

Quantification of longitudinal fastener stiffness and the effect on fastening system loading demand

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Abstract

Over the past 20 years, there have been at least 10 derailments due to spike fastener fatigue failures in North America. These fatigue failures have been considered a moderate to severe challenge that require manual walking inspections that are both time and labor intensive. These fatigue failures have been found to result from spike overloading due to lateral and longitudinal loads. To date, there has been limited quantification of the vertical, lateral, and longitudinal fastener forces in track. This paper quantifies the effect of fastener type on fastener load to account for various track types and locations. Laboratory experimentation was performed to quantify the stiffness of multiple fastening systems and this data was input into a previously validated analytical model to quantify the effect of stiffness on fastener loading. Additional laboratory experimentation was performed to quantify the stiffness between fastening systems, the model results indicate that the load transferred to the fastening system is less sensitive. However, spike strain data indicate the load path was affected by fastener type and vertical load. The characterization of longitudinal stiffness of multiple fastening systems and the relationship to spike load as presented can be used to advance track mechanistic-empirical design and improve rail neutral temperature prediction and track buckling models.

Keywords

Fastening systems, elastic, anchored, stiffness, laboratory, longitudinal load, uplift

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Introduction

94% of the world's railroad infrastructure is supported by ballast.¹ A ballasted track system is comprised of the rail, fastening systems, sleepers, ballast, sub-ballast, and sub-grade.² Rail fastening systems, in conjunction with the sleeper, secure the rail to maintain gauge, transmit thermal and service loads, and anchor the rail sleeper structure against lateral and longitudinal movements.² In doing so, fastening systems must transmit vertical, lateral, and longitudinal loads.

As wheel loads and resulting forces transferred to the track structure have increased, and/or geometric tolerances become more stringent, fastening systems are required to perform more rigorous tasks (maintain tighter tolerances, provide creep resistance, etc.). An example of how fastening system requirements respond to vertical, lateral, and longitudinal forces is presented to demonstrate how different components could be used in different track structures (Table 1).

Fastening systems, like many of the components in the rail infrastructure, have evolved iteratively over time, through a trial-and-error design approach aimed at addressing conditions symptomatic of track strength and force transfer deficiencies (e.g., plate cutting, rail seat deterioration, rail rollover, rail pad movement).³ These deficiencies

have also led to various track component failures (e.g., broken spikes, broken shoulders, broken threaded rods, etc.) that have caused derailments.^{4–9} Between 1999 and 2018 there were 250 Federal Railroad Administration (FRA) reportable derailments on mainlines and sidings in the United States caused by "defective or missing spikes or rail fasteners".¹⁰ This iterative approach has led to the installation of fasteners in track where demands exceed capacity, inefficient designs, and an insufficient understanding of the underlying mechanisms that govern the track system's response to changing conditions (e.g., input loads, component wear, support conditions) that has manifested itself in maintenance and safety problems as

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Table 1. Requirements of track fastening systems as a function of demands.

evident from the review of FRA track-caused derailment data.

Spike fatigue failure in timber sleepers using elastic fastening systems is one fastening system component failure example that over the past 20 years, has caused at least 10 derailments in North America.⁹ These spike fatigue failures reduce the safety and integrity of the track and require manual walking inspections. Recent studies have shown that spikes in elastic fastening systems, unlike anchored systems, transfer additional longitudinal load to the spike,⁹ leading to higher magnitude resultant spike stresses. Further, because the elastic fastener attaches the rail to the plate, the wave action of the rail creates separation of the plate from the sleeper, as explained through beam on elastic foundation principles.¹¹ This separation eliminates friction at this interface leading to additional transfer of longitudinal load to the spike.¹² Finally, these fatigue failures are almost exclusively found in environments that are subjected to high lateral loads (curves, special track work, etc.).¹³ Therefore, these failures are believed to be caused by a combination of lateral and longitudinal loads transferred to the spikes.¹³

It has also been shown that the quantification of lateral and longitudinal rail seat loads, though necessary, is not sufficient in estimating failure locations.¹⁴ The vertical, lateral, and longitudinal rail seat loads, as well as an estimate of the friction at the plate-sleeper interface, are required because the vertical rail seat load and fastener friction govern the failure threshold load.¹⁴ As such, Dersch et al.¹⁵ quantified the vertical, lateral, and longitudinal fastener loads in the field at a location that had experienced spike fatigue failures. However, these data only represent one location with a given fastening system and lacks the quantification of load transferred to the spike.

It is hypothesized that variability in fastening system can lead to variations in longitudinal track modulus and as track modulus increases the fastener load increases.¹⁶ Given the prevailing trial-and-error design approach and understanding that different components are required to meet varying demands (Table 1), fastening system designs vary greatly. For example, to distribute vertical and longitudinal loads, fastening systems can use plates of varying size, shoulder designs, or spike patterns. Further, to ensure adequate creep resistance, these systems can use either anchors and elastic fasteners, or the combination of both. Therefore, further quantification is required to account for additional fastening systems used in railroad infrastructure.

A preliminary analysis investigating the effect of fastening system stiffness on track modulus was performed. Longitudinal track modulus was calculated using both the fastener and ballast stiffnesses (equation (1)).¹⁷

$$k_a = \frac{1}{\left(\frac{1}{k_p}\right) + \left(\frac{1}{k_b}\right)} \tag{1}$$

Where k_a longitudinal track stiffness, k_p fastener stiffness, k_b ballast stiffness

Trizotto et al.¹⁶ showed that a quadrupling of longitudinal track modulus led to an increase in fastener force by approximately 50%. Assuming 50.8 cm (20 in.) sleeper spacing and a constant longitudinal ballast stiffness of 7010 kN/m (40 kips/in.)¹⁶ the data from this analysis (Figure 1) indicate that it is possible to quadruple track stiffness by changing fastener stiffness. For example, when fastener stiffness increased from 1750 kN/m (10 kips/in.) to 28,000 kN/m (160 kips/in.), the track stiffness increased from 55,200 to 222,000 kN/m (8-32 kips/in.). However, this preliminary analysis indicates that track stiffness is less sensitive to an increase in stiffness of a single component and is controlled by the component with the lowest stiffness value. That is, for the ballast condition considered, when the fastening system stiffness increases beyond 7010 kN/m (40 kips/in.) the track stiffness would not even double for all realistic fastener stiffnesses.

To estimate the longitudinal fastening system forces, this paper quantifies the longitudinal stiffness of four fastening systems in the laboratory and quantifies the effect of fastening system on load transfer using a previously validated model. This paper also quantifies the effect of vertical load (downward pressure and uplift) on the transfer of load to the cut spikes in fastening systems using instrumented spikes.

In addition to quantifying the relationship between spike load and fastener stiffness, quantification of fastening system stiffness can also provide an input for the proposed mechanistic-empirical (M-E) railroad track design procedure which is currently being developed.¹⁸ This M-E concept, which is similar to the mechanistic empirical pavement design guide methodology currently used in pavement design.¹⁹ The concept has been developed and advanced by researchers within the Rail Transportation and Engineering Center (RailTEC) at the University of Illinois at Urbana-Champaign (Illinois) since 2013.^{20,21}

It is believed that these data can be used to improve future fastening system designs to mitigate spike fatigue failures and other force transfer deficiency challenges as well as improve the accuracy of track models that predict the distribution of rail stress and track buckling strength.

Materials and methods

Laboratory experimentation overview

The longitudinal fastening system stiffness and load transferred to the spike was quantified for both elastic and anchored fastening systems. This quantification was conducted following a modified longitudinal load restraint test as recommended by AREMA.²² Experiments were performed on a single rail seat using a bi-axial load frame (Figure 2) which leveraged 136 RE rail, a $178 \times 229 \times$ 762 mm (7 × 9 × 30 in.) timber block, and four unique fastening systems. Although literature indicates dynamic load could prove to be most critical,²³ for fastener stiffness quantification within the laboratory, static loading is sufficient.

Longitudinal load was applied 19 mm (0.75 in.) above the bottom of the rail base, continuously, without shock, until slip of the rail occurred, as recommended by AREMA Committee 30, TIES,²⁴ to ensure there was no moment or angle induced in the rail during load application. The rail displacement and longitudinal load applied were recorded for the duration of the test. The stiffness of four fastening system configurations including an anchor only, eclip only, anchor and eclip, and tension clamp fastening system were quantified (Figure 2). Each configuration comprised of a timber sleeper block, spikes, plate, and either an anchor, eclips, anchor and eclips, or tension clamps. The eclips and tension clamps provided approximately a 2400 lb. toe-load.



Figure 1. Effect of fastener stiffness on track modulus.



Figure 2. Longitudinal stiffness experimentation setup and naming convention.

The anchor only, eclip only, and anchor and eclip systems used an 18 in. plate and four cut spikes. The tension clamp system used a 14 in. plate, four screw spikes and spring washers.

For the quantification of the relationship between spike load and fastening system type, a vertical compressive (vertical downward) load of 22.2 and 44.5 kN (5 and 10 kips) or tensile (vertical uplift) load sufficient to cause separation between the plate and sleeper, or approximately 22.2 kN (5 kips) was applied to the rail prior to the application of the longitudinal load. The vertical compressive load applied to the rail pre-engaged the plate to sleeper, thus developing friction at this interface. This pre-engagement is accomplished using screw spikes with spring washers and has been shown to reduce the spike strain by 70–80%.¹⁴ The vertical tensile load represents the wave-action of the rail that leads to the development of a gap between the plate and the sleeper.¹⁶

Fastening system longitudinal stiffness

The load and displacement recorded for the duration of the experiment were used to quantify the stiffness of each system. Stiffness is defined as the slope of the longitudinal load verses displacement data for each fastening system between 2.22 and 11.1 kN (500–2500 lb.). This range was used for stiffness quantification because it was within the elastic region of each fastening system (Figure 3).

Using the calculated stiffness, variance for each fastener was calculated using equation (2).²⁵ Additionally, the Mann-Whitney U-test was used to determine if the stiffnesses and rail seat loads were statistically different.²⁶

$$\sigma^2 = \frac{\sum\limits_{i=1}^{n} \left(\mathbf{x}_i - \boldsymbol{\mu} \right)^2}{n} \tag{2}$$

Where n quantity of data points, x_i relative data point, μ mean, σ^2 variance

Variance in spike load & uplift

To quantify the loads transferred to the spikes, four instrumented spikes were installed. Each instrumented spike had one strain gauge inset 1.5 mm (0.059 in.) into the tensile side of the spike located approximately 38.1 mm (1.5 in.) below the top of the sleeper surface once installed. This depth is known to be the location of maximum spike strain¹³ (Figure 4(a)). The strains from the four instrumented spikes were transformed into loads using:

- 1. strain and load data from the five-surface-straingauged instrumented spike (Figure 4(b)) as presented by Dersch et al.¹³ and
- knowledge that the strains measured within the inset would need to be multiplied by a factor to account for the linearly increasing strain from the neutral axis to the spike surface.

The strains from the five-surface-strain-gauged instrumented spike recorded at 29.5 and 42.9 mm (1.16 and 1.69 in.) below the top of the sleeper (Figure 4(a)) were linearly interpolated to quantify the relationship between spike surface strain and load at 38.1 mm (1.5 in.). Additionally, to account for the inset, and knowing the spike is 15.88 mm (0.625 in.) square, the strain recorded in the inset was multiplied by 1.25 to account for the linear increasing



Figure 3. Example longitudinal load versus displacement of fastening system.



Figure 4. Instrumented spike with strain gauge in 1.5 mm (0.059 in.) inset as used within this paper (a) and five-surface-strain-gauged instrumented spike used for calibration of strain to load (b).

strain from the neutral axis to the surface, as would be expected based on mechanics.²⁷ Multiplying the strain by each factor resulted in an equation relating strain measured to load (equation (3)).

$$P_s = 1.332 \times 10^6 \times \varepsilon \times 1.25 \tag{3}$$

Where P_s Spike Load (lbf.), ε Spike Strain

Analytical modeling overview

To quantify the fastener load, and thus the effect of fastener stiffness, an analytical model was used. Trizotto et al.¹⁶ developed a validated linear 1D analytical model that quantifies rail seat response when subjected to a train pass using a quantified number of locomotives, tractive effort, and longitudinal ballast modulus. This analytical model expanded upon the approach presented by Kerr.³ Laboratory fastening stiffness data were used as inputs into this model while maintaining the other inputs (ballast stiffness, tractive effort, etc.).

Results

Fastener stiffness variance

Five fastener-stiffness test replicates were performed using the protocol discussed previously (Figure 5). The data recorded from each replicate were then tabulated with quantified values for stiffness range, median and variance (Figure 5)

There is a statistical difference between each fastener type when using the Mann-Whitney U-test. Additionally, the data indicate the eclip only fastening system exhibited a median fastening system stiffness that was four times greater than the anchor only system (11,400 kN/m (65.3 kip/in.) vs 2910 kN/m (16.6 kips/in.), respectively). With the addition of an anchor, the anchor + eclip system

exhibited a median fastening system stiffness approximately three times greater than the anchor only system. The tension clamp system had comparatively lower median stiffness to the eclip only and anchor + eclip but was still two times greater than the anchor only configuration.

Considering the effect of fastener stiffness on track stiffness using median values, when increasing fastener stiffness from 2910 kN/m (16.6 kips/in.) to 11,400 kN/m (65.3 kips/in.), there is an increase in track stiffness of approximately 100%; 85,500–171,000 kN/m (12.4–24.8 kips/in.).

The quantification of fastener longitudinal stiffness led to the findings that these elastic fasteners result in both greater magnitudes and variance of fastener stiffness. Anchor only fastening systems had smaller stiffness variance and less than half the magnitude of stiffness compared to the elastic fasteners. This is likely because elastic fastener clamping force provides higher axial stiffness compared to the anchor bearing on the timber sleeper.

Rail seat load variance

The fastening system stiffness values were used as inputs for the analytical model developed by Trizotto.¹⁶ It was assumed that three locomotives with 44.5 kN/wheel (10 kips/wheel) of tractive effort applied longitudinal load to the track and the longitudinal ballast modulus was 13,800 kN/m/m (2 kips/in./in) (i.e., a stiffness of 7010 kN/m (40 kips/in.) at 508 mm (20 in.) sleeper spacing).

Using this information, the total longitudinal fastener (rail seat) load was quantified (Figure 6). The data recorded from each replicate were then tabulated with quantified values for fastener (rail seat) load range and median (Figure 6). The rail seat loads for each fastener were also determined to be statistically different using the Mann-Whitney U-test.



Figure 5. Fastener stiffness variance.



Figure 6. Rail seat loads with different fasteners.

The percent increase in fastener stiffness was not directly translated into a similar magnitude percent increase in rail seat load. That is, while the e-clip only system exhibited a median stiffness that was four times greater than the anchored system, the median force was only 8.3% greater (i.e. 0.48 kN (107 lb.)). This is likely because the ballast longitudinal stiffness of 7010 kN/m (40 kips/in.) controlled the longitudinal track modulus when the fastening system stiffness exceeded the ballast stiffness.

Spike load variance

To quantify the relationship between spike load and fastening system type, data from instrumented spikes were used. Spike load was found to be dependent on fastening system type (Figure 7). When a longitudinal rail seat load of 8.90 kN (2000 lb.) was applied to the anchor only system, zero spike load was quantified. Comparatively, the average spike load was 2.08 kN/spike (468 lb./spike) and 1.54 kN/ spike (347 lb./spike) when the eclip only and anchor + eclip system was used, respectively. This translates to approximately 94% and 69% of the longitudinal load being transferred to the spikes when the anchor + eclip fastening system and eclip only system was used, respectively.

The relationship between spike load and the presence of vertical load was quantified using the eclip only fastening systems and an application of 8.90 kN (2000 lb.) longitudinal load (Figure 8). Zero vertical load resulted in 2.08 kN/spike (468 lb./spike), or 94% of applied longitudinal load, being transferred to the spikes. Vertical compressive loads of 22.2 and 44.5 kN (5 and 10 kips) resulted in 596 and 462 N/spike (134 and 104 lb./spike) (i.e., 27% and 21% of applied longitudinal load), respectively. Comparatively, uplift resulted in 2.22 kN/spike (500 lb./ spike), or 100% of applied longitudinal load. Therefore, there was an inverse relationship between vertical compressive load applied and resulting spike load. That is, as the vertical compressive load increased, the spike load decreased. Further, the data indicate that the spike load is similar when no vertical load is applied (the control) and when a load creating uplift between the spike is applied.

The relationship between spike load and vertical load was quantified using eclip only fastening systems. The data indicated that downward pressure at the plate-to-sleeper interface resulted in less load being transferred to the spikes. More specifically, when a 22.2 kN (5000 lb.) vertical load was applied, there was a 72% reduction in loads transferred to the spike. Additionally, when a 44.5 kN (10,000 lb.) vertical load was applied, there was a 78% reduction when compared to the zero vertical (control) case. This generally aligns with the findings that a vertical plate load of 17.8 kN and 60.1 kN (4000 and 13,500 lb.) resulted in a 70% and 80% reduction in spike stress, respectively.¹⁴ Meanwhile, uplift resulted in 7% increase in spike loads which should be expected given there is minimal friction between the plate and sleeper after installation and minimal traffic that would cause spike uplift or plate cutting. This data indicates that when vertical load is not present, the longitudinal load is transferred by the spike but when a vertical compressive load is present, it is transferred by a combination of both friction and bearing.

The quantification of variance in spike loads within anchored and elastic fastening systems with zero vertical load present indicate that fasteners with elastic fasteners distribute more load to the spikes. Under these loading conditions, anchor only fastening systems distribute the longitudinal load through the anchor-to-sleeper interface. Whereas elastic fasteners distribute a significant portion of



Figure 7. Spike load variance between separate fasteners with 8.90 kN (2000 lb.) of applied longitudinal load.



Figure 8. Spike load variance from vertical load within eclip only fastening systems with 8.90 kN (2000 lb.) of applied longitudinal load.

the longitudinal load through the plate-spike interface. This results in the eclip only fastening system distributing load to the spikes as opposed to the anchors.

Conclusion

To reduce the risk of fastener failures due to overloading (e.g., spike fatigue failures) quantification of vertical, lateral, and longitudinal loads, and load transferred to individual components (e.g., the spike) is needed to improve current track design methods. Therefore, a laboratory study was undertaken to quantify the longitudinal stiffness of multiple fastening systems and how the stiffness affects the fastener loads. Additionally, instrumented spikes were used to quantify the relationship between spike loads and fastener type as well as plate compression and uplift. Laboratory data collected in this investigation indicate that:

- 1. Fastening system type significantly affects fastening system stiffness
 - Eclip only exhibited 4 times greater stiffness than anchor only
 - Anchor + eclip exhibited 2.75 times greater stiffness than anchor only
 - Tension clamp exhibited 2.25 times greater stiffness than anchor only
- 2. Fastening system stiffness did not result in similar percentage magnitude increases in load:
 - The eclip only system exhibited an 8.3% greater force (i.e., 0.48 kN (107 lb.))

- The anchor + eclip system exhibited a 6.7% greater force (i.e., 0.39 kN (87 lb.))
- The tension clamp system exhibited a 5.8% greater force (i.e., 0.34 kN (76 lb.))
- 3. Fastening system type significantly affects spike loads:
 - Applying a longitudinal load of 8.90 kN (2000 lb.) resulted in:
 - Zero load transferred to the spikes for anchor only system
 - 2.08 kN/spike (468 lb./spike), on average, for eclip only system
 - 1.54 kN/spike (347 lb./spike), on average, for anchor + eclip system, eclip only
- 4. There is an inverse relationship between plate compression and spike loads
 - Compared to the case where no vertical load is applied, when applying a vertical load prior to a longitudinal load of 8.90 kN (2000 lb.) there was a:
 - 72% reduction in spike load, on average, with 22.2 kN (5 kips) down
 - 78% reduction in spike load, on average, with 44.4 kN (10 kips) down
 - 7% increase in spike load, on average, with uplift

The characterization of longitudinal stiffness, quantification of spike load relationship with fastener type, and the effect of plate compression and uplift presented within this paper can be used to advance track mechanistic-empirical analysis and design principles as well as improve rail neutral temperature prediction and track buckling models.²⁸ However, there are some limitations of the 1D model as described by Trizotto.¹⁶ That is, the model only accounts for the linear approximation of the load-displacement response of the track, is applicable to open track, does not allow for any temperature gradients, and fixes the rail at both ends. Further, the stiffness is not variable throughout the model (i.e., the stiffness is constant for all fasteners). Keeping these limitations and simplifications in mind, the loads under revenue service traffic could be different. Thus, additional experimental work in the laboratory or field is needed to ensure that variance in fastener stiffness results in minimal variance in load distributed to a single rail seat.

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References

- Matias SR and Ferreira PA. Railway slab track systems: review and research potentials. *Struct Infrastruct Eng* 2020; 16(12): 1635–1653.
- Hay WW. Chapter 25: fastenings and other track materials. In: *Railroad engineering*. New York, NY: John Wiley & Sons, 1982, pp. 562–592.
- Kerr AD. Miscellaneous track problems and analyses. In: *Fundamentals of railway track engineering*. Omaha, NE: Simmons-Boardman, 2003, pp. 318–336.
- Choros J, Coltman MN and Marquis B. Prevention of derailments due to concrete tie rail seat deterioration. In: Proceedings of the 2007 Joint Rail Conference and Internal Combustion Engine Division Spring Technical Conference, Pueblo, CO, 13–16 March 2007: American Society of Mechanical Engineers (ASME), pp. 173–181.
- Wolf GP. Effects of wide gauge on derailment potential. In: Proceedings of 2014 Wheel Rail Interaction Heavy Haul Seminar Chicago, IL, January 2014, p. 4.
- Wu H and Kerchof B. Management of wheel/rail interface to prevent rail rollover derailments. *Proc Inst Mech Eng F: J Rail Rapid Transit* 2014; 228(6): 673–686.
- McHenry M and LoPresti J. *Tie and fastener system gage* restraint performance at FAST. Report No.: TD-15-013. Pueblo, CO: Association of American Railroads, Transportation Technology Center, Inc., 2015, p. 4.
- Edwards JR, Chavez L, Qian Y, et al. *Field study and an-alytical modeling of the performance of existing WMATA anchor bolt system*. Report No.: CQ17066. Washington, DC: Washington Metropolitan Area Transit Authority (WMATA), 2018. pp. 1–140.
- Roadcap T, Kerchof B, Dersch MS, et al. Field experience and academic inquiry to understand mechanisms of spike and screw failures in railroad fastening systems. In: Proceedings of the 2019 AREMA Annual Conference with Railway Interchange, Minneapolis, MN, 22–25 September 2019. AREMA.
- Wang BZ, Barkan CP and Rapik Saat M. Quantitative analysis of changes in freight train derailment causes and rates. *J Transp Eng A Syst* 2020; 146(11): 04020127.
- Hetenyi M. Beams on elastic foundation; theory with applications in the fields of civil and mechanical engineering. Ann Arbor, MI: The University of Michigan Press, 1946, Vol. XVI.
- Gao Y, McHenry M and Kerchof B. Investigation of broken cut spikes on elastic fastener tie plates using an integrated simulation method. In: Proceedings of the 2018 Joint Rail Conference, Pittsburgh, PA, 18–20 April 2018.

- Dersch M, Roadcap T, Edwards JR, et al. Investigation into the effect of lateral and longitudinal loads on railroad spike stress magnitude and location using finite element analysis. *Eng Fail Anal* 2019; 104: 388–398.
- Dersch MS, Khachaturian C and Edwards JR. Methods to mitigate railway premium fastening system spike fatigue failures using finite element analysis. *Eng Fail Anal*, 2020; 121: 105160.
- Dersch MS, Trizotto M, Edwards JR, et al. Quantification of vertical, lateral, and longitudinal fastener demand in broken spike track: Inputs to mechanistic-empirical design. *Proc Inst Mech Eng F: J Rail Rapid Transit* 2021; 236: 557–569, DOI: 10.1177/09544097211030736
- Trizotto M, Dersch MS, Edwards JR, et al. Analytical elastic modeling of rail and fastener longitudinal response. *Transp Res Rec* 2021; 2675: 164–177.
- Marquis B, Liu S and Stuart C. Longitudinal rail load distribution: an analytical solution. In: Proceedings of the 2020 Joint Rail Conference, St. Louis, MO, 20–22 April 2020. American Society of Mechanical Engineers (ASME), p. V001T08A010.
- Edwards JR, Quirós-Orozco RJ, Bastos JC, et al. A vision for mechanistic-empirical railway track system and component analysis and design. *Transp Res Rec* 2021; 2675: 41–55.
- American Association of State Highway and Transportation Officials (AASHTO). *Mechanistic-empirical pavement design guide*. Washington, DC: American Association of State Highway and Transportation Officials (AASHTO), 2008, p. 218.
- 20. Quirós-Orozco RJ. Prestressed concrete railway crosstie support variability and its effect on flexural demand. Master's

Thesis. Urbana, IL: University of Illinois at Urbana-Champaign, Department of Civil and Environmental Engineering, 2018.

- Edwards JR. Quantification of prestressed concrete railway crosstie flexural response: implications for mechanistic design. Doctoral Thesis. Urbana, IL: University of Illinois at Urbana-Champaign, Department of Civil and Environmental Engineering, 2019.
- American Railway Engineering and Maintenance-of-Way Association (AREMA). Chapter 30, Ties. In: *Manual for railway engineering*. Landover, MD: The American Railway Engineering and Maintenance-of-Way Association, 2017.
- Van Dyk BJ, Edwards JR, Dersch MS, et al. Evaluation of dynamic and impact wheel load factors and their application in design processes. *Proc Inst Mech Eng F: J Rail Rapid Transit* 2017; 231(1): 33–43.
- American Railway Engineering and Maintenance-of-Way Association (AREMA). Chapter 30, Ties. In: *Manual for railway engineering*. Landover, MD: The American Railway Engineering and Maintenance-of-Way Association, 2021.
- 25. Furey E. Variance calculator. CalculatorSoup, 2020. https://www.calculatorsoup.com/
- Knapp H. Mann-Whitney U-test. In: *Intermediate statistics using SPSS*. Thousand Oaks, CA: Sage, 2017.
- Hibbeler RC. *Mechanics of materials*. 6th ed. Upper Saddle River, NJ: Pearson Prentice Hall, 2005.
- Gräbe PJ and Jacobs D. The effects of fastening strength on the variation in stress-free temperature in continuous welded rail. *Proc Inst Mech Eng F: J Rail Rapid Transit* 2016; 230(3): 840–851.