Railroad Tie Spacing Related to Wheel-Load Distribution and Ballast Pressure

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Abstract: Currently, there is no formula for tie and direct fixation fastener (DFF) spacing. Because the rail does not bend between the tie/DFF, it is not possible to calculate spacing by the bending stress formula. Usually, rail-stress analysis confirms a trial spacing. Such analysis may support a longer spacing that might not address issues related to load distribution, pressure on formation, torsional rigidity, noise, and vibration. Thus, tie spacing is formulated from the perspective of wheel-load distribution and load dispersion by ballast, considering the fact that ties under the Benkel beam support the vertical load. The DFF/tie spacing is related to the characteristic length of the track, and this study shows that the desired DFF/tie spacing should be less than the characteristic length. Codes and practical examples validate spacing formulated for DFFs, concrete ties, and wood ties. **DOI:** 10.1061/(ASCE)SC.1943-5576.0000243. *© 2014 American Society of Civil Engineers.*

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Introduction

Tie spacing is formulated from the perspective of wheel-load distribution and load dispersion by ballast on the subballast. Owing to the elasticity of the railroad, rails sag under the wheels and act as distributors of the load. Load distribution is dependent on the tie and axle spacing, track modulus, and flexural strength of the rails. The distribution of the reaction force may be calculated by using the formula of the beam on elastic foundation model. Mathematically, the distributed reaction force is given as (Esveld 2001)

$$r(x) = \frac{V}{2L}\eta(x)$$

$$\eta(x) = e^{-x/L}\left(\cos\frac{x}{L} + \sin\frac{x}{L}\right)$$

in which $L =$ characteristic length given by $\sqrt[4]{4EI/u}$.

Under the wheel load $\eta(x) = 1$

$$r(x = 0) = \frac{V}{2L}$$

Without considering the reducing effect of the adjacent wheel, the reaction force under the wheel load is

$$R(x = 0) = \frac{VS}{2L} = 0.5V\frac{S}{L} \tag{1}$$

If $S = L$, $R(x = 0) = 0.5V$.

In reality, the reaction under the wheel will be less than 50% of the wheel load owing to the reducing effect of the adjacent wheel. Thus, the use of Eq. (1) is conservative; the equation is used in this paper for simplicity. Tie spacing less than the characteristic length is a good choice, because it reduces the load [cf. Eq. (1)] on the tie under the axle, resulting in a reduced tie section and less ballast pressure under the tie and on the subballast. This fact is used to determine tie spacing by developing assumptions about the number of ties under the Benkel beam. Fig. 1 shows a typical distribution of the wheel load and the ballast pressure (Lichtberger 2005).

Fig. 1 shows that the tie on which the wheel is standing usually takes up 40% of the wheel load; the neighboring ties together take up 50%, and the following ties together take up 10%. It is possible that five ties will take up 100% of the wheel load with some similar distribution. The stiffer the subsoil conditions and the thinner the ballast bed, the higher the load applied to the central tie (Lichtberger 2005). From Fig. 1

$$h = \frac{S}{2\tan\alpha} \tag{2}$$

For new, sharp-edged ballast, $\alpha = 42°$; for used ballast, $\alpha = 39°$ (Lichtberger 2005). In other literature, for new, sharp-edged ballast, $\alpha = 45°$ (Agarwal 2002). Ballast loses elasticity with contamination. For contaminated ballast, $\alpha = 30°$ (Lichtberger 2005).

The optimum ballast thickness is given by Eq. (2). It is optimum because the thickness ensures the overlap of load-dispersion lines between two ties equal to the extent of the bottom width of the tie (w). Pressure-distribution lines should intersect; otherwise, the subballast would be pressed up between the ties into heaps (Lichtberger 2005). If tie spacing increases by the width of the tie (w) or ballast thickness decreases by $w/(2\cdot\tan\alpha)$ from the optimum values shown in Fig. 1, the pressure on the formation would remain uniform, according to Fig. 2.

Overlap of load-dispersion lines does not exist in Fig. 2; however, this load dispersion is not unacceptable, because overlap of load-dispersion lines is not the only factor that helps prevent soil from heaping up between the ties. Other major factors are the surcharge load of ballast, the shearing strength of ballast and soil, the bearing capacity of soil, and the contact pressure between ballast and

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the subballast. Thus, the pressure distribution in Fig. 2 is also acceptable. From Fig. 2

$$h^* = \frac{S^* - w}{2\tan\alpha} \tag{3}$$

The spacing in Eq. (3) may be considered the maximum allowable spacing. It appears that it is not strictly necessary to provide the optimum ballast bed thickness shown in Fig. 1. Either ballast thickness [Eqs. (2) or (3)] is acceptable.

Eqs. (2) and (3) are not used in the North American rail industry to compute the ballast thickness from the tie spacing or the tie spacing from the ballast thickness. However, a uniform pressure on the subballast is necessary to prevent uneven settlement. American Railway Engineering and Maintenance-of-Way Association (AREMA) suggests a formula to compute the combined thickness of ballast and the subballast and recommends a ballast thickness of 300 mm (12 in.) below the tie for practical reasons (AREMA 2013).

The optimum thickness of the ballast bed is usually 250–300 mm (10–12 in.) measured from the lower side of the tie (Esveld 2001). Fig. 3 shows a deflected shape of a rail under a wheel load.

Deflection at any distance (x) from the wheel load is given by (Esveld 2001)

$$\Delta(x) = \frac{V}{2uL}\eta(x)$$

in which u = track modulus, which is defined as the distributed track stiffness. Typical track modulus values for some tie

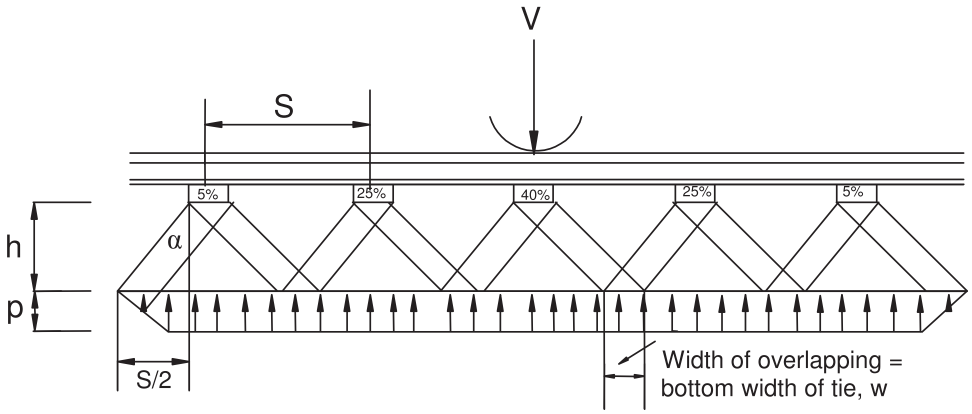

Fig. 1. Schematic representation for calculation of optimum ballast bed thickness

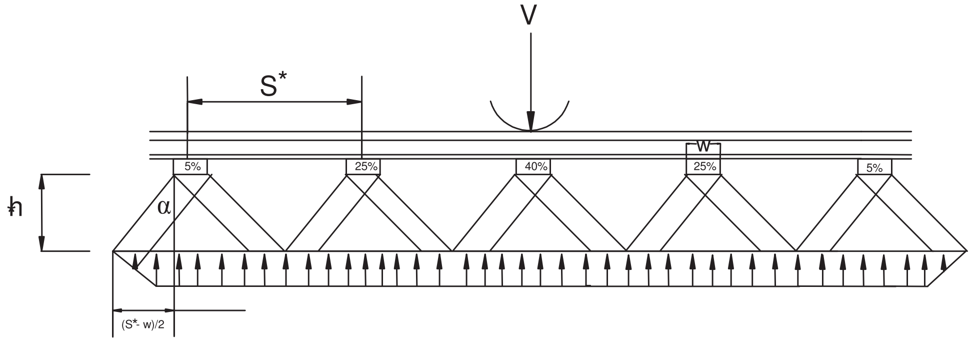

Fig. 2. Alternate schematic representation for calculation of ballast bed thickness

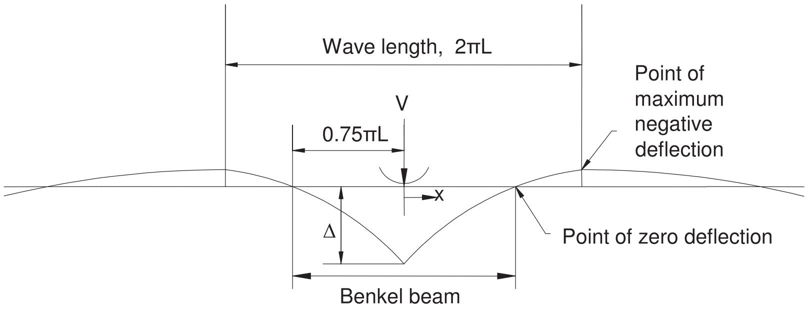

Fig. 3. Deflected shape of rail under load

configurations are given by AREMA (2013): $u = 20.7$ kN/mm (3,000 lbs/in./in.) for a wood tie track compacted by traffic; $u = 41.4$ kN/mm (6,000 lbs/in./in.) for a concrete tie track compacted by traffic. The $\nu(x)$ is a shape function which determines the form of the elastic line and is given by

$$\eta(x) = e^{-x/L}\left(\cos\frac{x}{L} + \sin\frac{x}{L}\right)$$

The span between the points of zero deflection of a rail on either side of the load, as shown in Fig. 3, is termed the Benkel beam. At $x = (3/4)\pi L$, $\eta(x) = 0$ and $\Delta(x) = 0$. Thus, the span of the beam is given by (Esveld 2001)

$$2 \cdot \frac{3}{4}\pi L = 1.5\pi L = 1.5\pi\sqrt[4]{\frac{4EI}{u}} \tag{4}$$

Beyond the Benkel beam, the rail lifts up, as shown in Fig. 3. The uplift load is taken by the anchor and/or clip fixed with the tie. The vertical downward load is taken by the ties under the Benkel beam (Fig. 3). The ballast compression value is a characteristic parameter indicating the stability of track geometry. Therefore, desirable load distribution on the ties is a top priority. Thus, at least seven ties should be assumed under the Benkel beam: two ties under the points of zero deflection, which theoretically take almost no vertical load, and five ties sharing the full wheel load, according to Figs. 1 or 2. Tie spacing equal to the characteristic length suggests six $(1.5\pi + 1)$ ties under the Benkel beam [Eq. (4)]. Tie spacing should be less than the characteristic length to ensure the desirable distribution of the load on the ties and to minimize the load on each tie. Thus, there should be at least seven ties. The spacing given by seven ties is labeled as the absolute maximum spacing of the concrete tie and is given by

$$S^C_{\max}(\text{absolute}) = \frac{1.5\pi L}{(7-1)} = 0.25\pi\sqrt[4]{\frac{4EI}{u}} = 0.78L \tag{5}$$

Eq. (5) suggests tie spacing equal to approximately 80% of the characteristic length, which implies that the tie under the rail seat would share 40% of the wheel load [Eq. (1)].

For the same rail section, the Benkel-beam span for a wood-tie track [assuming $u = 17.25$ kN/mm (2,500 lbs/in./in.)] is $\sqrt[4]{2}$ ($\sqrt[4]{41.4/20.7}$) times the Benkel-beam span for a concrete-tie track [assuming $u = 34.5$ kN/mm (5,000 lbs/in./in.)]. Thus, the number

Table 1. Absolute Maximum Tie Spacing [Eqs. (5) and (6)]

Rail section	Moment of inertia, I [mm^4 (in.^{4})]	Young's modulus, E [N/mm^2 (psi)]	Track modulus, u [kN/mm (lbs/in./in.)]	$S^W_{\max}$ (absolute) [mm (in.)]	Track modulus, u [kN/mm (lbs/in./in.)]	$S^C_{\max}$ (absolute) [mm (in.)]
RE100	20,191,386 (48.51)	206,800 (30.10^6)	20.7 (3,000)	635 (25)	41.4 (6,000)	635 (25)
RE115	27,263,158 (65.5)	206,800 (30.10^6)	20.7 (3,000)	686 (27)	41.4 (6,000)	686 (27)
RE119	29,718,924 (71.4)	206,800 (30.10^6)	20.7 (3,000)	686 (27)	41.4 (6,000)	686 (27)
RE132	36,586,742 (87.9)	206,800 (30.10^6)	20.7 (3,000)	737 (29)	41.4 (6,000)	737 (29)
RE133	35,879,149 (86.2)	206,800 (30.10^6)	20.7 (3,000)	711 (28)	41.4 (6,000)	711 (28)
RE136	39,209,000 (94.2)	206,800 (30.10^6)	20.7 (3,000)	737 (29)	41.4 (6,000)	737 (29)
RE140	39,916,594 (95.9)	206,800 (30.10^6)	20.7 (3,000)	737 (29)	41.4 (6,000)	737 (29)

Note: $S^C_{\max}(\text{absolute}) = 0.25\pi\sqrt[4]{4EI/u}$; $S^W_{\max}(\text{absolute}) = 0.21\pi\sqrt[4]{4EI/u}$.

Table 2. Absolute Maximum Tie Spacing [Eqs. (5) and (6)]

Rail section	Moment of inertia, I [mm^4 (in.^{4})]	Young's modulus, E [N/mm^2 (psi)]	Track modulus, u [kN/mm (lbs/in./in.)]	$S^W_{\max}$ (absolute) [mm (in.)]	Track modulus, u [kN/mm (lbs/in./in.)]	$S^C_{\max}$ (absolute) [mm (in.)]
RE100	20,191,386 (48.51)	206,800 (30.10^6)	17.25 (2,500)	660 (26)	34.5 (5,000)	660 (26)
RE115	27,263,158 (65.5)	206,800 (30.10^6)	17.25 (2,500)	711 (28)	34.5 (5,000)	711 (28)
RE119	29,718,924 (71.4)	206,800 (30.10^6)	17.25 (2,500)	711 (28)	34.5 (5,000)	711 (28)
RE132	36,586,742 (87.9)	206,800 (30.10^6)	17.25 (2,500)	762 (30)	34.5 (5,000)	762 (30)
RE133	35,879,149 (86.2)	206,800 (30.10^6)	17.25 (2,500)	762 (30)	34.5 (5,000)	762 (30)
RE136	39,209,000 (94.2)	206,800 (30.10^6)	17.25 (2,500)	762 (30)	34.5 (5,000)	762 (30)
RE140	39,916,594 (95.9)	206,800 (30.10^6)	17.25 (2,500)	787 (31)	34.5 (5,000)	787 (31)

Note: $S^C_{\max}(\text{absolute}) = 0.25\pi\sqrt[4]{4EI/u}$; $S^W_{\max}(\text{absolute}) = 0.21\pi\sqrt[4]{4EI/u}$.

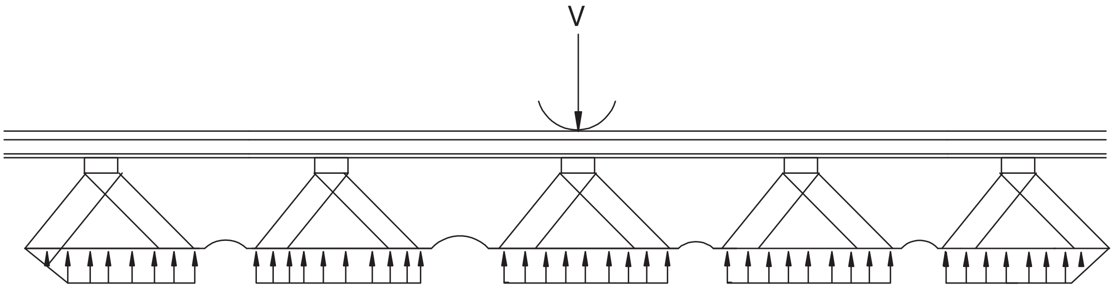

Fig. 4. Undesirable load dispersion pattern for heavy axle load

of ties under the Benkel beam is increased to eight ($7 \cdot \sqrt[4]{2} \approx 8$) for deciding the absolute maximum spacing of the wood tie. The spacing given by eight ties is labeled as the absolute maximum spacing of the wood tie and is given by

$$S^{W}_{\max}(\text{absolute}) = \frac{1.5\pi L}{(8-1)} = 0.21\pi\sqrt[4]{\frac{4EI}{u}} = 0.67L \qquad (6)$$

Eq. (6) suggests a tie spacing equal to approximately 67% of the characteristic length, which implies that the tie under the rail seat would share 33.5% of the wheel load [Eq. (1)].

Tables 1 and 2 show computations for the absolute maximum tie spacing for wood and concrete ties. From Table 1, the absolute maximum wood and concrete-tie spacing is 635–740 mm (25–29 in.), with the assumed track moduli of $u = 20.7$ kN/mm (3,000 lbs/in./in.) and $u = 41.4$ kN/mm (6,000 lbs/in./in.), respectively. From Table 2, the absolute maximum wood and concrete-tie spacing is 660–790 mm (26–31 in.), with the assumed track moduli of $u = 17.25$ kN/mm (2,500 lbs/in./in.) and $u = 34.5$ N/mm/mm (5,000 lbs/in./in.), respectively. Interestingly, the maximum wood and concrete-tie spacing shown in Fig. 30-1–1 in the AREMA manual (AREMA 2013) is 813 mm (32 in.). The maximum concrete-tie spacing shown in Fig. 30-4–3 in the AREMA manual (AREMA 2013) is 790 mm (31 in.). The absolute maximum spacing computed in Tables 1 and 2 does not exceed the maximum value shown in the preceding AREMA figures.

The characteristic lengths of 700 and 1,300 mm (28 and 51 in.) represent the good and poor conditions of the track foundation, respectively (Esveld 2001). Thus, if a characteristic length of 1,000 mm [(700 + 1,300 mm)/2] is assumed as a typical representative value of a concrete-tie-ballasted track, the absolute maximum concrete-tie spacing is 800 mm [80% of 1,000 mm (32 in.)]. The typical value of the characteristic length of a wood-tie-ballasted track should be 1,190 mm ($1{,}000\ \text{mm} \cdot \sqrt[4]{2}$), and the absolute maximum wood-tie spacing is 798 mm (67% of 1,190 mm) (31 in.).

To ensure safety on Class 4 and 5 tracks, the Federal Railroad Authority (FRA 2008) requires a minimum of 12 serviceable ties in a 39-ft segment of rail. This provision is not a design requirement; it is a safety requirement. From this requirement, a spacing of 1,080 mm (42 in.) may be labeled as the absolute maximum spacing from a safety point of view. Thus, the absolute maximum tie spacings given by Eqs. (5) and (6) seem to be acceptable.

Many factors may cause the undesirable load-dispersion patterns shown in Fig. 4, such as contaminated ballast and longer tie spacing with less ballast cushion. The load-dispersion pattern in Fig. 4 is generally undesirable, because it may lead to uneven track settlement. Of course, the desirability of the load-dispersion pattern depends on the wheel load. On a light rail transit (LRT) track, the ballast cushion is much less than it is on a heavy haul or passenger train track, which might lead to a load-dispersion pattern like the one shown in Fig. 4, even with high-quality ballast. The load-dispersion pattern shown in Fig. 4 may not be undesirable for LRT, where the nominal axle load is about 100 kN (10 t). Thus, the absolute maximum spacing with seven ties under the Benkel beam may be used for a LRT-ballasted track.

Table 3. DFF Spacing [Eq. (7)]

Rail section	Moment of inertia, I [mm^4 (in.^{4})]	Young's modulus, E (MPa)	Sitffness of DFF, K (N/mm)	DFF spacing, S_{DFF} [mm (in.)]
RE 100	20,190,000 (48.51)	206,800	20,000	682 (27)
RE115	27,263,158 (65.5)	206,800	20,000	754 (30)
RE119	29,718,924 (71.4)	206,800	20,000	776 (31)

Note: $S_{\text{DFF}} = (\pi/4)^{4/3}\sqrt[3]{4EI/K}$.

Development of Assumption

The absolute maximum spacing that corresponds to seven direct fixation fasteners (DFFs) under the Benkel beam is recommended for a LRT direct fixation track, because it is ballastless. For many reasons, the absolute maximum spacing may not be desirable for a ballasted track. Thus, it is necessary to develop an assumption of the number of ties under the Benkel beam that would help achieve some desirable objectives.

Under a tie with an absolute maximum spacing of 750 mm (30 in.), new, sharp-edged ballast ($\alpha = 42°$) with a thickness of 425 mm (16.65 in.) [Eq. (2)] should be used to achieve a desirable pressure-distribution pattern, as shown in Fig. 1. In North America, the preferred ballast thickness is 300 mm (12 in.). It is not feasible to achieve a desirable pressure-distribution pattern, as shown in Fig. 1, at the cost of excess ballast. Thus, it is necessary to reduce the spacing to achieve a pressure-distribution pattern, as shown in Fig. 1, with 300-mm-thick ballast (12-in.-thick ballast). Furthermore, during service, ballast loses elasticity because of contamination, resulting in a smaller load-dispersion angle, for example, $\alpha = 30°$. The load-dispersion pattern may shift gradually from that depicted in Figs. 1 and 2 to that shown in Fig. 4, which is undesirable. The situation would be more undesirable under the absolute maximum tie spacing, because the length of the nonoverlapping pressure line would be longer. The absolute minimum number of seven ties under the Benkel beam leads to the absolute maximum spacing, equal to 80% of the characteristic length, implying that the tie under the rail seat would share 40% of the wheel load. To reduce the load on the tie and the ballast pressure under the tie and on the subballast, it is desirable to increase the number of ties under the Benkel beam. Reducing the load on a tie is important because of the repeated nature of the wheel load. A concrete tie is capable of supporting more load than a wood tie is capable of supporting. Thus, industry standards usually recommend more spacing for concrete ties than for wood ties. For example, the Burlington Northern Santa Fe (BNSF) standard recommends 545 mm (21.5 in.) of spacing for wood ties and 710 mm (28 in.) of spacing for concrete ties. Tie spacing has an appreciable effect on many other parameters, such as the rail deflection, the rail break gap, the torsional rigidity, and the natural frequency. If the spacing is doubled, the deflection and reaction on ties are increased by 1.68 ($2^{0.75}$) times. For the same rail section, the Benkel-beam span for a wood-tie track [assuming $u = 20.7$ kN/mm (2,500 lbs/in./in.)] is $\sqrt[4]{2}$ ($\sqrt[4]{41.4/20.7}$) times the Benkel-beam

Table 4. Flexural Stress in Rail under Wheel Load, $K = 20{,}000$ N/mm (113,925 lbs/in.)

Rail section	Section modulus, Z (mm^3)	Axle load (kN)	Characteristic length, L [mm (in.)]	Impact factor	Moment reducing factor	Net moment (N-mm)	Stress [kPa (psi)]
RE100	244,000	100	869 (34)	1.3	0.85	12,000,736	49,000 (7,105)
RE115	358,877	100	960 (38)	1.3	0.85	13,264,406	37,000 (5,365)
RE119	373,625	100	988 (39)	1.3	0.85	13,651,283	37,000 (5,365)

span for a concrete-tie track [assuming $u = 41.4$ kN/mm (5,000 lbs/in./in.)]. Thus, the number of ties under the Benkel beam is increased to 9 for concrete ties and 11 ($9 \cdot \sqrt[4]{2} \approx 11$) for wood ties. This assumption helps to achieve the following objectives:

- Derive desirable tie spacing.
- Reduce the load shared by the tie under the axle, which would reduce the ballast pressure under the tie and, subsequently, on the subballast. This is important because of the repeated nature of the wheel load.
- Avoid a thicker ballast bed to attain the desirable pressure, as shown in Fig. 1.
- Avoid undesirable load dispersion, as shown in Fig. 4, due to the contamination of ballast during service.
- Increase the torsional rigidity of the track, helping to prevent rail rollover derailments.
- Reduce the natural frequency of the track.
- Increase the dead weight of the track [it is generally assumed that the dead weight of the track compensates for the negative deflection (Esveld 2001)].
- Reduce the rail deflection (high rail deflection results in the excessive mechanical action of all track components, leading to reduced tie life; the rapid deterioration of the surface and line; ballast degradation; and the accelerated wear of joints, fittings, turnouts, and loose bolts).
- Reduce the gap in case of a rail break, which is a requirement for safe operation.
- Allow some percentage of unserviceable ties without speed reduction during service.

The assumptions of 9 concrete ties and 11 wood ties offer the preceding benefits. With time, some ties would become unserviceable. Using the preceding assumption, one can compute the allowable number of unserviceable ties. It has been shown that seven and nine concrete ties under the Benkel beam lead to the absolute maximum and desirable concrete-tie spacing. Thus, the acceptable percentage of unserviceable ties in a concrete-ballasted track would be 22% $[(9-7)/9]$, which is nearly one in every four ties. Theoretically, 22% unserviceable ties [ties incapable of bearing load and bending moment (e.g., broken ties or split ties)] may be allowed in a concrete-ballasted track without speed reduction if the track is designed with the desirable tie spacing. Similarly, the acceptable percentage of unserviceable ties in a wood-ballasted track would be 27% $[(11-8)/11]$, which is nearly one in every three ties. Theoretically, 27% unserviceable ties [ties incapable of bearing load and bending moment (e.g., broken ties, split ties, or ties cut by the tie plate through more than 40% of a tie's thickness)] may be allowed in a wood-ballasted track without speed reduction if the track is designed with the desirable tie spacing. The current practice allows more unserviceable ties on tracks.

Table 5. DFF Spacing [Eq. (7)]

Rail section	Moment of inertia, I [mm^4 (in.^{4})]	Young's modulus, E (MPa)	Stiffness of DFF, K (N/mm)	DFF spacing, S_{DFF} [mm (in.)]
RE 100	20,190,000 (48.51)	206,800	30,000	596 (23)
RE115	27,263,158 (65.5)	206,800	30,000	659 (26)
RE119	29,718,924 (71.4)	206,800	30,000	678 (27)

The BNSF standards recommend 545 mm (21.5 in.) of spacing for wood ties and 710 mm (28 in.) of spacing for concrete ties; the numbers of concrete and wood ties in a 11.9-m (39-ft) segment would be 18 $(1 + 39 \cdot 12/28)$ and 23 $(1 + 39 \cdot 12/21.5)$, respectively. To ensure safety on Class 4 and 5 tracks, the FRA (2008) requires a minimum of 12 serviceable ties in a 39-ft segment of rail. The FRA requirement (FRA 2008) is not a design requirement; the requirement is a safety requirement. According to FRA regulations (FRA 2008), the maximum allowable percentages of unserviceable concrete and wood ties are 33% $[(18-12)/18]$ and 48% $[(23-12)/23]$, respectively, on Class 4 and 5 tracks. Thus, BNSF may allow a maximum of 33% (cf. 22%) and 48% (cf. 27%) unserviceable concrete and wood ties, respectively, on Class 4 and 5 tracks. The author does not know the current BNSF practice regarding unserviceable ties on tracks. It is understood that more unserviceable ties may be allowed with speed restrictions. On track classes lower than Class 4, the number of allowable unserviceable ties may be greater owing to lower speeds.

Because the pressure distribution shown in Fig. 2 is also acceptable, the maximum allowable spacing is obtained by adding the desirable spacing and the bottom width of the tie. The absolute maximum spacing assumption of seven and eight ties under the Benkel beam is explained in the preceding section. The assumption of 7 DFF and 9 or 11 ties is validated by practical examples and codes in the "Validation of Assumption" section.

Table 6. Flexural Stress in Rail under Wheel Load, $K = 30{,}000$ N/mm

Rail section	Section modulus, Z (mm^3)	Axle load (kN)	L [mm (in.)]	Impact factor	Moment reducing factor	Net moment (N-mm)	Stress [kPa (psi)]
RE100	244,000	100	759 (30)	1.3	0.85	10,483,608	43,000 (6,235)
RE115	358,877	100	839 (33)	1.3	0.85	11,587,526	32,000 (4,640)
RE119	373,625	100	863 (34)	1.3	0.85	11,925,494	32,000 (4,640)

Table 7. Desirable Concrete-Tie Spacing [Eq. (8)]

Rail section	Moment of inertia, I [mm^4 (in.^{4})]	Young's modulus, E [N/mm^2 (psi)]	u [kN/mm (lbs/in./in.)]	L [mm (in.)]	S_C [mm (in.)]	Ties/m (ties/km)
RE100	20,191,386 (48.51)	206,800 (30.10^6)	41.4 (6,000)	787 (31)	457 (18)	2,129 (3,427)
RE115	27,263,158 (65.5)	206,800 (30.10^6)	41.4 (6,000)	864 (34)	508 (20)	1,975 (3,179)
RE119	29,718,924 (71.4)	206,800 (30.10^6)	41.4 (6,000)	889 (35)	508 (20)	1,933 (3,112)
RE132	36,586,742 (87.9)	206,800 (30.10^6)	41.4 (6,000)	914 (36)	533 (21)	1,835 (2,954)
RE133	35,879,149 (86.2)	206,800 (30.10^6)	41.4 (6,000)	914 (36)	533 (21)	1,843 (2,968)
RE136	39,209,000 (94.2)	206,800 (30.10^6)	41.4 (6,000)	940 (37)	559 (22)	1,803 (2,903)
RE140	39,916,594 (95.9)	206,800 (30.10^6)	41.4 (6,000)	940 (37)	559 (22)	1,795 (2,890)

Note: $S_C = (3/16)\pi\sqrt[4]{4EI/u}$.

Formulation of DFF/Tie Spacing

DFF Spacing

Although most tracks still use traditional ballast, recent applications tend increasingly toward nonballasted track. In ballasted track, each tie can contribute to distortions in track geometry, because ballasted track is a low-fixity track. In the case of high-fixity slab track, rail fasteners are in fixed positions. Currently, slab track is used mainly for high-speed lines and light rails. The DFF spacing is given by

$$S_{\mathrm{DFF}} = \frac{1.5\pi L}{(7-1)} = \frac{\pi}{4}\sqrt[4]{\frac{4EI}{u}} = 0.79L$$

Table 8. Optimum Ballast Thickness under Concrete Tie

Rail section	S_C [mm (in.)]	α (degrees)	h [mm (in.)] [Eq. (2)]	α (degrees)	h [mm (in.)] [Eq. (2)]
RE100	457 (18)	39	279 (11)	45	229 (9)
RE115	508 (20)	39	300 (12)	45	254 (10)
RE119	508 (20)	39	330 (13)	45	254 (10)
RE132	533 (21)	39	330 (13)	45	279 (11)
RE133	533 (21)	39	330 (13)	45	279 (11)
RE136	559 (22)	39	330 (13)	45	279 (11)
RE140	559 (22)	39	356 (14)	45	279 (11)

The DFF spacing is about 80% of the characteristic length, implying that each DFF under a wheel would share 40% of the wheel load [Eq. (1)].

For a direct fixation track

$$u = \frac{K}{S_{\mathrm{DFF}}}$$
$$S_{\mathrm{DFF}} = \left(\frac{\pi}{4}\right)^{4/3}\sqrt[3]{\frac{4EI}{K}} \tag{7}$$

Concrete-Tie Spacing

The desirable concrete-tie spacing is given by

$$S_C = \frac{1.5\pi L}{(9-1)} = \frac{3}{16}\pi\sqrt[4]{\frac{4EI}{u}} = 0.59L \tag{8}$$

The desirable concrete-tie spacing is about 60% of the characteristic length, implying that the concrete tie under the rail seat would share 30% of the wheel load [Eq. (1)].

The maximum allowable concrete-tie spacing is derived by adding the bottom width of the tie to the desirable spacing, as given by

$$S^C_{\max}(\text{allowable}) = S_C + w \leq S_{\max}(\text{absolute}) \tag{9}$$

The absolute maximum concrete-tie spacing is given by Eq. (5).

Table 9. Flexural Stress in Rail under Wheel Load

Rail section	Section modulus, Z [mm^3 (in.^{3})]	Axle load [kN (t)]	L [mm (in.)]	Impact factor	Moment reduction factor	Net moment [kN·m (t-in.)]	Stress [kPa (psi)]
RE100	244,167 (14.9)	110 (10)	838 (33)	1.3	0.85	11.53 (45.37)	42,000 (6,094)
RE115	358,877 (21.9)	150 (15)	889 (35)	1.3	0.85	18.64 (73.36)	46,000 (6,700)
RE119	373,625 (22.8)	150 (15)	914 (36)	1.3	0.85	19.04 (74.96)	45,000 (6,576)
RE132	449,006 (27.4)	200 (20)	965 (38)	1.3	0.85	26.75 (105.28)	53,000 (7,685)
RE133	440,812 (26.9)	200 (20)	965 (38)	1.3	0.85	26.62 (104.77)	54,000 (7,790)
RE136	462,115 (28.2)	200 (20)	991 (39)	1.3	0.85	27.72 (107.12)	52,000 (7,597)
RE140	468,670 (28.6)	350 (35)	991 (39)	1.3	0.85	47.84 (188.30)	91,000 (13,168)

Table 10. Ballast Pressure under Concrete Tie (psi)

Axle load kN (t)	Distribution factor	Impact factor	Footprint area of tie	Ballast pressure under tie [kPa (psi)]
100 (10)	0.6	1.3	2,515 × 250 mm (8 ft 3 in. × 10 in.)	170 (24)
150 (15)	0.6	1.3	2,515 × 250 mm (8 ft 3 in. × 10 in.)	250 (36)
200 (20)	0.6	1.3	2,515 × 300 mm (8 ft 3 in. × 12 in.)	270 (39)
250 (25)	0.6	1.3	2,515 × 300 mm (8 ft 3 in. × 12 in.)	330 (48)
350 (35)	0.6	1.3	2,515 × 330 mm (8 ft 3 in. × 13 in.)	390 (53)

Table 11. Desirable Wood-Tie Spacing [Eq. (10)]

Rail	Moment of inertia, I [mm^4 (in.^{4})]	Young's modulus, E [N/mm^2 (psi)]	u [kN/mm (lbs/in./in.)]	L [mm (in.)]	S_W [mm (in.)]	Ties/m (ties/km)
RE100	20,191,386 (48.51)	206,800 (30.10^6)	20.7 (3,000)	940 (37)	457 (18)	2,186 (3,520)
RE115	27,263,158 (65.5)	206,800 (30.10^6)	20.7 (3,000)	1,016 (40)	483 (19)	2,071 (3,335)
RE119	29,718,924 (71.4)	206,800 (30.10^6)	20.7 (3,000)	1,041 (41)	483 (19)	2,071 (3,335)
RE132	36,586,742 (87.9)	206,800 (30.10^6)	20.7 (3,000)	1,092 (43)	508 (20)	1,968 (3,168)
RE133	35,879,149 (86.2)	206,800 (30.10^6)	20.7 (3,000)	1,092 (43)	508 (20)	1,968 (3,168)
RE136	39,209,000 (94.2)	206,800 (30.10^6)	20.7 (3,000)	1,118 (44)	533 (21)	1,874 (3,017)
RE140	39,916,594 (95.9)	206,800 (30.10^6)	20.7 (3,000)	1,118 (44)	533 (21)	1,874 (3,017)

Note: $S_W = 0.15\pi\sqrt[4]{4EI/u}$.

Wood-Tie Spacing

The desirable wood-tie spacing is given by

$$S_W = \frac{1.5\pi L}{(11-1)} = 0.15\pi\sqrt[4]{\frac{4EI}{u}} = 0.47L \tag{10}$$

The desirable wood-tie spacing is about 50% of the characteristic length, implying that the wood tie under the rail seat would share 25% of the wheel load [Eq. (1)].

The maximum allowable spacing is derived by adding the bottom width of the tie to the desirable spacing, as given by

$$S^W_{\max}(\text{allowable}) = S_W + w \leq S_{\max}(\text{absolute}) \tag{11}$$

The absolute maximum wood-tie spacing is given by Eq. (6).

Validation of Assumption

For a DFF, the assumption of seven DFFs on a DF track under the Benkel beam is validated by the outcome of the formulas in the contexts of DFF spacing and rail stress. Tables 3–6 calculate the DFF spacing and the rail stress. Some examples are subsequently given of DFF spacing from real-world projects.

On Millennium Line in Vancouver, Canada

For a 115 RE rail, the DFF stiffness, K, is 20 kN/mm (113,925 lbs/in.); the spacing on the curve with $R < 1{,}500$ m is 750 mm (30 in. for $R < 457$ ft); and the spacing on a tangent track or curve with $R \geq 1{,}500$ m is 1,000 mm (40 in. for $R \geq 457$ ft).

Table 12. Optimum Ballast Thickness under Wooden Tie

Rail	S_W [mm (in.)]	A (degrees)	h [mm (in.)] [Eq. (2)]	α (degrees)	h [mm (in.)] [Eq. (2)]
RE100	457 (18)	39	279 (11)	45	229 (9)
RE115	483 (19)	39	300 (12)	45	229 (9)
RE119	483 (19)	39	300 (12)	45	254 (10)
RE132	508 (20)	39	330 (13)	45	254 (10)
RE133	508 (20)	39	330 (13)	45	254 (10)
RE136	533 (21)	39	330 (13)	45	254 (10)
RE140	533 (21)	39	330 (13)	45	254 (10)

Low flexural stress in the rail might cause structural engineers to increase the DFF spacing. However, a spacing of 1,000 mm is not a good choice, because it exceeds the characteristic length, and DFF under a wheel would share more than 50% of the wheel load. The track modulus of a direct fixation track is given by $u = K/S_{\text{DFF}}$. Thus, with greater spacing, the track modulus would decrease, and the characteristic length would increase. Consequently, the wavelength of the track would increase (Fig. 3). With greater spacing, more noise would be generated by the rail, because the greater length of the rail would vibrate with each wheel of a train, and rail vibration propagates for a long distance along the rail. In case of a failure of a DFF for any reason, the problem would be aggravated, because the spacing would be doubled. In case of a rail break, the gap would be larger if there were more space between DFFs, which might be unsafe. The design of spacing by engineering, procurement, and construction (EPC) companies may be driven by cost, to maximize the likelihood of successful bidding.

On Canada Line in Vancouver, Canada

For a 115 RE rail, the DFF stiffness, K, is 20 kN/mm (113,925 lbs/in.), and the spacing on the tangent track is the same as in the preceding example. The comments are the same as in the preceding example.

Concrete Ties

The assumption of nine concrete ties under the Benkel beam is validated by the outcome of the formulas in the contexts of tie spacing, ballast thickness, rail stress, and ballast pressure under ties. These parameters are computed in Tables 7–10, respectively.

As a general rule, the minimum number of ties one might expect to find on all but the lightest industrial lines or transit lines would be 1,863 ties/km (~3,000 ties/m). However, some standards specify more ties, with tie spacing of 510 mm (20 in.); others require as many as 24 ties per 11.9-m (39-ft) panel, or 2,019/km (~3,250/m). One new line standard requires tie spacing as close as 495 mm (19.5 in.). However, this is about the limit for a nominal-size tie, because spacing smaller than this may pose difficulties for surfacing equipment. Thus, this tie spacing (cf. Table 7) is acceptable practically.

Table 13. Flexural Stress in Rail under Wheel Load

Rail	Section modulus, Z [mm^3 (in.^{3})]	Axle load [kN (t)]	L [mm (in.)]	Impact factor	Moment reduction factor	Net moment [kN·m (t-in.)]	Stress [MPa (psi)]
RE100	244,167 (14.9)	100 (10)	991 (39)	1.3	0.85	13.71 (53.96)	50,000 (7,247)
RE115	358,877 (21.9)	150 (15)	1,067 (42)	1.3	0.85	22.16 (87.24)	55,000 (7,967)
RE119	373,625 (22.8)	150 (15)	1,092 (43)	1.3	0.85	22.65 (89.15)	54,000 (7,820)
RE132	449,006 (27.4)	200 (20)	1,143 (45)	1.3	0.85	31.81 (125.20)	63,000 (9,139)
RE133	440,812 (26.9)	200 (20)	1,143 (45)	1.3	0.85	31.65 (124.59)	64,000 (9,139)
RE136	462,115 (28.2)	200 (20)	1,168 (46)	1.3	0.85	32.37 (127.39)	62,000 (9,139)
RE140	468,670 (28.6)	350 (35)	1,168 (46)	1.3	0.85	56.89 (223.93)	108,000 (9,139)

Table 14. Ballast Pressure under Wood Tie (psi)

Axle load [kN (t)]	Distribution factor	Impact factor	Footprint area of tie	Ballast pressure under tie [kPa (psi)]
100 (10)	0.5	1.3	2,590 × 230 mm (8 ft 6 in. × 9 in.)	140 (21)
150 (15)	0.5	1.3	2,590 × 230 mm (8 ft 6 in. × 9 in.)	220 (32)
200 (20)	0.5	1.3	2,745 × 230 mm (9 ft × 9 in.)	280 (41)
250 (25)	0.5	1.3	2,745 × 230 mm (9 ft × 9 in.)	350 (51)
350 (35)	0.5	1.3	2,745 × 330 mm (9 ft × 13 in.)	370 (53)

The optimum thickness of the ballast bed is usually 250–300 mm (10–12 in.) measured from the lower side of the sleeper (Esveld 2001). The current specified depth of 300 mm (12 in.) of ballast below the track ties precludes maintenance tamping from penetrating and damaging the subballast layer (AREMA 2013). In North America, the preferred ballast thickness is 300 mm (12 in.). The load-dispersion angle would change under service. Thus, two load-dispersion angles are used in the computation. Variation from the computed optimum values is not significant (cf. Table 8). The load-dispersion pattern would remain somewhere between those patterns shown in Figs. 1 and 2.

With the assumption of nine concrete ties under the Benkel beam, the rail stress (cf. Table 9) is considerably below AREMA's maximum allowable bending stress of 172 MPa (25,000 psi). Tie spacing does not have an appreciable effect on the rail stress. The bending moment of the rail is proportional to the characteristic length, L, which is proportional to $\sqrt[4]{S}$ (Esveld 2001). Thus, the bending stress in the rail is proportional to $\sqrt[4]{S}$.

It appears that the assumption of nine ties under the Benkel beam leads to acceptable ballast pressure under ties (cf. Table 10). For pressure calculation, two-thirds of the footprint area (AREMA 2013) of ties is used. To accommodate pressure within acceptable limits, the tie length and bottom width may be increased to 2,743 mm (9 ft) and 330 mm (13 in.), respectively, as recommended by AREMA.

Wood Ties

The assumption of 11 wood ties under the Benkel beam is validated by the outcome of the formulas in the contexts of tie spacing, ballast thickness, rail stress, and ballast pressure under ties. These parameters are computed in Tables 11–14, respectively. Comments are the same as those for concrete ties.

Conclusions

Tie/DFF spacing is expressed in terms of the characteristic length of the track. Tie spacing should be less than the characteristic length of the track to ensure desirable load distribution on the ties/DFFs and to minimize the load on ties/DFFs under the axle. Three types of tie spacing are suggested: desirable, maximum allowable, and absolute maximum.

Notation

The following symbols are used in this paper:

E = Young's modulus of rail steel;
h = optimum ballast thickness [Eq. (2)];
h^* = optimum ballast thickness [Eq. (3)];
I = moment of inertia of rail section about x-axis;
K = stiffness of DFF;
L = characteristic length;
S = tie spacing [Eq. (2)];
S_C = desirable concrete-tie spacing;
$S^C_{\max}(\text{absolute})$ = absolute maximum concrete-tie spacing;
$S^C_{\max}(\text{allowable})$ = maximum allowable concrete-tie spacing;
S_{DFF} = DFF spacing;
S_W = desirable wood-tie spacing;
$S^W_{\max}(\text{absolute})$ = absolute maximum wood-tie spacing;
$S^W_{\max}(\text{allowable})$ = maximum allowable wood-tie spacing;
S^* = tie spacing [Eq. (3)];
u = track modulus;
V = wheel load;
w = bottom width of tie;
x = distance from wheel load;
α = ballast load-dispersion angle with vertical; and
$\eta(x)$ = relative deflection.

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