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DEVELOPMENT OF A MULTILAYER ANALYSIS  
MODEL FOR TIE/BALLAST TRACK STRUCTURES

By

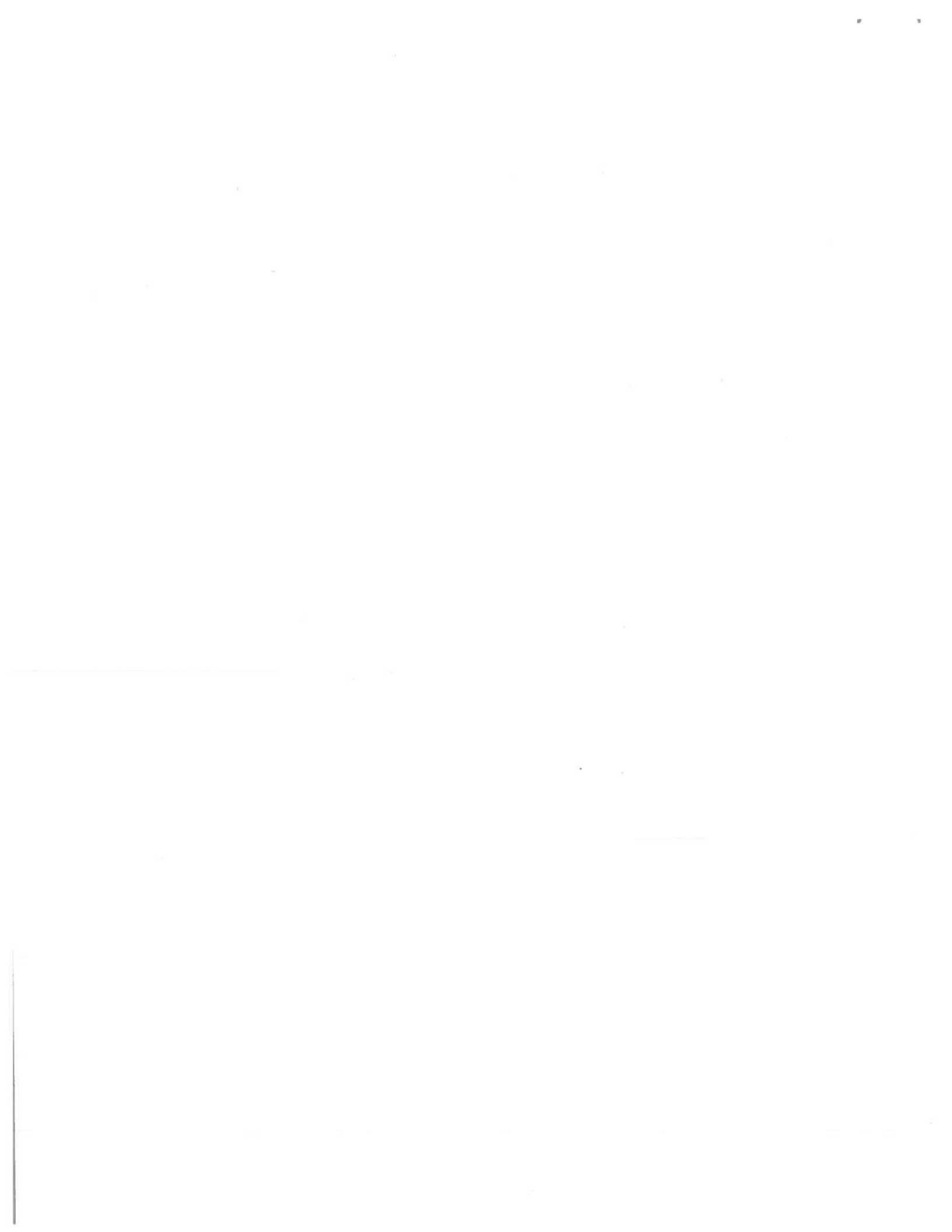
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ABSTRACT

A multilayer analysis model for tie/ballast track structures has been developed. The model includes the effects of rail bending, rail fastener stiffness, tie bending, variable ballast and subgrade material type, and variable tie spacing and ballast depth. Predicted results from the model are compared with experimental results and excellent agreement is shown. The model offers the advantages of simplicity of use and reduced computer run time compared with current finite element codes.

## INTRODUCTION

The evaluation of track performance and track design for vertical loads requires the ability to predict realistic pressure distributions at the tie/ballast interface and at the ballast/subgrade interface. This requires a model which includes the effect of tie bending, rail fastener stiffness, and changes in ballast depth, roadbed material properties, and tie spacing in a unified manner. In such a model, changes in roadbed configuration that affect track modulus and the distribution of loads from the rails to individual ties are apparent.

A track model and computer code that incorporates the above features have been developed. This paper compares the ease of use, computer time required per run, and accuracy of results with other existing analysis codes. Analytical validation as well as comparison of computer predictions with experimental results are also presented.

The Multi Layer Track Analysis computer routine, or MULTA, discussed herein is a two stage numerical procedure for determining the three-dimensional load and stress distribution in a railroad track system subjected to static loads.

MULTA can be used to evaluate new and existing track system configurations for various combinations of concentrated vertical loads or moments exerted on either or both rails.

### Typical Methods of Analysis of Track Structures

Currently the analysis of track structures usually follows one of two paths: (1) the track structure is represented very simply such as a beam on an elastic foundation wherein the substructure is represented as a series of discrete springs, or (2) the track structure is modeled in great detail with a

finite element representation. In the first case, the system is represented so simply that individual contributions such as ballast material type and depth, subgrade material type, and tie bending are not sufficiently detailed or easily evaluated. On the other hand, the detail characteristic of most finite element codes requires preparation of input data and running time for computer analysis of such magnitude that extensive analyses are quite often prohibited.

A finite element code was picked that could simulate variable ballast depth and material type and subgrade depth and material type so that a comparison with the results obtained using MULTA could be made. MULTA is not a finite element code as such and the differences between MULTA and a typical finite element code will be pointed out in the following paragraphs. The finite element code used for this comparison was the Prismatic Solid Analysis (PSA) code originally developed at Berkeley and modified by the Association of American Railroads (AAR). The comparison of the results from the MULTA and PSA codes showed negligible difference in predicted stresses and displacements. A complete description of the PSA code and the above mentioned comparison can be found in [1].

Typically, preparation of input data for use in MULTA requires considerably less time than seemingly equivalent finite element codes. In the results that are discussed subsequently, eleven ties are used in the simulation of the track structure. Preparation of input data for MULTA, including punched data cards, required about three hours time for one person. Running time required about 400 total computer seconds. On the other hand, preparation of input data for the alternate analysis using the PSA finite element code required about eight hours preparation time for one person and about 750 seconds computer run time. Thus the MULTA program has the advantage of being able to simulate and evaluate the effects of parameters such as ballast depth and material types, subgrade material type, tie bending, and rail fastener stiffness where other similar analysis codes such as beam on elastic foundation formulation do not.

On the other hand, the relative ease of input data preparation and considerably less computer run time offers a definite advantage over the more detailed finite element codes without compromising the results for a vertical linear elastic track analysis tool.

Predicted results from the MULTA code have also been compared with the ILLI-TRACK structures code. This is a two dimensional finite element code developed at the University of Illinois [2]. This comparison shows that ballast pressure, rail deflection, and rail bending moment predicted values can be in serious error if the effective bearing area of the tie is not properly chosen in ILLI-TRACK. This is a key difference between the two models. It is necessary to assume an initial tie bearing area for ILLI-TRACK whereas tie deformation and contact area are included directly in the MULTA model. The comparison of MULTA and ILLI-TRACK predictions is shown in [3].

### Track Model Development

The two stages of solution in MULTA are modifications to two previously developed computer codes. The first stage is a modified version of the computer program developed by the AAR and described in [4]. The program in [4] is a ballast/subgrade model using a multilayered elastic system. The theoretical basis for the multilayered elastic system was first presented by Burmister [5] and revised for use in [4]. The second stage of solution in MULTA is a modified version of part of the program described in [6]. The loads combination phase is that portion of the program in [6] that was revised for use in MULTA. This second stage of MULTA includes rail loads, rail bending, rail fastener stiffness and tie bending. The schematic for MULTA is shown in Figure 1.

### Model Description

The first stage of MULTA analyzes the track substructure (ballast and subgrade) and provides displacement and stress influence information as input to the second stage. The basic theory in the first stage assumes the ballast/subgrade structure to be that of an elastic half space and as such the horizontal and vertical (downward) dimensions of the track structure are infinite in extent. This precludes the simulation of actual ballast profile geometry such as sloping shoulders. However, the effects of infinite dimensions in the horizontal and vertical directions on the stress and displacement predictions for vertical loads have been evaluated and are presented in [1]. The conclusion therein is that the finite dimension of the ballast shoulder had a negligible effect on the ballast and subgrade pressure under the ties.

The calculation of the stress and displacement influence functions occurs in the form of the stress and displacement response of the ballast/subgrade structure to unit vertical loads applied to specific locations on the horizontal surface of the ballast. These specific locations are at the tie-ballast interface for the particular tie-track system being simulated. Critical in the simulation of how the loads are transmitted from the tie onto the ballast is the choice of effective load distribution area on the ballast. This distribution is in the form of load circles that distribute the tie loads onto the ballast, see Figure 2. Load circle size (radius) and number of load circles necessary to achieve simulation efficacy and solution accuracy are discussed in [1].

The second stage of MULTA is basically an equation solver. The equations that are solved in the second stage include the magnitude and position of a wheel load on each rail, rail displacement, rail force and moment equilibrium, rail fastener stiffness, and tie bending.

MODEL ASSUMPTIONS, FEATURES AND LIMITATIONS

The track system model includes the following assumptions:

- a. The entire system behaves in a linear fashion.
- b. Loads and moments applied to the rails are static and concentrated.
- c. The material of each component of the system is homogeneous, isotropic and linear elastic.
- d. The depth of the last soil layer is infinite.
- e. The tie spacing is constant for all ties.
- f. The track gage is constant.
- g. The rail-tie system (including first and last tie) deforms compatibly on the elastic foundation.

MULTA has available to the user the following options and features:

- a. The ballast/subgrade system can be modeled by as few as two and by as many as seven layers of homogeneous, isotropic elastic material, each having distinct material properties and depths. However, the last layer must have an infinite depth.
- b. The vertical stiffness of the spring used to represent the combined stiffness of a rail fastener and tie pad can be selected arbitrarily, but must be greater than zero.
- c. Unequal loads are permitted for each rail and at any position along a rail.



Use of MULTA is subject to the following limitations:

- a. All ties must have identical material and geometric properties.
- b. The track roadbed representation as an elastic half space with infinite horizontal dimensions does not permit modeling the actual cross-section of a ballast section having sloping shoulders.
- c. The model does not permit missing ties.
- d. The model does not allow external loading in the lateral or longitudinal directions and thermal loads cannot be included.

As previously discussed, MULTA is equivalent analytically to other more detailed codes. It was also desirable to compare MULTA results with experimental results. The following is a brief description of the test sites used to obtain data for comparison and evaluation of MULTA predicted results.

#### Test Site Description

The test sites selected for this measurement program were on the Florida East Coast Railway (FEC) about 20 miles north of West Palm Beach. The FEC track was selected from the several available concrete tie test sites such as the Kansas Test Track, Streator (Santa Fe), Lorrain (Chessie), and Roanoke (Norfolk and Western), because it provided the best combination of track variables required for this program. The test sites included two concrete tie tangent track sections, one having a nominal tie spacing of 24 inches (Site 1), the other having a nominal tie spacing of 20 inches (Site 2), and a concrete tie curve site with 24-inch tie spacing (Site 3), see Figure 3. The concrete ties

were produced by the Railroad Concrete Crosstie Corporation (RCCC) and are a modification of the original MR-2 tie design. All three sites included a main instrument array which extended over seven ties. The purpose of this continuous section was to obtain a complete set of track load and response data over a nominally uniform track section.

Specific locations for instrumentation were selected on the tangent track sections to provide uniform subgrade conditions away from any embankments and at locations shown by track geometry charts to be free of any anomalies in profile, alignment or gage. Results from a complete set of measurements from the DOT track geometry car showed that the track was in excellent condition throughout the entire test section. A detailed description of the test sites and the instrumentation used to record the various track quantities of interest are presented in [1]. Only the instrumentation that pertains to the validation of the analysis code (MULTA) is described here.

#### Measurement of Vertical Track Loads

Rail Seat Loads. The main array of each test section contained 6 instrumented tie plates, with 5 along one rail. The instrumented tie plates were used to record rail seat loading throughout the influence zone of the center tie. Each instrumented tie plate had a pair of load cell washers. The signals from the two load cell washers were summed to measure total vertical rail seat load.

Tie/Ballast Pressures. The Federal Rail Administration/Portland Cement Association (FRA/PCA) load-cell ties developed for the Kansas Test Track were used to measure tie support reactions at the tie/ballast interface. These steel ties have ten separate segments along the bottom to convert bearing pressures to discrete loads. Each rail seat is instrumented to measure vertical

rail seat loads. A detailed description of the construction of the FRA/PCA load-cell tie and a comparison of the bending stiffness between the load-cell tie and the RCCC tie can be found in [1].

Two of the load-cell ties were installed at Site 1, and one load-cell tie was installed on the curve at Site 3. These ties were placed in track and hand-tamped approximately one month before beginning the measurement program to allow for reconsolidation of the disturbed ballast. The purpose of using these load-cell ties was to simultaneously measure vertical rail seat loads and the resulting distribution of tie/ballast pressure on the 10 instrumented segments along the tie length. It was recognized that inserting a single tie in the track might result in that tie supporting less than the normal percentage of wheel load. Therefore, data from the pressure distribution on the load-cell ties have been normalized by the rail seat load to minimize this influence.

Figure 4 shows a photograph of the load-cell tie installed in track. The load-cell ties were refurbished at BCL prior to use on the measurement program, and each of the bottom pressure cells and the rail seat load cells were calibrated in a static load machine.

Track Deflections. The main array included displacement transducers to measure absolute vertical rail deflection, lateral deflection of the rail head relative to the tie, and absolute lateral displacement of the tie. All measurements were made adjacent to the center tie using Direct Current Differential Transformers (DCDT) having a displacement range of  $\pm 0.5$  inches. The absolute vertical and lateral displacement of the rail or tie were referenced to a "ground stake" which consisted of a 1-inch diameter steel rod driven through a concentric hollow casing through the ballast into the subgrade. The casing was about 4 feet long to isolate the rod from ballast movements. The 1-inch diameter steel rod was 8 feet long and it was driven into the roadbed until about only 8 inches projected above the ballast surface.

Generation of Input Data for MULTA

Input data requirements for the MULTA track analysis model include the elastic properties for a layered representation of the ballast and subgrade. The following plate bearing test procedure was used to obtain representative data for the elastic properties:

(1) Two adjacent ties were removed, sufficiently far away to avoid any effect on the instrumentation, and load-deflection plate bearing measurements were made on the ballast surface in the footprint of one tie, as shown in Figure 5. An 8-inch diameter circular loading plate was used on the ballast surface, and this area was covered with plaster-of-paris (dental cement) so that the loading plate would bear uniformly on the ballast. A fixed wooden reference beam supported outside the track was used as a displacement reference for two displacement transducers (DCDT) attached to the plate. Displacements were recorded for ballast loading up to about 125 psi, which exceeds the ballast pressure encountered in service by a considerable margin. Typical ballast pressures in service rarely exceeded about 50 - 60 psi.

(2) The ballast crib was excavated at the location of the two removed ties to determine the actual ballast depth. The ballast depth under the bottom of the tie was 6.5 inches at both Site 1 and Site 2. The plate bearing tests were repeated on the subgrade without using the dental cement. Data from Steps (1) and (2) were then used with the multilayer track analysis model to determine representative values of Young's modulus for the ballast and subgrade layers.

The loading cycle was repeated three consecutive times at each of three positions along the length of the tie. As shown in Figure 6, the initial load cycle has a much lower slope (force versus displacement) value than the second

loading cycle. In fact, after the initial load cycle, the subsequent load cycles have almost the same slope. Data shown in Figure 6 are for the Site 1 subgrade at 6.5 inches on the gage side of the rail. Data for the other locations are characteristically similar.

Initial and final slope values from the subgrade tests were used to estimate Young's modulus ( $E_2$ ) for the subgrade, using theory of elasticity solutions for the deflection of an elastic half-space loaded by a rigid, circular plate. Having determined  $E_2$ , the ballast stiffness data were used to estimate Young's modulus ( $E_1$ ) for the ballast. This estimate was made using the multi-layer program in an interactive scheme until predicted load-deflection values for the circular plate load were sufficiently close to the experimental values. It was hoped that using initial and final stiffness values would place a bound on the value of  $E_2$  so that the predicted value of track modulus ( $U$ ) would compare favorably with the measured data for track modulus.

Values of Poisson's ratio for the subgrade and ballast layers are also needed as input to the MULTA program. Typical values of  $\nu_1 = 0.4$  for ballast and  $\nu_2 = 0.4$  for subgrade Poisson's ratio were picked from the subgrade property data obtained from the results of soil tests conducted by Pittsburgh Testing Laboratories as reported in [1].

Table 1 shows the values of ballast modulus ( $E_1$ ), subgrade modulus ( $E_2$ ), and track modulus  $U$  based on initial and final plate bearing test stiffness data in conjunction with the MULTA program. Track modulus  $U$  is defined as the force per inch of rail required to depress the track roadbed 1-inch. This parameter has been used historically to quantify the effective stiffness, or

resilience, of a track structure and it is a key parameter in the beam-on-elastic-foundation analysis procedure used for conventional track design. The predicted modulus values  $U$  are based on the beam-on-elastic-foundation equation for vertical rail seat load in the form:

$$U = 4 EI \left[ \frac{2}{l_t} \left( \frac{Q}{P} \right) \right]^4$$

where

$Q$  = maximum rail seat load predicted by MULTA

$P$  = wheel load

$l_t$  = tie spacing

$EI$  = rail bending stiffness.

Comments regarding the magnitude of the modulus values shown in Table 1 and subsequent modulus calculations will be made later.

#### COMPARISON OF MEASURED AND PREDICTED LOADS

##### Effect of Track Modulus on Rail Seat Loads

Data on vertical rail-seat loads\* from a slow roll-by of the work train were used to determine the track modulus,  $U$ . The work train consisted of one empty and one loaded 100-ton hopper car with a 4-axle locomotive. The effect of tie-to-tie variations in the main array was minimized by averaging the maximum rail seat loads for a known wheel load during a slow traverse of the work train.

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\* Rail seat load is the load that is absorbed by a tie in a track structure. For example, if a static wheel load of 35 kips is placed on a rail directly over a particular tie, that particular tie will absorb approximately 40 percent to 60 percent of the applied wheel load.

The average ratio of rail seat load to wheel load ( $Q/P$ ) was used with the theoretical relationship from the beam-on-elastic-foundation formulation to determine an experimental track modulus. This is the same formula that was used to calculate the values of  $U$  presented in Table 1.

Table 2 lists maximum measured vertical rail seat to wheel load ratio in percent. These data show a considerable load-dependent effect as well as large tie-to-tie variations. The average rail seat load for heavy cars on track with 20-inch tie spacing was 12.5 percent lower than that for 24-inch tie spacing. A 16 percent reduction would normally be expected based on conventional guides for track design. However, individual ties in both sections carried as much as 65 percent of the heavy car wheel load and as much as 76 percent of the light car wheel load.

Figure 7 shows a comparison of measured and predicted rail seat loads for light and heavy car wheels centered in the main array of Site 2. The model parameters corresponding to a track modulus of 30.4 ksi per rail (final values from Table 1) were used for the predictions. It is evident from the load distribution shape that the actual track was stiffer than the analysis model.

As discussed previously, it was hoped that data from the initial and final load cycles of the plate-bearing load-deflection tests would provide a bound to the estimate for the roadbed parameters. However, the comparison in Figure 7 shows that the plate-bearing test data did not provide a reliable prediction of roadbed stiffness even though the values for subgrade and ballast modulus appear reasonable when compared to the laboratory subgrade measurements and to typical values for ballast.

Since the FEC roadbed is stiffer than that predicted using the plate bearing data, the following procedure was adopted in an attempt to synthesize the model parameters that determine roadbed stiffness and track modulus. The ratio of ballast to subgrade modulus determined from the plate-bearing tests was

retained, and the actual ballast ( $E_1$ ) and subgrade ( $E_2$ ) modulus values were increased so that the maximum predicted rail seat load equals the average maximum experimental rail seat load for the heavy car. The heavy car was chosen to reduce the effect of any nonlinearities. This procedure was used to adjust  $E_1$  and  $E_2$  values so that the maximum predicted vertical rail seat load was within 1.2 percent of the average experimental data for the 20-inch tie spacing (Site 2) and within 1.6 percent for the 24-inch tie spacing (Site 1). The adjusted values of foundation properties were:

$$E_1 = 60 \text{ ksi and } E_2 = 35.65 \text{ ksi, with } \nu_1 \text{ and } \nu_2 \text{ equal to } 0.4.$$

Figures 8 and 9 compare measured and predicted rail-seat loads with a heavy car wheel centered in the main array of the track for the 24-inch and 20-inch tie spacings, respectively. The case of a very stiff track (high value of  $U$ ) is characterized by the loaded tie absorbing a large percentage of the applied load (>50%) while the loads absorbed by adjacent ties drop off rapidly. The average maximum experimental rail seat-load was 18.9 kips for an applied load  $P = 33.9$  kips in the 24-inch tie spacing ( $Q/P = 55.8$  percent). This gives a track modulus of  $U = 47.7$  ksi. The maximum predicted rail seat load was 18.6 kips, and the predicted track modulus was 44.7 ksi. The lower predicted modulus is apparent from the comparison of the rail-seat load distribution shapes shown in Figure 8.

These comparisons show that the actual track structure is at least as stiff as the value predicted using the adjusted modulus values of  $E_1$  and  $E_2$ . The tie/ballast pressure distribution data in the following section also support this conclusion.

#### Tie/Ballast Pressure Distribution

Tie bending moments at the rail seat, and bending and torsional moments at the tie center have been identified as the major causes of concrete



tie failures. The distribution of the support reaction between the tie and ballast is the principal unknown factor in validating the bending moments predicted by analytical models. Therefore, measurements of tie/ballast pressure distribution along the length of the tie were needed to validate fully the analytical prediction of bending moments at the tie rail seat and at the center.

The vertical tie/ballast pressures along the length of one load-cell tie for heavy, medium, and light cars are shown in Figure 10. These pressure profiles indicate that this particular tie was noticeably center-bound for light car loads. That is, the tie center bears almost the entire load while the outer ends of the tie are carrying almost no load. As the magnitude of the load is increased, the peak pressures moved outward from the tie center toward the rail seat regions. The experimental data show that the peak pressure shift from the tie center to the rail seat region reaches a maximum on the gage side of the rail seat. Pressures up to about 40 psi were measured in the rail seat region for normal heavy cars.

Predicted results from the MULTA program for the medium car weight are shown for comparison in Figure 10. The MULTA program assumes a uniform elastic support for the roadbed. The resulting tie-ballast pressure distribution is a maximum under the applied load (rail seat), and reaches a minimum at the tie center. The maximum predicted pressure of 33 psi is within 14 percent of the measured data for the medium load despite the center-binding effect for this tie.

The experimental data shown in Figure 10 were normalized and replotted in Figure 11 so that peak pressures per unit rail seat loads can be easily determined. The MULTA results show that the ratio of peak pressure to applied rail-seat load is approximately 3.0 psi/kip, and that the normalized peak pressure occurs under the rail seat region. The experimental results show that

the ratio of peak pressure to applied rail-seat load had an approximate maximum value of 3.2 psi/kip at the tie center for center binding under light loads and a maximum value of 2.5 psi/kip at the rail-seat region for heavy loads.

The experimental data from the load-cell tie in the curved track section (Site 3) are shown in Figures 12 and 13. Tie-ballast pressure distributions along the length of the tie for light, medium, and heavy wheel loads are shown in Figure 12. An integration of the pressure distributions showed that vertical equilibrium was satisfied to within 3 percent of the respective applied loads.

The results from the MULTA program shown in Figure 12 for medium wheel loads show good agreement with the experimental data. Maximum pressures are predicted within 5 percent, and the shape of the pressure distribution is very similar. It is also evident that the vertical load is considerably greater on the high rail and the case of unequal loads can be used as input to the model.

The normalized pressure distributions for the three cases of light, medium, and heavy wheel loads are shown in Figure 13. The small variation shows that the support reactions for this tie behaved in a very linear manner, and that the uniform elastic foundation used in the MULTA program gave very good predictions for the pressure distributions for all wheel loads.

The results from the analytical model can also be used to predict bending moments for the rail seat and tie center. The pressure distributions for the medium wheel load shown in Figure 13 were used to calculate the shear and bending moment distributions along the tie length that are shown in Figures 14 and 15. Since the analytical and experimental pressures were in good agreement, the predicted bending moments should be equally accurate. Thus it is concluded that the MULTA model is capable of predicting rail seat and tie center bending moments that are typical of service loads except when ties have a very

serious center binding condition. However, the data from the load cell tie, which did have severe center binding for light wheel loads, were even in reasonably good agreement with predicted results for heavy wheel loads.

### Track Displacement Predictions

Results from the MULTA program were used to determine how the track displacement compares to that for a Winkler foundation. The data shown in Figure 16 show that predicted displacements are distributed over a greater length of track than the tie load distribution. The difference in the displacement shape predicted by MULTA and the tie load distribution indicates that the rail is not behaving like a beam on a Winkler type foundation. The two distributions would be identical for a Winkler foundation.

The same conclusions regarding the displacement being distributed over a greater distance than the tie loads is evident in the data from Site 2 (20-inch tie spacing). Comparison of Figures 16 and 17 show the influence of tie spacing on tie load and displacement. The predicted peak tie load and displacement values are reduced by 14 percent and 15 percent, respectively, when the tie spacing is reduced 16 percent from 24 to 20 inches.

Vertical rail displacements were measured at two locations at each test site. These vertical displacements were measured at the middle tie of the main array and at a tie about 35 feet outside the main array. Since only two locations were instrumented at each test site for vertical displacement data, it was difficult (in view of the local variations previously discussed) to characterize the track structure with experimental displacement values. However, some comparisons can be made with the results from the model.

Table 3 shows a comparison between measured track displacement values and values predicted from the model. In Table 3,  $\Delta Y$  = differential displacement for heavy and light wheel loads. This differential load,  $\Delta P$ , was 24,750 pounds.

These experimental values show the variation in displacement values from site to site. In view of this variation, it is believed that more values of displacement (per test site) are required so that average maximum displacement values could be used to better predict track modulus. However, the alternative approach of averaging data from five instrumented tie plates gave good results.

#### Track Modulus Measurements

It was originally planned that rail bending strains measured under heavy and light loads similar to those used for measuring displacement would provide a check on the track modulus determined from the displacement data. However, the lack of a sufficient number of strain gages (i.e., at many positions along the length of the rail) prevents the sort of averaging process that subsequently was determined essential to minimize local variations. Difference (heavy load minus light load) stress and displacement values and corresponding track moduli are listed in Table 4.

The values of track modulus shown in Table 4 indicate that the track structure is quite stiff. However, the data in Table 4 are for one or two discrete points along a rail at a particular test site, and they do not represent any sort of averaged values. As such, they should not be considered as truly representative of the overall track modulus.

Table 5 gives a summary of the track modulus values that were implicitly or explicitly generated from the test data. This summary directly compares the predicted and experimental modulus values discussed previously in other sections.

As stated earlier, the predicted calculations of track modulus shown in Tables 1 and 5 are based on the beam-on-elastic-foundation equation for vertical rail seat load,  $Q$ , as shown in equation (1). Equation (1) is one of two forms used to calculate track modulus  $U$ . The other form is based on maximum rail displacement  $Y_0$ . Both forms are derived from beam-on-elastic-foundation theory. If the track system in reality behaves as a beam-on-elastic-foundation, then either form can be used to calculate  $U$  and the answers will be identical. However, if the shear coupling in the roadbed is significant, the track does not behave according to the assumptions used for the beam-on-elastic foundation, and the results from estimates of track modulus using measured data for  $Q$  and  $Y_0$  will not give equivalent values for  $U$ . This is also true for the MULTA model where the shear coupling is appreciable in the simulation of the roadbed.

As we have seen, the measurements on the Florida East Coast Railway showed that using the average maximum rail seat load to calculate  $U$  gives results that are more consistent with the loads and moments than using rail displacements. As mentioned previously, the rail seat load distribution predicted by MULTA is qualitatively similar to the results from the beam-on-elastic-foundation solution and the Florida East Coast measurements, whereas the displacement distribution is different from beam-on-elastic-foundation solution because of coupling in the roadbed. However, if predicted modulus values are calculated using the rail displacements, the values will be approximately 1/2 to 1/3 those from MULTA using rail seat loads, and will be in the range of typical measured track modulus data for concrete tie track.

SUMMARY OF RESULTS AND CONCLUSIONS

The comparison of predicted and measured track response parameters in the previous section shows that the MULTA track analysis program is capable of making good predictions of tie loads and tie/ballast pressures. The inclusion of tie bending has been shown to be quite important in predicting ballast pressures. The program can also be used to predict rail bending stresses and tie bending moments.

No experimental data on stresses in the ballast and subgrade below the tie were measured for comparison. However, the good agreement with the predicted ballast pressures immediately under the tie gave confidence that pressures predicted elsewhere in the roadbed will be sufficiently accurate for track design evaluations. Predictions of soil behavior are limited by the assumptions of linear elasticity in the MULTA model, so inelastic behavior of highly loaded soils could not be predicted accurately.

The major difficulty in using MULTA, or any other track analysis program, is in the accurate modeling of the ballast and subgrade. The elastic continuum used in the MULTA model does show that the transfer of shear in the roadbed produces appreciable tie-to-tie coupling in displacements. This effect is also observed in track response measurements but it is not included in conventional beam-on-elastic-foundation models. However, the real difficulty is in establishing the material properties for a layered model of the ballast and subgrade that match the overall track modulus measurements. The plate bearing tests on the ballast and subgrade and independent vibroseismic measurements of subgrade properties did not give sufficiently accurate predictions of track modulus for predicting track loads with heavy wheel loads even though pressures in excess of maximum pressures under traffic were used for the plate bearing tests. This difficulty cannot be explained at this time. In the

meantime, it is recommended that the ballast and subgrade properties be adjusted to match experimental measurements of track modulus under heavy wheel loads using representative soil data for the relative ballast/soil stiffness. Predictions of tie loads, track deflections, and roadbed pressures will not be greatly influenced by changes in the relative ballast and soil stiffnesses as long as the track modulus is matched. Inaccurate estimates of these parameters will have the greatest effect on predicting relative deflections in the ballast and subgrade layers.

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This paper presents the views and positions of the authors and does not necessarily reflect those of the Department of Transportation.

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Table 5. Summary of Track Modulus Values



TABLE 1. MODEL PARAMETERS FROM PLATE BEARING TESTS<sup>(1)</sup>

	Young's Modulus (ksi)		Predicted Track Modulus (ksi) <sup>(2)</sup>
	Initial	Final	
I. Tangent Site, 24-inch tie spacing (Site 1)	$E_1 = 24.$ $E_2 = 8.9$	$E_1 = 30.$ $E_2 = 17.8$	15.2 - 25.5
II. Tangent Site, 20-inch tie spacing (Site 2)	$E_1 = 15.$ $E_2 = 4.8$	$E_1 = 28.$ $E_2 = 17.8$	10.5 - 30.4

Notes:

(1)  $E_1$  = ballast modulus,  $E_2$  = subgrade modulus, Ballast depth = 6.5 inch., Poisson's ratio = 0.4

(2) Range for initial to final values for model parameters based on predicted maximum tie plate load.

TABLE 2. MAXIMUM MEASURED RAIL-SEAT TO WHEEL-LOAD RATIO (Q/P) IN PERCENT (%)

	Tie Number					Average
	1	2	3	4	5	
I. Tangent Track, 24-inch tie spacing (Site 1)						
a. Light Car	43	71	31	-	33	44.5
b. Heavy Car	47	58	53	-	65	55.8
II. Tangent Track, 20-inch tie spacing (Site 2)						
a. Light Car	22	38	64	-	76	50
b. Heavy Car	44	31	56	-	64	48.8

TABLE 3. COMPARISON OF MEASURED AND PREDICTED  
TRACK DISPLACEMENTS

Site Description	Measured Values $\Delta Y \sim \text{in.}$	Predicted Values $\Delta Y \sim \text{in.}$
I. Tangent Site 24-inch Tie Spacing (Site 1)		
Main Array	0.015	0.018
Outside Main Array	0.0135	0.018
II. Tangent Site 20-inch Tie Spacing (Site 2)		
Main Array	0.029	0.017
Outside Main Array	0.008	0.017
III. Curved Site 24-inch Tie Spacing		
Main Array	0.034	0.018
Outside Main Array	0.044	0.018

TABLE 4. MEASURED VALUES OF TRACK MODULUS

Site Description	$\Delta$ Stress, (Psi)	$\Delta$ Disp. (in.)	Measured Track Modulus	
			Disp. <sup>(1)</sup> (lb/in./in.)	Strain <sup>(2)</sup> (lb/in./in.)
I. Tangent Site, 24-inch Tie Spacing Main Array	4575	0.015	39,100	45,900
		0.0135	41,000	
II. Tangent Site, 20-inch Tie Spacing Main Array	3850	0.029	18,300	87,000
		0.008	82,000	

(1) Calculated track modulus using rail differential displacement for light and heavy wheel loads.

(2) Calculated track modulus using differential rail bending strains for light and heavy loads.

TABLE 5. SUMMARY OF TRACK MODULUS VALUES

Site Description	Measured Track Modulus ~ ksi			Predicted Track Modulus ~ ksi		
	Displacement (1)	Strain (2)	Average Tie (3)		Foundation Parameters from Plate Bearing Tests (4)	Adjusted Values of $E_1$ and $E_2$ (5)
			Light	Heavy		
I. Tangent Site 24-inch Tie Spacing	39.1	45.9	18.9	47.6	15.2 - 25.5	44.7
Outside Main Array	41				15.2 - 25.5	
II. Tangent Site 20-inch Tie Spacing	18.3	87	62.6	58.2	10.5 - 30.4	55.4
Main Array Outside Main Array	82				10.5 - 30.4	

- (1) Calculated track modulus using rail displacement for light and heavy wheel loads.
- (2) Calculated track modulus using rail bending strains for light and heavy wheel loads.
- (3) Based on average maximum tie plate loads on 4 ties, light load  $\approx$  8 kips, heavy load  $\approx$  34 kips.
- (4) Range for initial to final values for model parameters based on predicted maximum tie plate load. (See Table 1)
- (5)  $E_1$  = ballast modulus and  $E_2$  = subgrade modulus, adjusted so that maximum predicted rail seat load equals average maximum experimental rail-seat load at Site 1.





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- Figure 2. Basic Geometry for First Stage of MULTA
- Figure 3. Experimental Test Site Layout
- Figure 4. Load-Cell Tie
- Figure 5. Ballast Plate Bearing Measurements
- Figure 6. Force-Displacement Curve for Subgrade Plate Bearing Test at 6.5 in. from Rail Seat - Site 1
- Figure 7. Comparison of Experimental and Analytical Rail Seat Loads for Site 2 (Tangent track with 20-inch tie spacing)
- Figure 8. Comparison of Measured and Predicted Vertical Rail Seat Loads at Site 1
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- Figure 10. Tie/Ballast Pressure Data at Site 1
- Figure 11. Ballast/Tie Vertical Pressure Normalized to Respective Rail Seat Reaction, at Site 1
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- Figure 15. Tie Bending Moment, LCT-0 (Site 3)
- Figure 16. Predicted Tie Load and Displacement Distribution (Site 1)
- Figure 17. Predicted Tie Load and Displacement Distributions (Site 2)



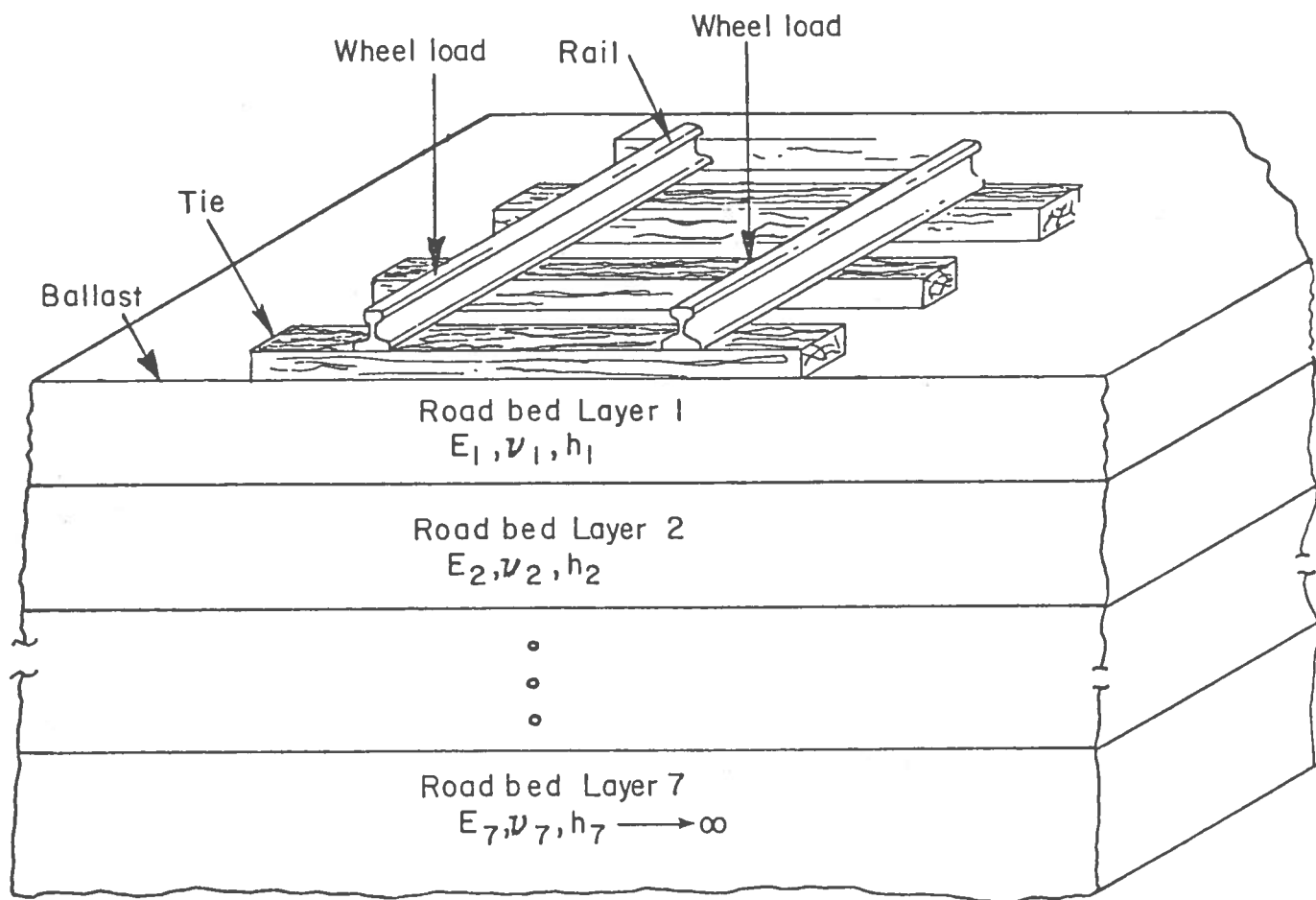


FIGURE 1. TRACK MODEL FOR MULTA PROGRAM

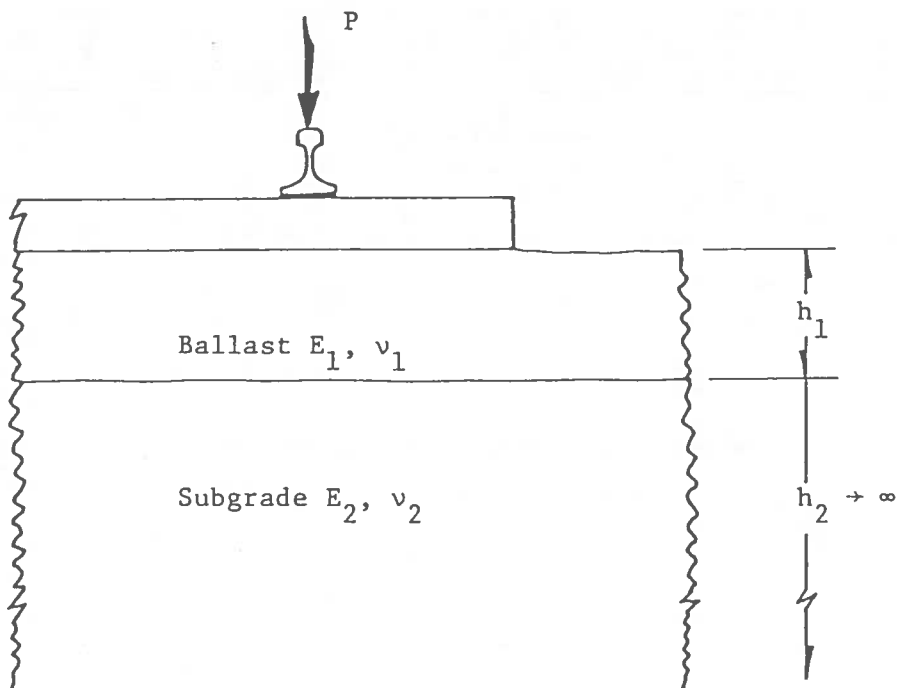
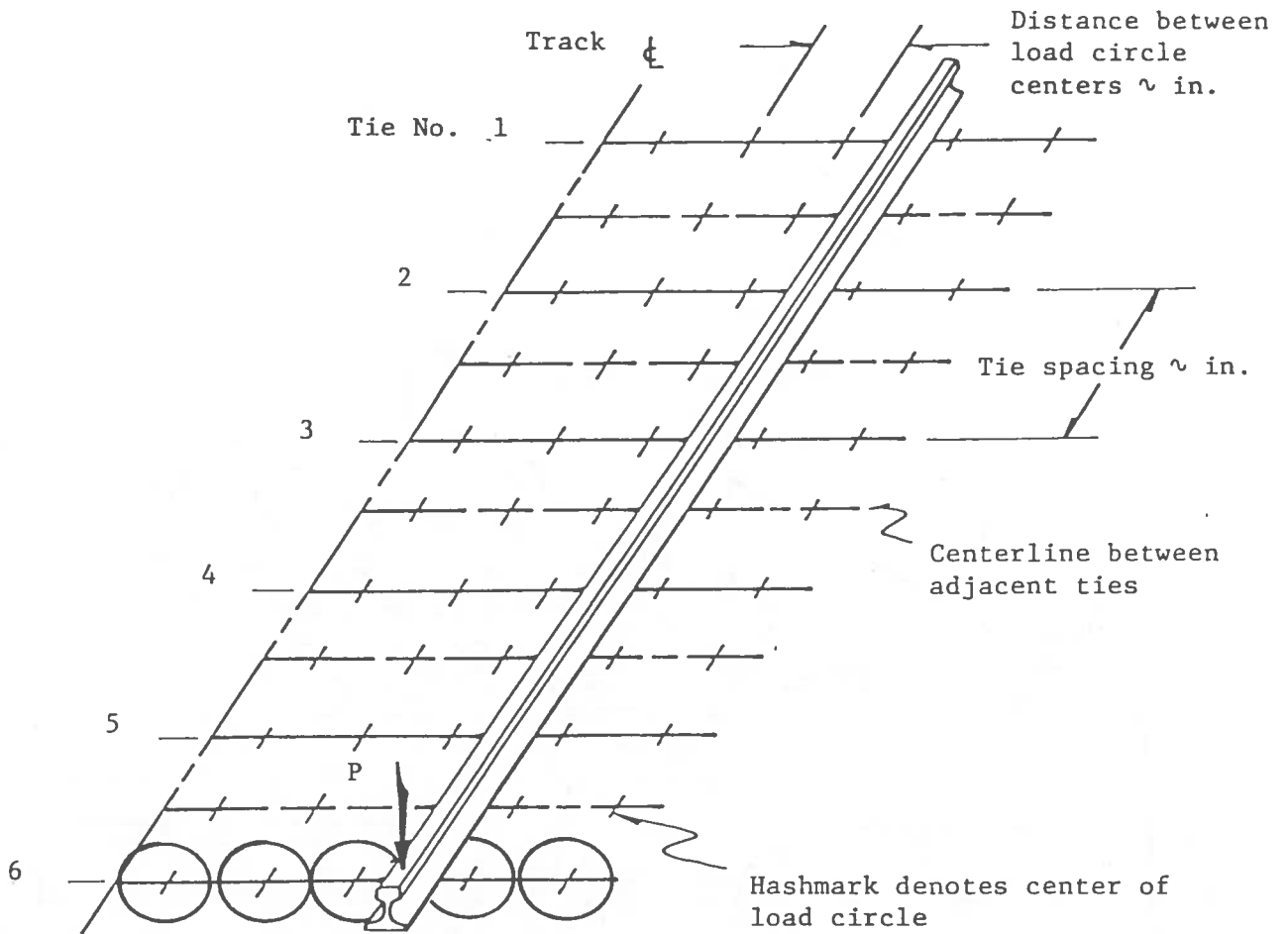
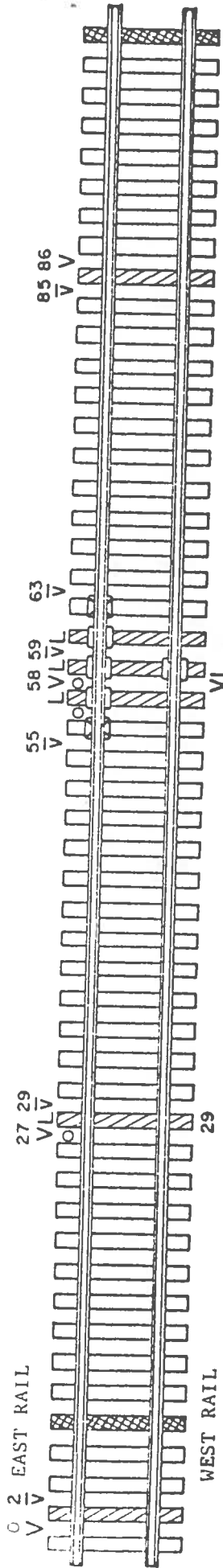
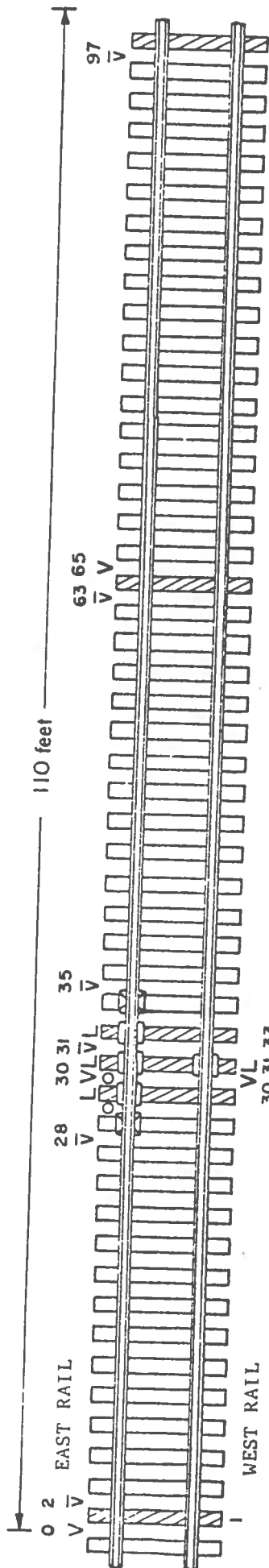


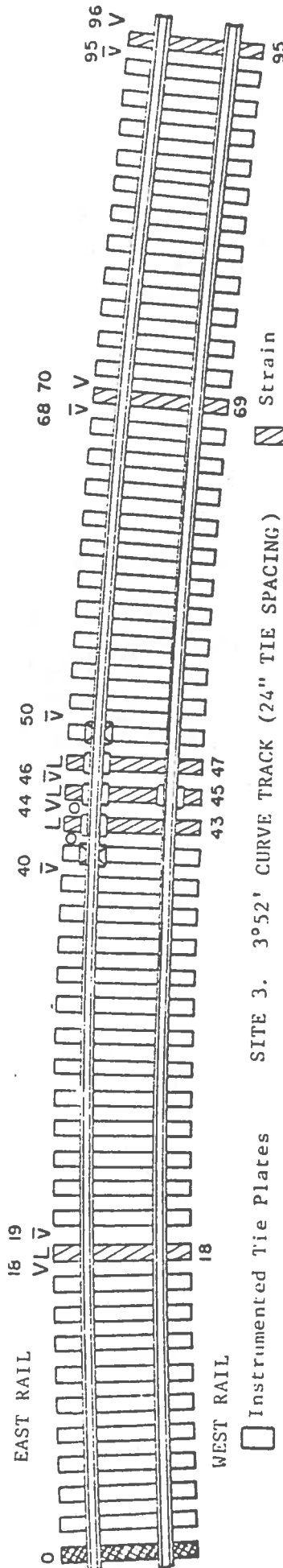
FIGURE 2. BASIC GEOMETRY FOR FIRST STAGE OF MULTA





SITE 1. TANGENT TRACK (24" TIE SPACING)



SITE 2. TANGENT TRACK (20" TIE SPACING)



SITE 3. 3°52' CURVE TRACK (24" TIE SPACING)

-  Strain Gaged Ties
-  Load Cell Ties







-  Instrumented Tie Plates
-  Vertical Circuits
-  Lateral Circuits
-  Axle Detector Circuits
-  Deflection Circuits
-  Tie Plate Spacers

FIGURE 3. SITE LAYOUT

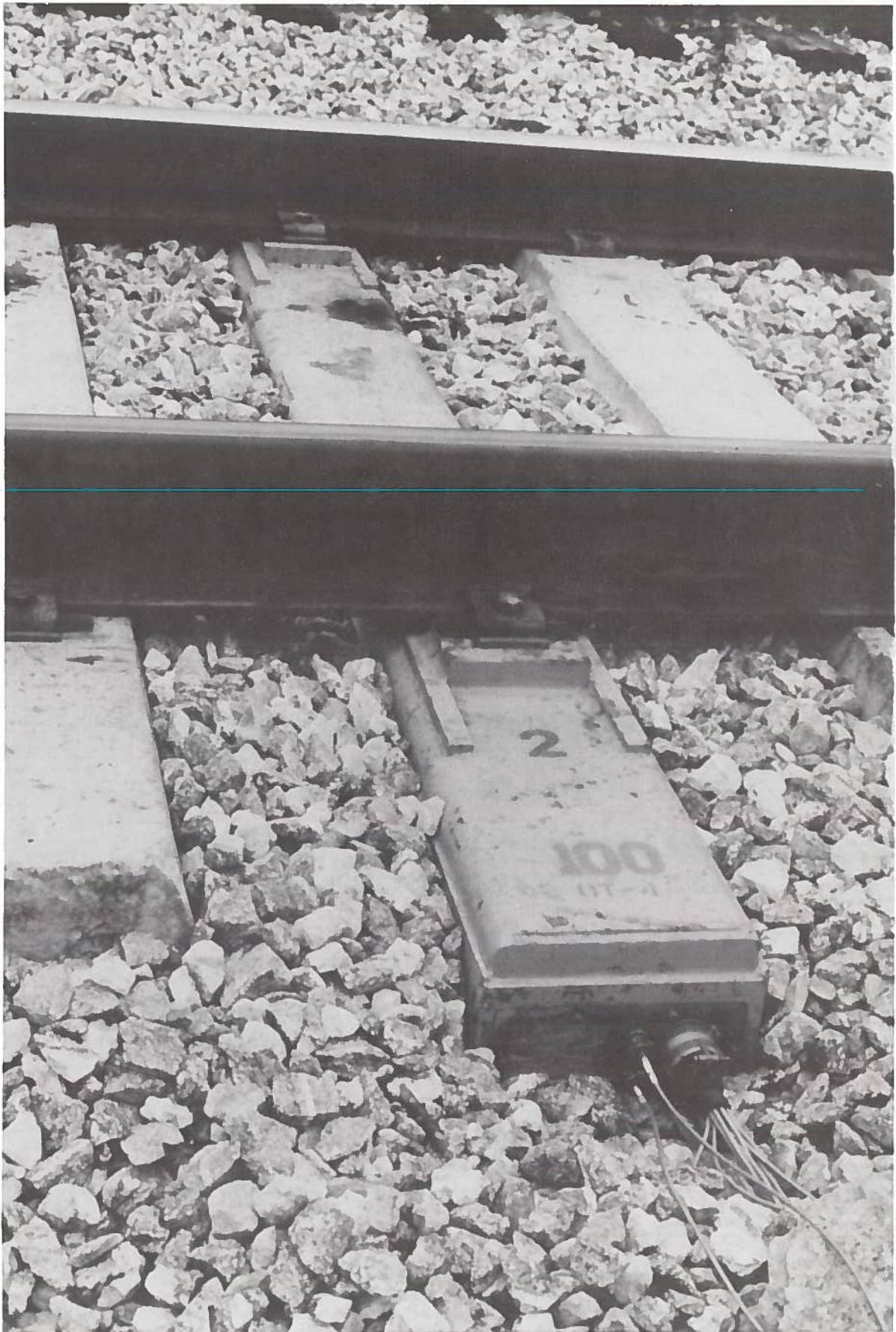


FIGURE 4. LOAD-CELL TIE

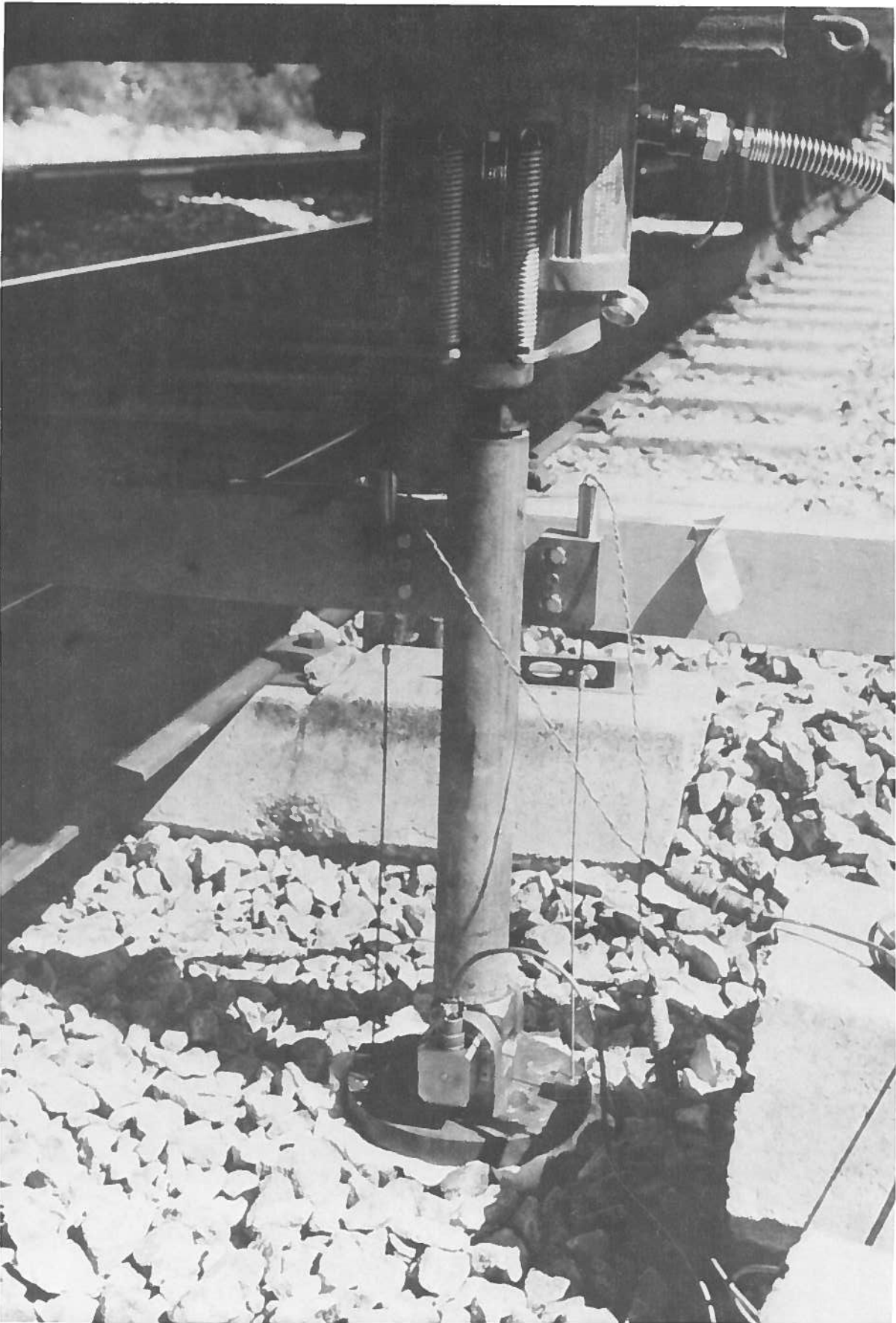


FIGURE 5. BALLAST PLATE BEARING MEASUREMENTS

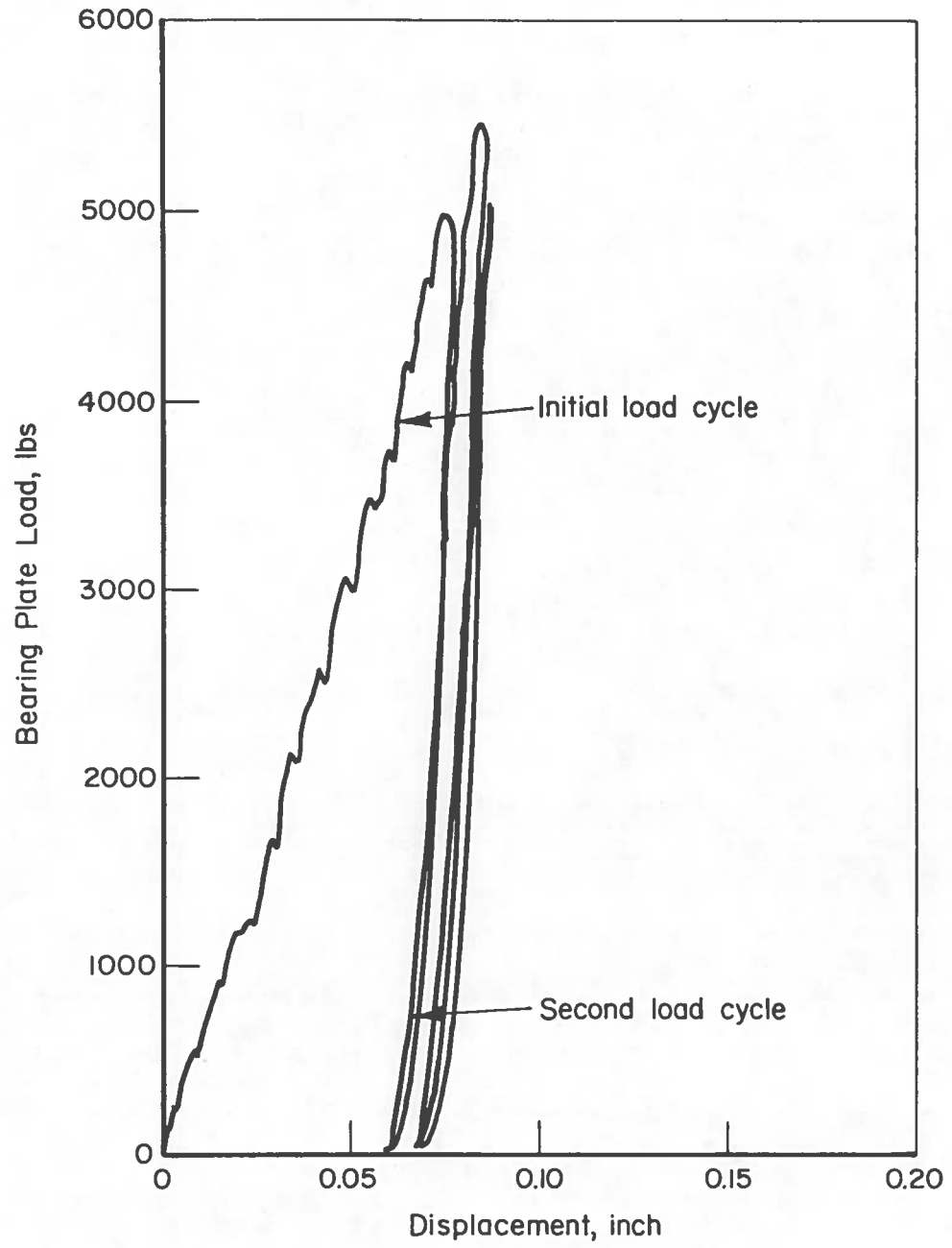


FIGURE 6 . FORCE-DISPLACEMENT CURVE FOR SUBGRADE  
 PLATE BEARING TEST @ 6.5 in. FROM RAIL  
 SEAT - SITE 1



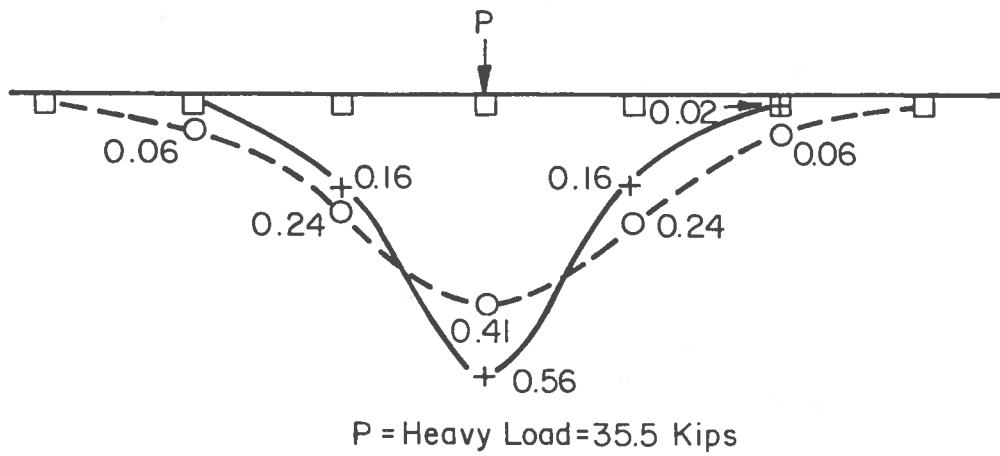
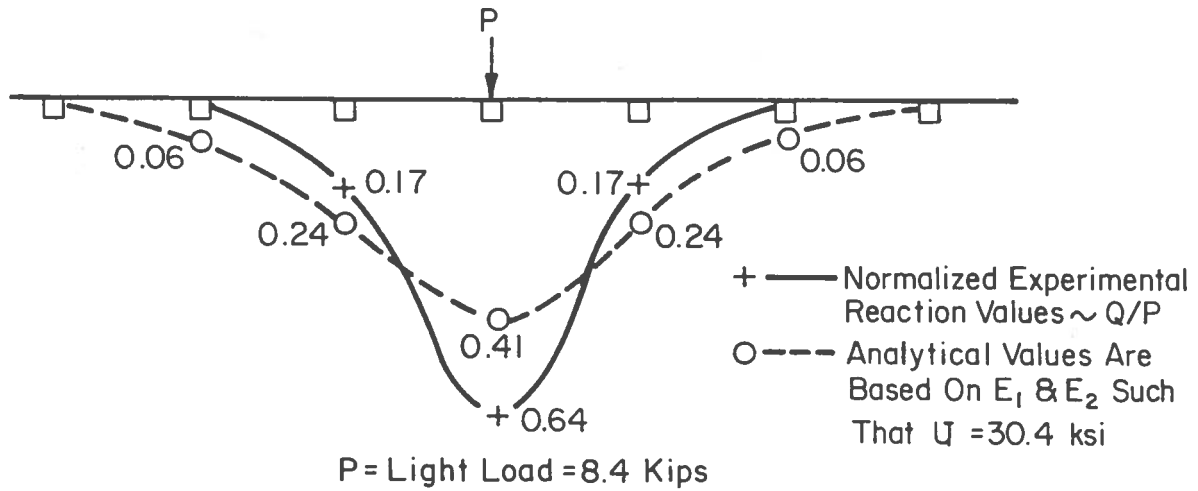


FIGURE 7. COMPARISON OF EXPERIMENTAL AND ANALYTICAL RAIL SEAT LOADS FOR SITE 2 (Tangent track with 20-inch tie spacing)

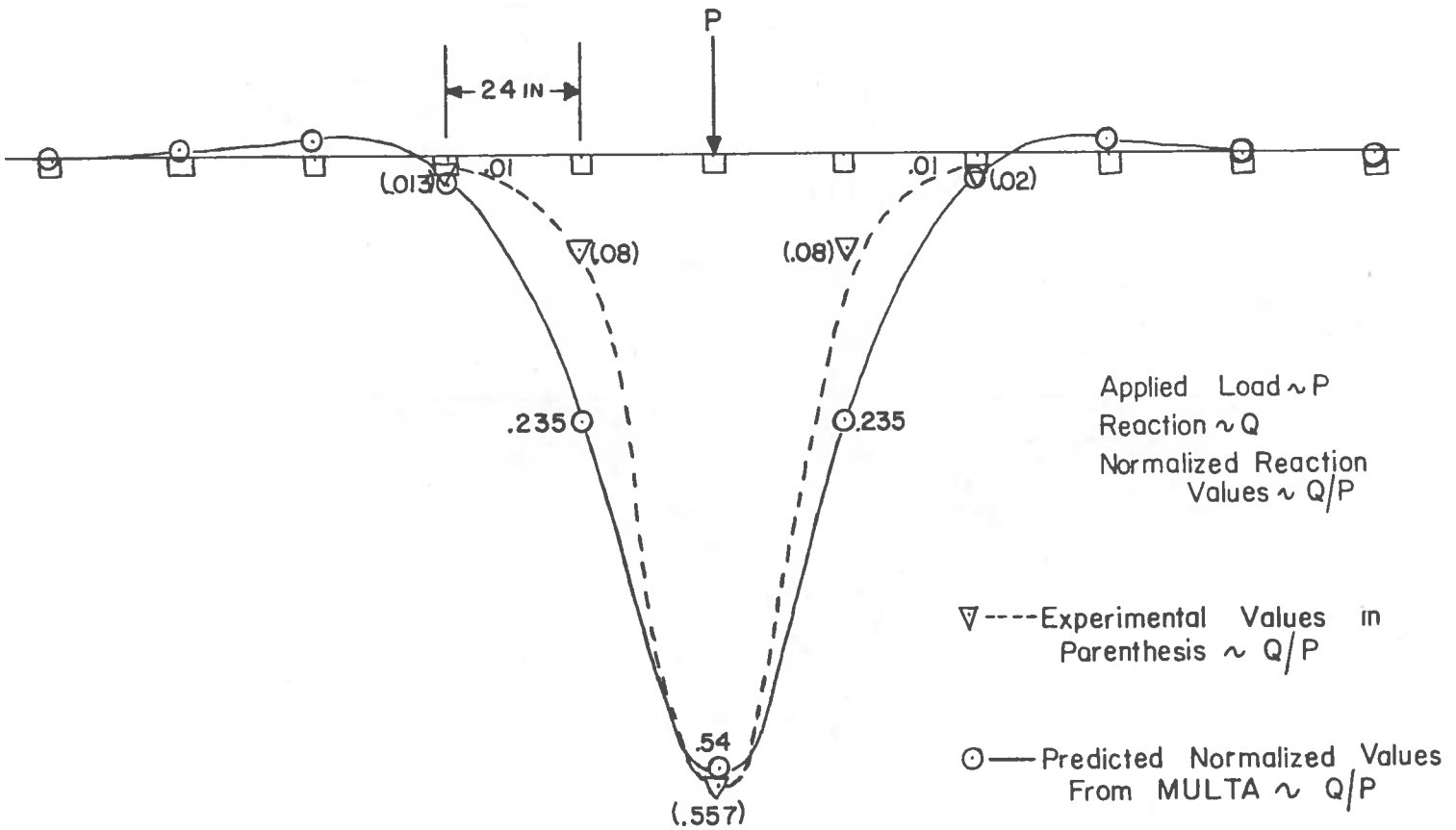


FIGURE 8. COMPARISON OF MEASURED AND PREDICTED VERTICAL RAIL SEAT LOADS AT SITE 1

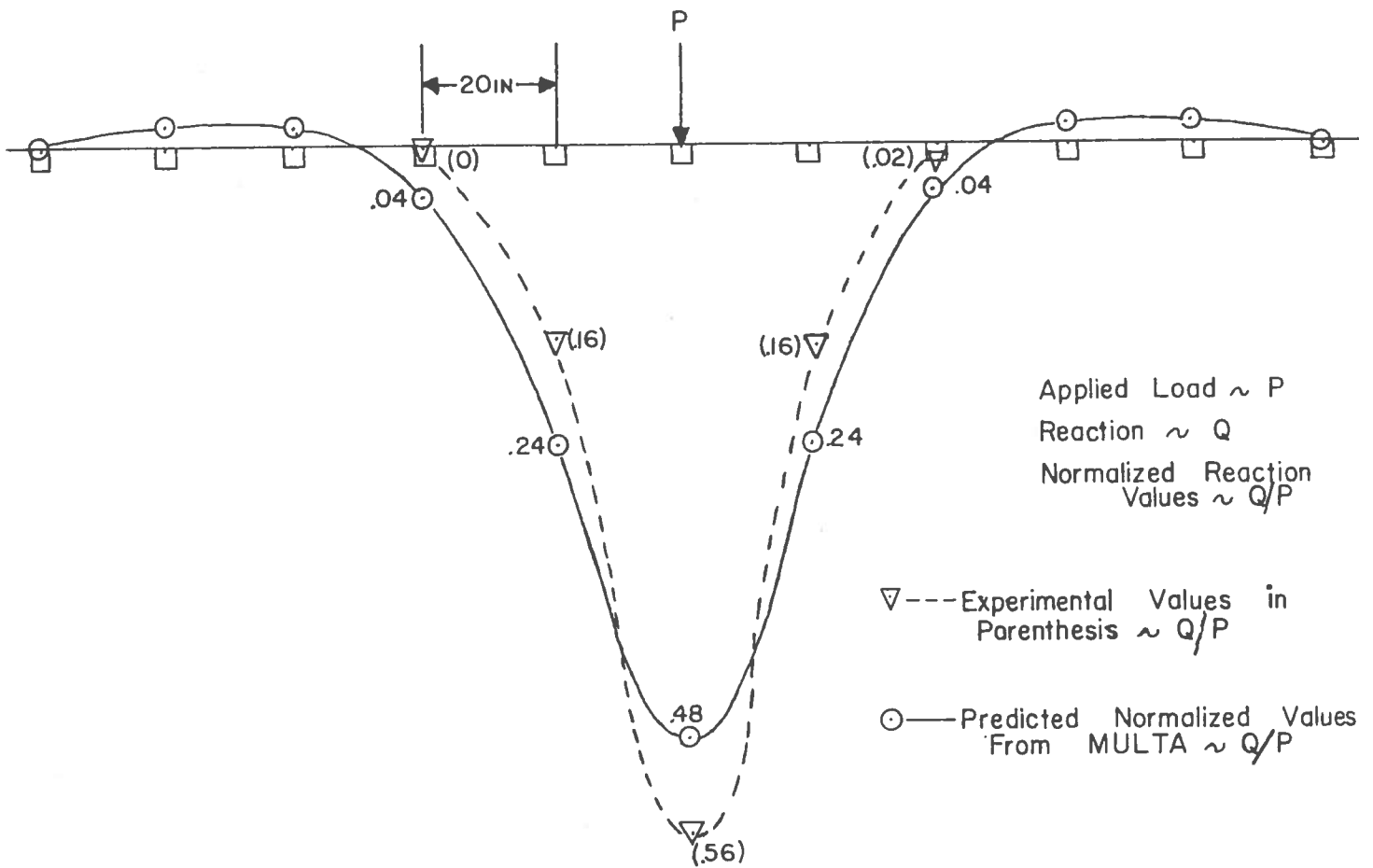


FIGURE 9. COMPARISON OF MEASURED AND PREDICTED VERTICAL RAIL SEAT LOADS AT SITE 2

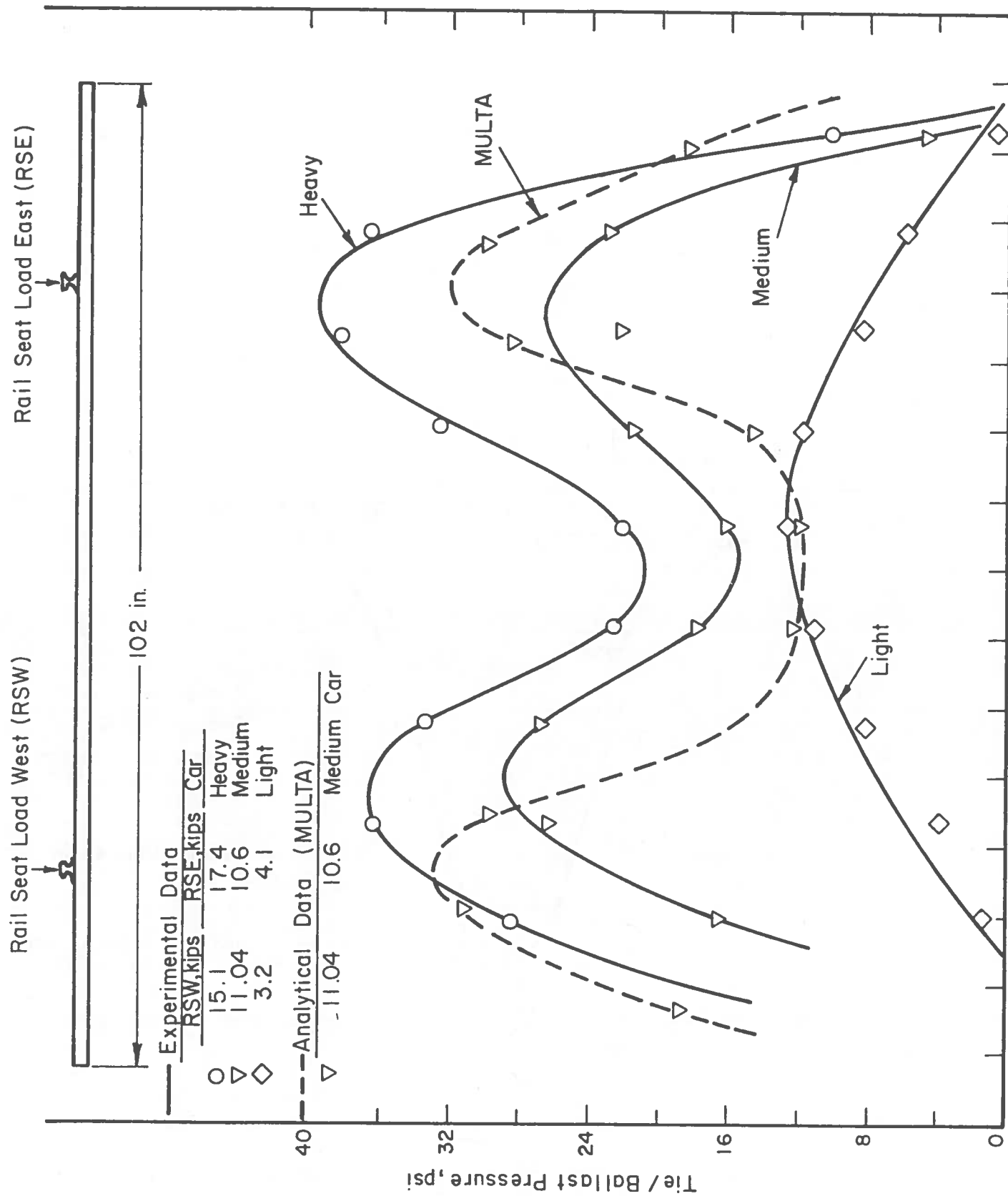


FIGURE 10. TIE/BALLAST PRESSURE DATA AT SITE 1

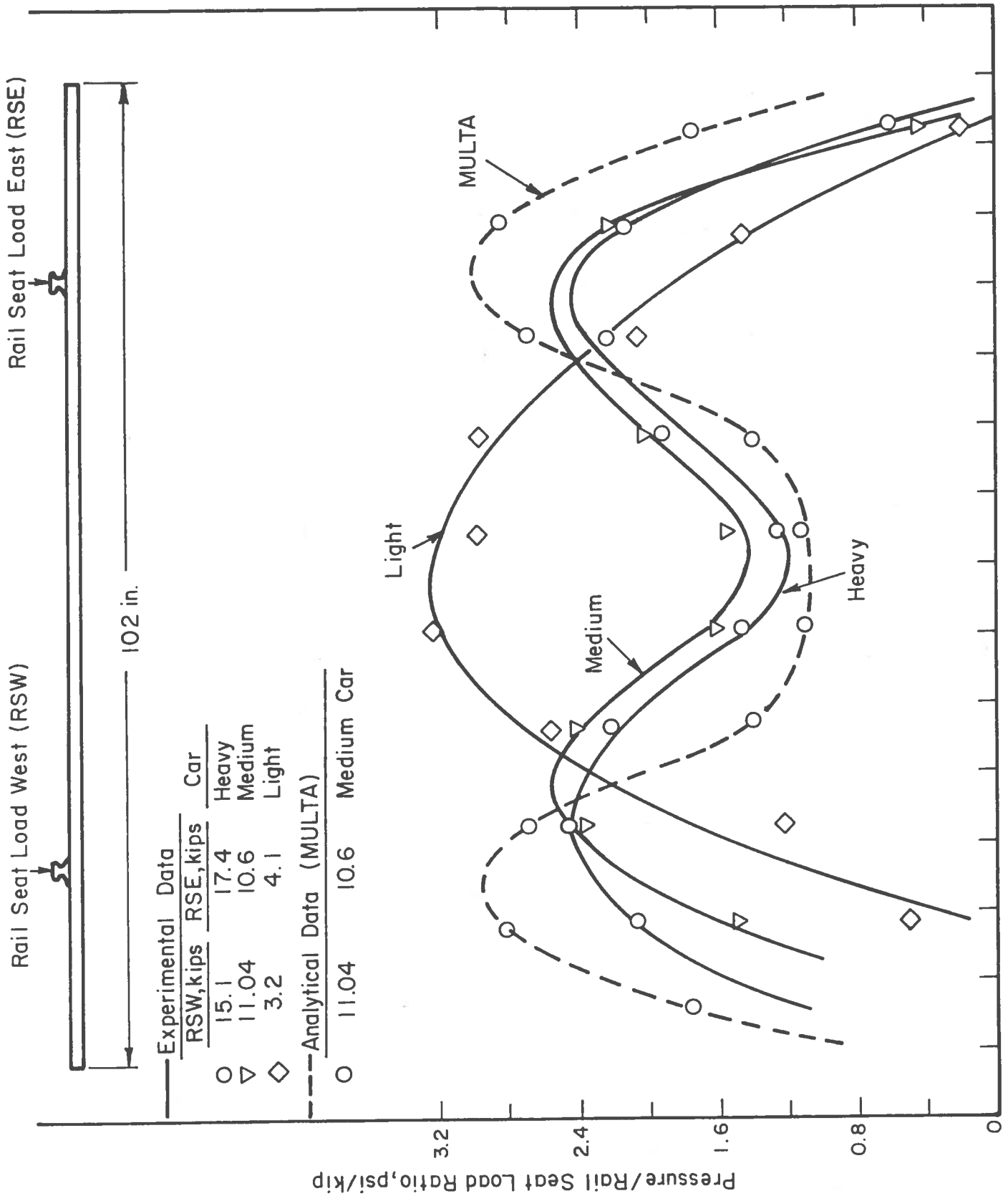


FIGURE 11. BALLAST/TIE VERTICAL PRESSURE NORMALIZED TO RESPECTIVE RAIL SEAT REACTION, AT SITE 1

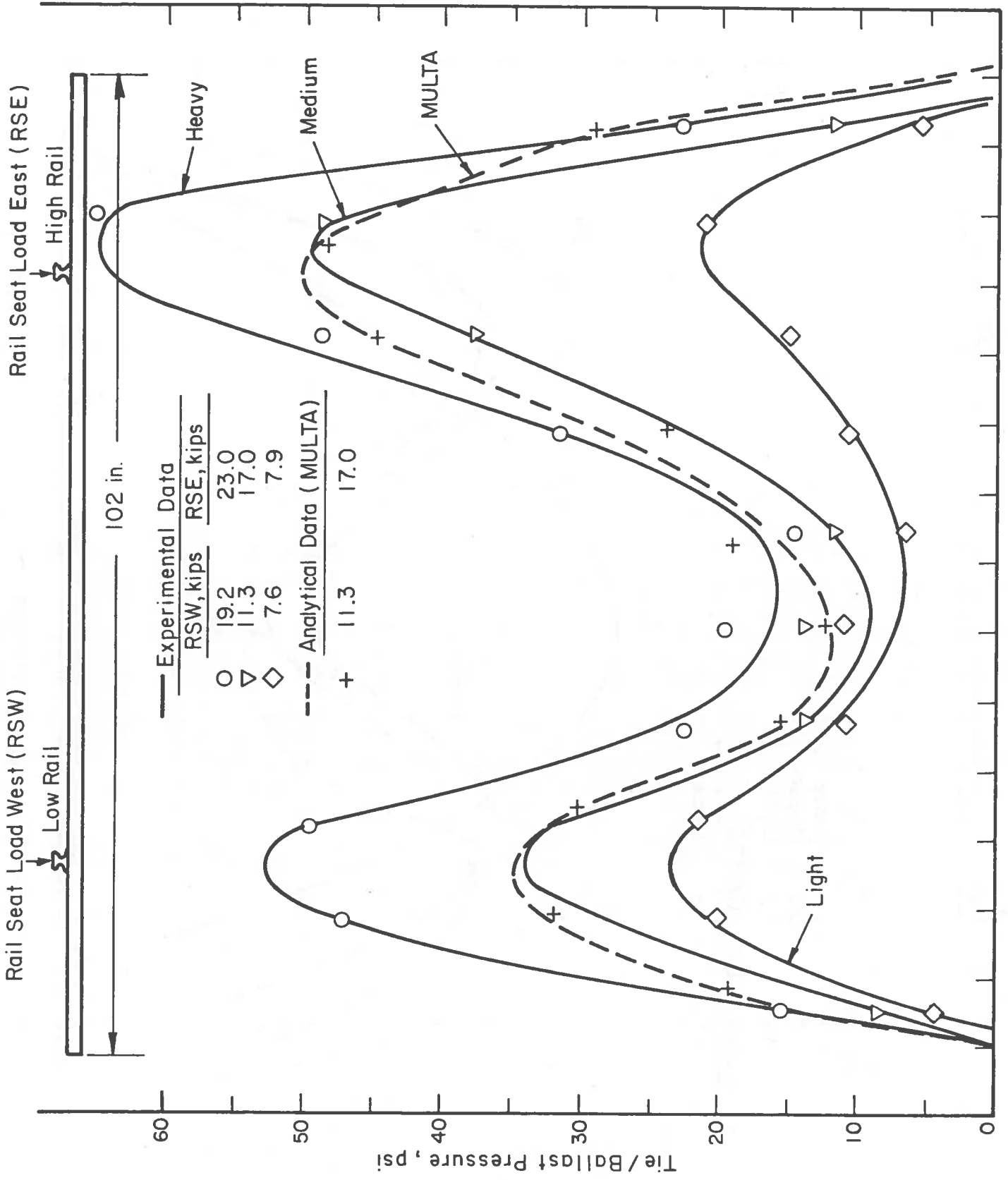


FIGURE 12. TIE/BALLAST VERTICAL PRESSURE, DATA AT SITE 3

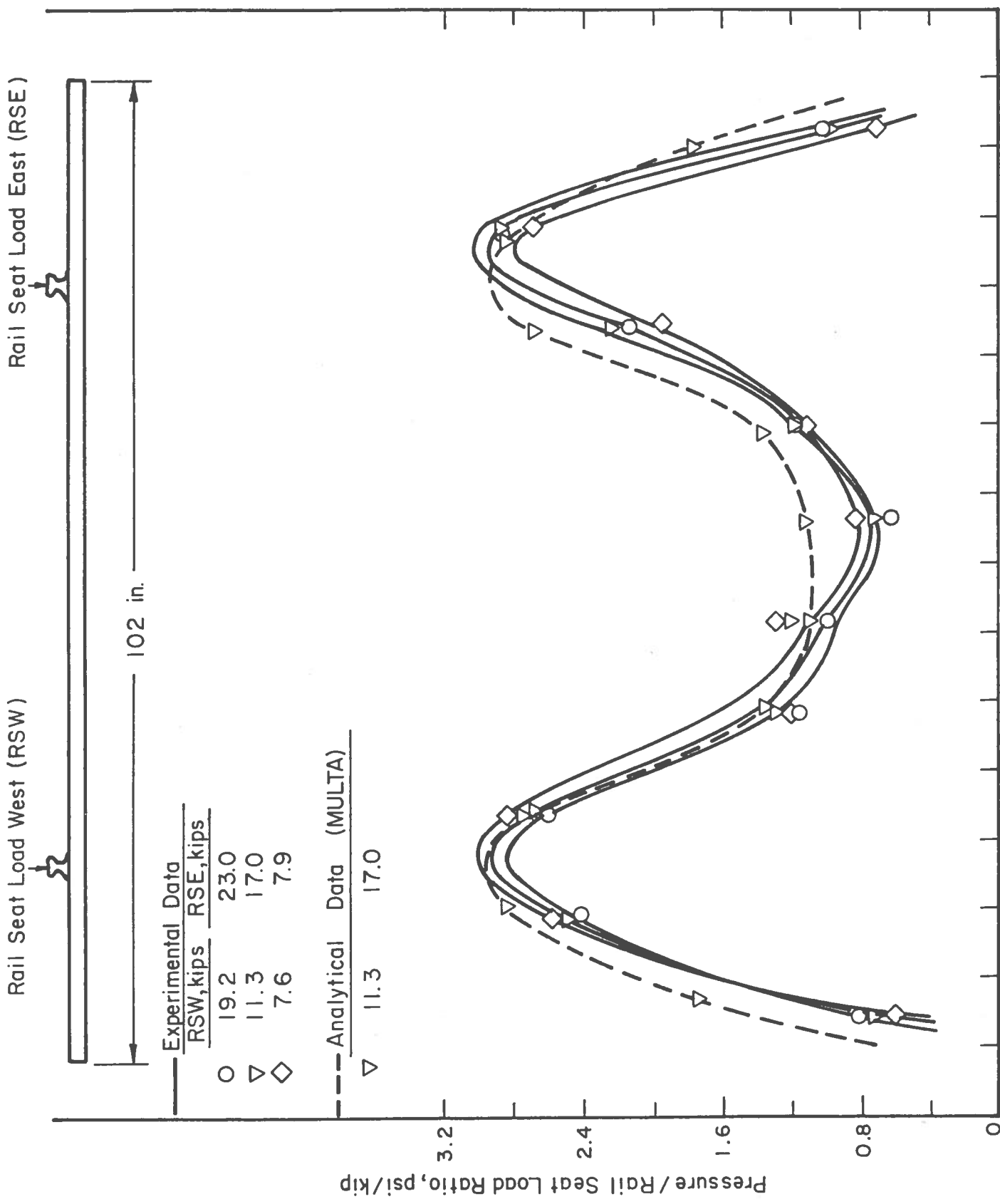


FIGURE 13. BALLAST/TIE VERTICAL PRESSURE NORMALIZED TO RESPECTIVE RAIL SEAT REACTION, DATA AT SITE 3

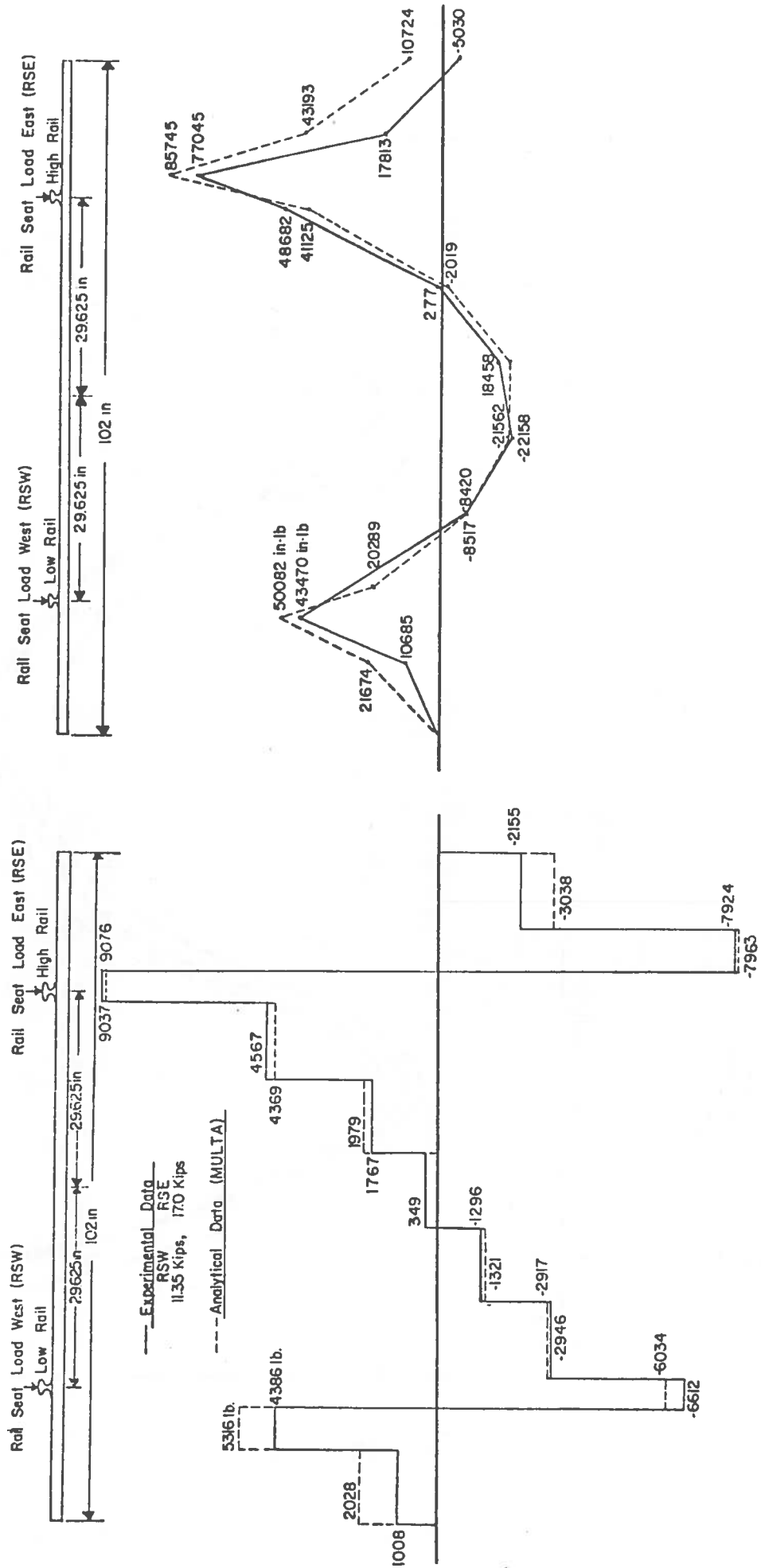


FIGURE 14. TIE SHEAR FORCE, LCT-0 DATA (SITE 3)

FIGURE 15. TIE BENDING MOMENT, LCT-0 DATA (SITE 3)



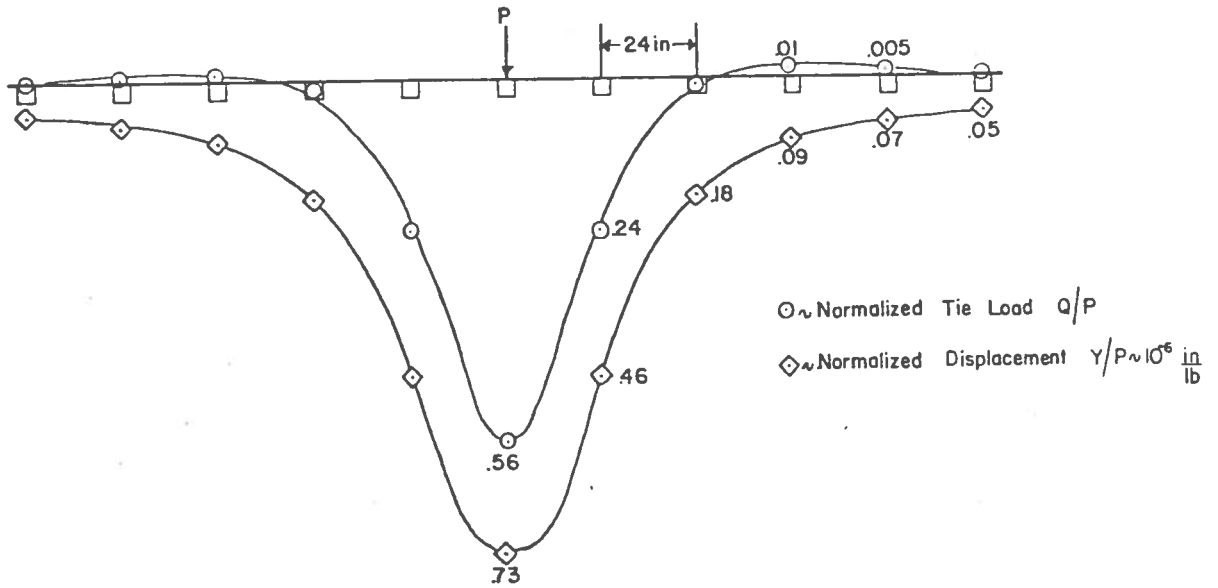


FIGURE 16. PREDICTED TIE LOAD AND DISPLACEMENT DISTRIBUTION (SITE 1)

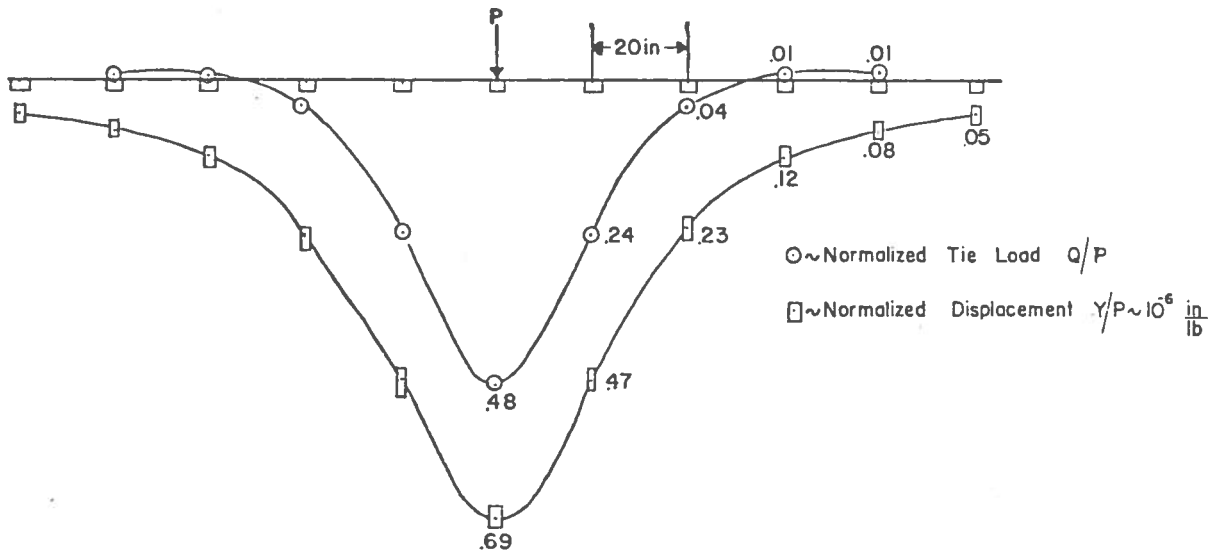


FIGURE 17. PREDICTED TIE LOAD AND DISPLACEMENT DISTRIBUTIONS (SITE 2)

