalized Extreme Value.

Table 3. Defined random variables used in the capacity and demand models.

| Variable | Symbol | Distribution type | Units | Mean | Standard deviation |
|--------------------------------|----------|-------------------|---------------|--------------|--------------------|
| Concrete compressive strength | f'_c | Lognormal | MPa (ksi) | 48.26 (7.00) | 7.24 (1.05) |
| Jacking force | P_j | Normal | kN (kips) | 31.13 (7.00) | 1.89 (0.42) |
| Prestressing losses | $loss$ | Lognormal | % | 15.00 | 3.00 |
| Center field bending moment | M_c | Weibull | kN-m (kip-in) | 1.15 (10.17) | 0.40 (3.57) |
| Rail seat field bending moment | M_{rs} | Gamma | kN-m (kip-in) | 2.14 (18.92) | 0.65 (5.76) |

compressive strength (f'_c) as given by the manufacturer. On the other hand, the tensile stress is limited to the modulus of rupture, which is defined by ACI as 7.5 times of the square root of the concrete compressive strength (f'_c) for normal weight concrete.²⁶ For HSC, defined by ACI as concrete with compressive strength over 6000 psi, the modulus of rupture is accepted to range between 7.5 and 12 times of the square root of f'_c .²⁹ The lower bound of this formula was used for this investigation as information regarding the studied sleeper's concrete design mix, manufacturing process or testing procedures was limited. Furthermore, the design compressive strength of the concrete used in the manufacture of the studied sleepers should be compared to be on the low range (7000 psi) of HSC as defined by ACI. Equations (4) and (5) represent the top and bottom fiber stresses at the center cross-section, respectively

$$g_1(x) = 7.5\sqrt{f'_c} + \frac{P_i}{A_c}(1 - loss) + \frac{P_i e y_{t_c}}{I_c}(1 - loss) - \frac{M_{field} y_{t_c}}{I_c} \quad (4)$$

$$g_2(x) = 0.6f'_c - \frac{P_i}{A_c}(1 - loss) + \frac{P_i e y_{b_c}}{I_c}(1 - loss) - \frac{M_{field} y_{b_c}}{I_c} \quad (5)$$

where

- $\frac{P_i}{A_c}(1 - loss)$ represents the compressive stresses induced by the prestressing axial load after losses;
- $\frac{P_i e y_{t_c}}{I_c}(1 - loss)$ represents the stresses (compressive at the top, tensile at the bottom) induced by the moment after losses created by the prestressing eccentricity;
- $\frac{M_{field} y_{b_c}}{I_c}$ represents the stresses induced by the bending moment due to revenue service loads.

As the demand model depicts negative center bending moments, the stresses at the top fiber are limited to the tensile capacity of the concrete as defined by ACI 318.²⁶ Stresses at the bottom are limited to the maximum allowable compressive strength in prestressed structures, as defined by ACI 318.²⁶

Similarly, equations (6) and (7) represent the stresses at top and bottom, respectively, at the rail seat

cross-sections. Positive bending moments are found at the rail seats; hence, the limit-state equations (6) and (7) represent compressive stresses at top fiber and tensile stresses at bottom fiber, respectively. Symmetry is considered in this analysis; thus, the capacity of both rail seats is analyzed jointly

$$g_3(x) = 0.6f'_c - \frac{P_i}{A_{rs}}(1 - loss) + \frac{P_i e y_{t_{rs}}}{I_{rs}}(1 - loss) - \frac{M_{field} y_{t_{rs}}}{I_{rs}} \quad (6)$$

$$g_4(x) = 7.5\sqrt{f'_c} + \frac{P_i}{A_{rs}}(1 - loss) + \frac{P_i e y_{b_{rs}}}{I_{rs}}(1 - loss) - \frac{M_{field} y_{b_{rs}}}{I_{rs}} \quad (7)$$

Reliability analysis – results and discussion

To conduct the reliability analysis of the defined model, first-order reliability method (FORM), second-order reliability method (SORM), and Monte Carlo simulation (MCS) were used. Structural reliability software developed as a MATLAB toolbox at the University of California Berkeley³⁰ was used to conduct the analysis. Table 4 presents the obtained results for the four limit-state functions when analyzed using the three different methods.

Several conclusions can be drawn from the results shown in Table 4. First, given the existing design and current flexural demand on the sleepers, the rail seat section is closer to failure than the center, both at top and bottom fibers. Even though the capacity at the rail seat is higher, the larger demands at these sections demonstrate freshly tamped conditions, as discussed earlier. It is also remarkable how the probability of failure at top and bottom fibers on each of the critical cross-sections is similar. This reveals a balanced design, where the two design failure modes (cracking by excessive tensile stress and crushing by excessive compressive stress) are found at similar load levels. Failure by excessive compressive stress represents a very brittle failure; hence, this should be seen as a more critical case. This is considered by limiting the maximum compressive stress to the 60% of the

Table 4. Reliability index and probability of failure results for the studied limit-state functions.

| Limit-state function | FORM | | SORM | | MCS | |
|----------------------------|---------|-------|---------|----------|---------|----------|
| | β | P_f | β | P_f | β | P_f |
| Center top (g^1) | 10.5035 | – | – | 1.57E-26 | – | 4.03E-26 |
| Center bottom (g^2) | 9.5058 | – | – | 3.90E-22 | – | 4.24E-19 |
| Rail seat top (g^3) | 7.9068 | – | 7.8910 | – | 7.8910 | – |
| Rail seat bottom (g^4) | 7.7798 | – | 7.7695 | – | 7.7642 | – |

compressive strength of the concrete in prestressed structures, as defined by ACI 318²⁶ and referenced by AREMA.² Table 4 also shows that the results generated by the different methods (FORM, SORM, MCS) are similar. As concrete sleepers are designed not to crack, a linear elastic analysis is sufficient for capturing the mechanics of the problem. For this reason, FORM provides sufficiently accurate results, not requiring more involved and computationally heavier methods as SORM or MCS.

Additionally, the results reveal a large overdesign for the realized flexural demand. The probability of failure obtained at the four different limit-state functions is large, especially at the center, where the probability of failure and the reliability index obtained through the different methods cannot be translated to the other variable (equation (2)) as it exceeds the limits of the standard normal distribution. However, these results shall be compared with current research and industry practice. Limiting cracking is a type of Service Limit State, which is defined as a common check used in structural engineering. As previously discussed here, sleepers are typically designed using an ASD approach, where stresses are bounded to prevent crack initiation. The formulated model is considered to represent flexural failure in concrete sleepers, even though it does not match a traditional failure of a flexural Ultimate Limit State (ULS) in structural engineering. Ongoing research aims to address this topic; however, for the purpose of this study, the current design basis for concrete sleepers is considered. Previous research has developed the equivalent reliability indices to the safety factors used in current structural engineering design codes.^{31,32} As AREMA² refers to ACI 318,²⁶ this design code is taken as reference for comparison with the obtained results.

According to previous research in the field,³¹ the safety factors proposed for the ULS flexural design of prestressed beams by ACI 318²⁶ have an equivalent β ranging from 4.2 to 4.4, calculated through iteration of different material, geometry and load values with a target reliability index of 3.5. This equivalent β defines what is an acceptable design following the concrete structures design code in the US. Hence, when compared with the obtained β for each of the four limit-state functions developed for this model, the analyzed design proves largely conservative. This can be a consequence of the large uncertainty both in the input loads and the support conditions, which pushes the designer to overly conservative approaches to obtain a durable product.

Conclusions

A set of five prestressed monoblock concrete sleepers were instrumented in a light rail transit system. Flexural data were obtained at the two critical sections over a period of 14 months. The five sleepers

presented similar results at center and rail seats, showing a consistent support across the entire field installation. The atypical nature of the results stemming from relatively uniform support conditions on MetroLink presented a unique opportunity to develop a new approach for design. Field results were compared with the most relevant design standards, indicating that some of the assumptions within those standards are inaccurate. Consequently, uncertainty is introduced in the design process, requiring the industry to take a very conservative design approach.

Using the light rail transit data collected, the center and rail seat bending moment demands were fit using standard distributions in order to perform a probabilistic analysis of an existing light rail concrete sleeper design. Despite the large size of the data sets, which captured environmental and loading variability, the data closely fit the proposed probabilistic distributions.

A probabilistic approach through the use of SRA concepts was introduced, analyzing the flexural capacity of the light rail concrete sleeper design deployed at MetroLink. Stresses at the top and bottom fibers of the critical cross-sections were defined as the limit-state functions. Three methods were employed to analyze the probability of failure of the different limit-state functions: FORM, SORM, and MCS. The results revealed an overly conservative design that is unlikely to fail under the current loading conditions. Despite this, based on the documented level of demand and existing field support conditions, the rail seat cross-section demonstrated higher probability of failure. Furthermore, FORM presented similar results to the other, more computationally demanding, methods.

In conclusion, the noticeable discrepancy between the design capacity and demand that MetroLink sleepers experience presents a sub-optimal design solution. This lack of accuracy of the design due to the misunderstanding of the actual demands often results in over-dimensioned track components, leading to excessive and unnecessary infrastructure investment. The introduction of probabilistic concepts in the design of concrete sleepers aims to reduce the amount of uncertainty in the design, helping to close the existing gap between design and field performance.

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