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THE MECHANICS OF RAIL FASTENERS FOR CONCRETE SLAB TRACK

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SUMMARY

The author seeks to show what conditions must be fulfilled by rail fasteners for use in slab track. Vertical elasticity has to compensate for resilience of cross ties on ballast, and elastic restraint in other modes of deformation is discussed. Considerations of noise and vibration raise problems of damping which must influence the design of elastic elements. Requirements for adjustability during installation and later maintenance are important aspects of practical design having repercussions on cost and line occupation. Where slab track is proposed for very high speeds, fine adjustment becomes essential. Some desirable features of practical design are indicated.

1. INTRODUCTION

Fasteners for securing rails directly to a concrete base were developed initially for use on bridges and in subway tunnels. Elimination of a ballast layer had obvious advantages in minimising construction depth, and overall dimensions of structures, particularly of tunnels in city areas with a labyrinth of existing underground service pipes and conduits.

Gradually the prospect of using slab track has appeared as a means of reducing maintenance time on main lines carrying dense traffic, or of achieving the very fine geometrical accuracy required for high speeds. The fasteners must provide elastic connection between the rail and the concrete base, to distribute train loading and avoid damaging impact on the concrete. To this extent, the rail on a series of elastic fasteners is mechanically a beam on elastic supports and analogous to the slab itself on a yielding soil base. The theories developed by Zimmermann [1], Kerr [2,3] and others are, therefore, relevant to the mechanics of the rail and fastener system.

Using these theories, some authors [4] have advocated making a slab sufficiently stiff to combine pressure pulses from axles of a bogie into a single pulse as illustrated in Fig. 1. In regard to the rail and fastener system, however, bending stresses in the rails must be limited and the pressure pattern on top of the slab is more like the first diagram than the last.

In fact the rail movement relative to the slab may be in three mutually perpendicular directions of translation and three mutually perpendicular planes of rotation. These six modes of relative displacements should be resisted resiliently by the fastenings.

The rail fastener is an important component of slab track and its elastic characteristics should be designed in relation to the rail and to some extent in relation to the type of slab.

2. VERTICAL ELASTICITY OF FASTENERS

So far, conventional rails have been used in slab track. Their bending stresses limit the permissible spread of loading, and hence the allowable flexibility of supporting pads.

Using available rails as supplied for conventional track, spacing of fasteners has tended to adhere to usual practice, and support elasticity adopted to give more or less the customary distribution of wheel loading. Thus, in the evaluations from tests shown in Table 1, the maximum reaction per fastener is about half the applied wheel load for a variety of different types of fasteners used on slab base track.

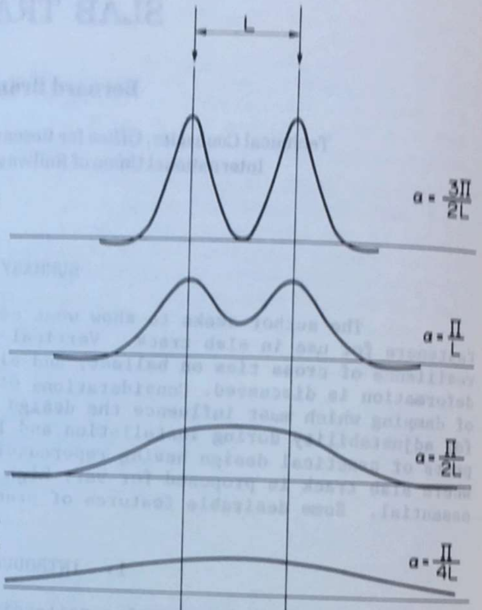

Fig. 1 Track Signature of Bogie
$\alpha = \sqrt[4]{k/(4EI)}$, k = Foundation Stiffness per Unit Length, EI = Bending Stiffness of Track Structure

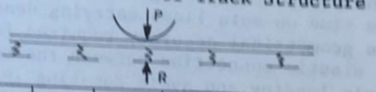

TABLE 1

Type of track		BR Std.	LTE	NS (Soft)	NS (Stiff)	SNCF Monaco	CFF Sonne-ville	BRDL (Stiff)	BRDL (Soft)	BRCT	B on C (Ballast on concrete)
K'	MN/m	90	135	111		76	75	153	111		90
L	m	0.76	0.65	0.65	0.65	0.65	0.60	0.65	0.65	0.65	0.65
I	cm^4	2346	1698	2346	2346	2346	2346	2346	2346	2346	2346
EI	$MN.m^2$	4.833	3.498	4.833	4.833	4.833	4.833	4.833	4.833	4.833	4.833
R/P		0.504	0.549	0.462		0.407	0.374	0.514	0.462		0.430

Note: Data taken from Table 1, ORE Report D 87/RP 6.
K' = Apparent track stiffness, L = Distance between fastenings, I = Second moment of area of rail, E = Elastic modulus of rail = 2,100 T/cm^2 = 20.6 MN/cm^2 and $R/P = (L/4)\sqrt[3]{K'/(EI)}$.

In fact the vertical support elasticity was intended to be equivalent to that commonly provided by a cross tie resting on ballast.

The types of fasteners shown in Fig.'s 2.1 to 2.5 had been tested in a laboratory under pulsating load, having a vertical component of half the wheel load plus impact allowance. They withstood 2.5 million load cycles and Fig.'s 3.1, 3.2, and 3.3 show in summary form the distortions occurring under load and residual deformations [8].

For such spacing and stiffness of fasteners, the differences in calculating maximum reactions by the theory of discrete supports [5] or by the continuous support theory are negligible.

There have, in fact, been serious attempts to develop a continuous support for the rail. Instances are illustrated in Fig.'s 4 and 5. The British version is still not perfectly continuous, since rail clips are retained at a spacing of 65 cm. The Dutch experiment, on a bridge, uses an elastomer enveloping a large part of the rail.

Fig. 6 illustrates typical flexure of rail and slab, having regard to the local spread of reaction between rail and slab and the greater spread on the soil accomplished by the slab. Comparison is given between a yielding foundation and an absolutely rigid base. Self evidently, the relative deflection between rail and slab is practically alike in both cases. Hence the reaction intensity between continuous rail and continuous slab might in practice be calculated as if the slab did not deflect.

Any lack in continuity must, of course, modify the equilibrium. The fastening and the slab must for instance be capable of withstanding reactions at a rail joint. A hinged joint in the slab has less serious consequences than a joint which is unable to transmit shear, provided that the slab has continuous bearing on the soil.

If, however, the slabs are laid as a series of simply supported bridges, an abrupt change of slope occurs at their ends under loading. Discussion rages about permissible deformation of repetitive bridge spans for high speed lines, but even for the static bending of rails, a limitation of deflection should be observed. In Fig. 7 a comparison is offered between the effects of grade change and the effects of a 10 ton wheel load. Basis of calculating end grade is the proposition that the deflection of a span be limited to span length divided by 800 under a uniformly distributed loading. Then the end slope would be 1/250.

UIC 65 rail with elastic supporting stiffness of the fasteners amounting to 1 ton per centimeter of length per centimeter of relative deflection, give quite typical parameter values. The effect of end slope on rail bending is seen to be quite serious. Some extra reaction on the fasteners is also engendered by the extra relative deflection between rail and slab. They must, therefore, be designed to accommodate this extra compression without damage.

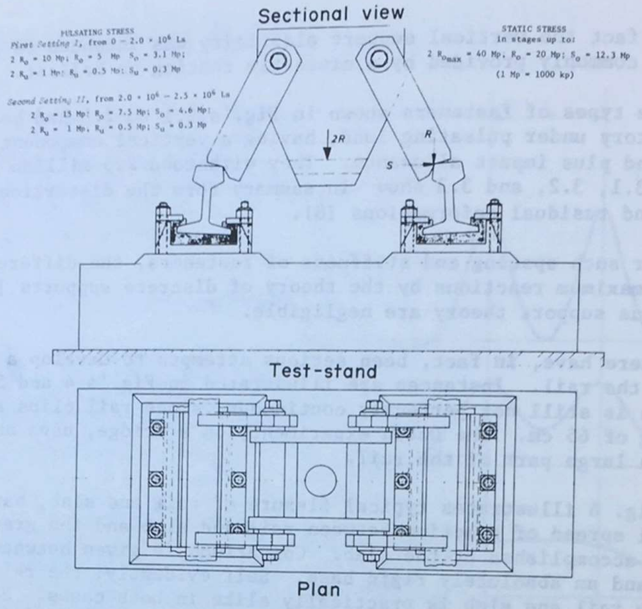

Fig. 2.1 Pulsator for Testing the NS Fastenings

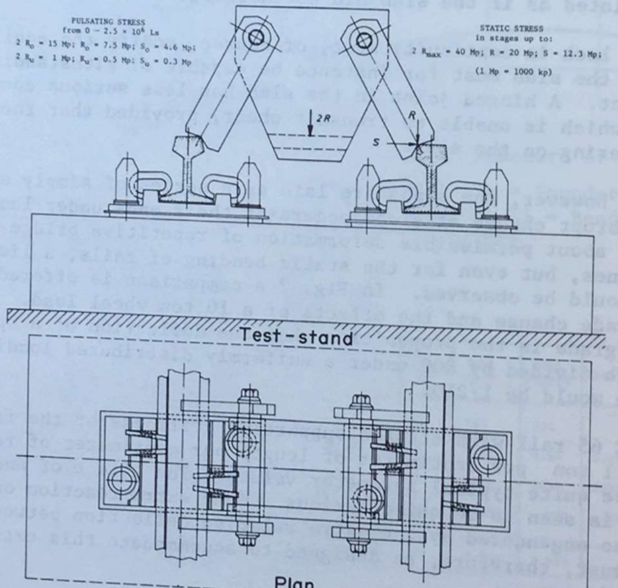

Fig. 2.2 Pulsator for Testing LTB Experimental Fastenings

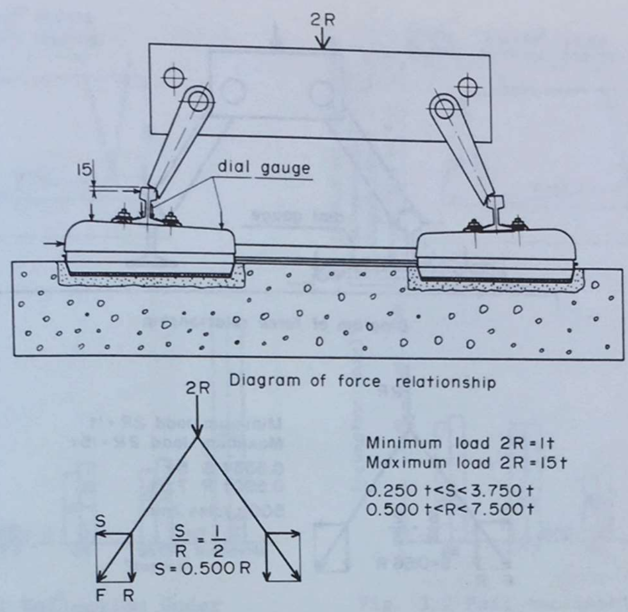

Fig. 2.3 Pulsator for Testing DB Fastenings for Underground Lines

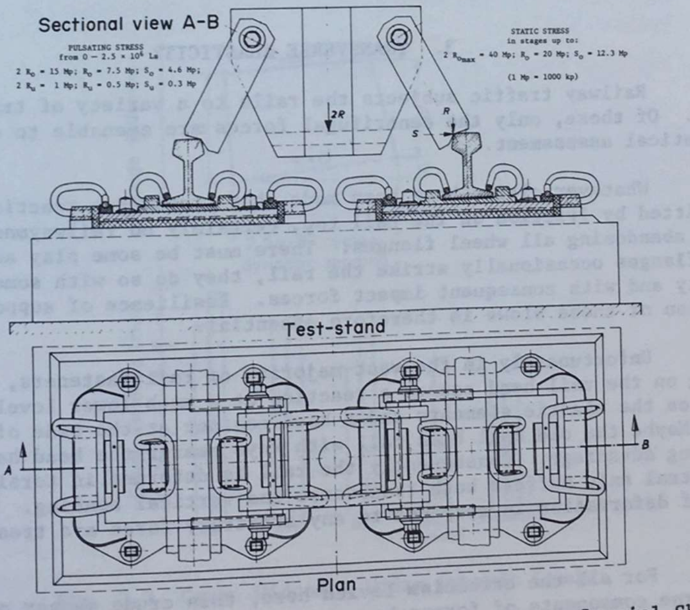

Fig. 2.4 "Sonneville" Direct-Fastening System Using Special Shoe

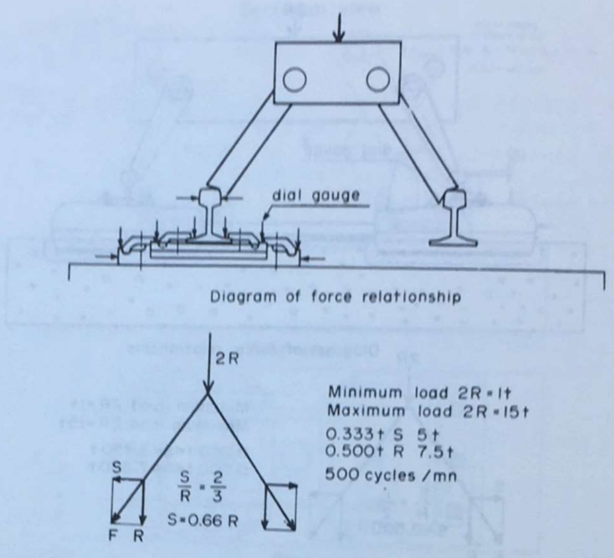

Fig. 2.5 "Monaco" Direct-Fastening System

3. TRANSVERSE ELASTICITY

Railway traffic subjects the rails to a variety of transverse forces. Of these, only the centrifugal forces are amenable to convincing mathematical assessment.

Whatever claims have been made that transverse reactions can be transmitted by friction on the rail top, certainly no railwayman has yet suggested abandoning all wheel flanges. There must be some play and hence when these flanges occasioually strike the rail, they do so with some transverse velocity and with consequent impact forces. Resilience of support in the direction of these blows is therefore essential.

Unfortunately in the vast majority of rail fasteners, transverse loading on the rail head causes a reaction at a much lower level by a couple acting on the elastic elements and a hard contact at the side of the rail foot. Maybe the old bull head rail with keys nearer the head has some lingering advantage. Consequently the rail is deformed in torsion whilst its neutral axis suffers both transverse and vertical bending. The complex modes of deformation in response to any arbitrary force are treated in Ref. [6].

For all the criticism levied here, this crude manner of countering transverse components of forces has stood the test of time in thousands of miles of conventional track. Perhaps it would be truer to say that railwaymen have learned to live with the problem of progressive gauge widening. An

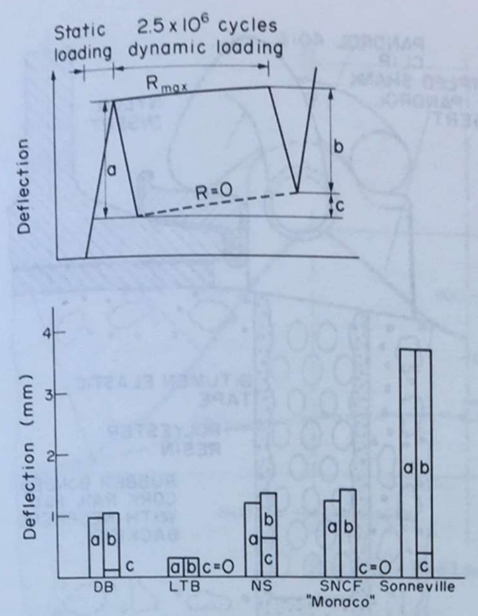

Fig. 3.1 Rail Deflection Under Dynamic Loading

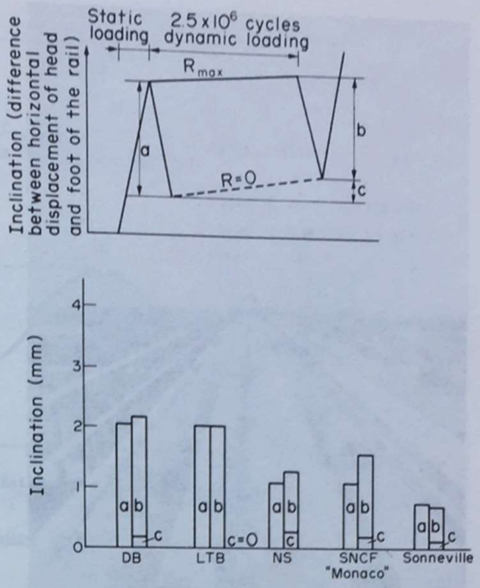

Fig. 3.2 Rail Inclination Under Dynamic Loading

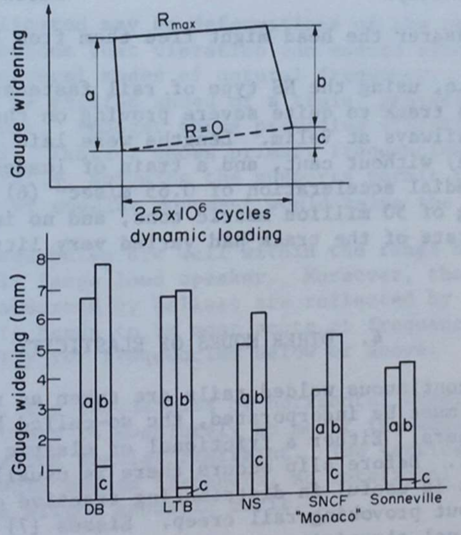

Fig. 3.3 Gauge Widening Under Dynamic Loads

Fig. 4 Continuous Support - British Railways

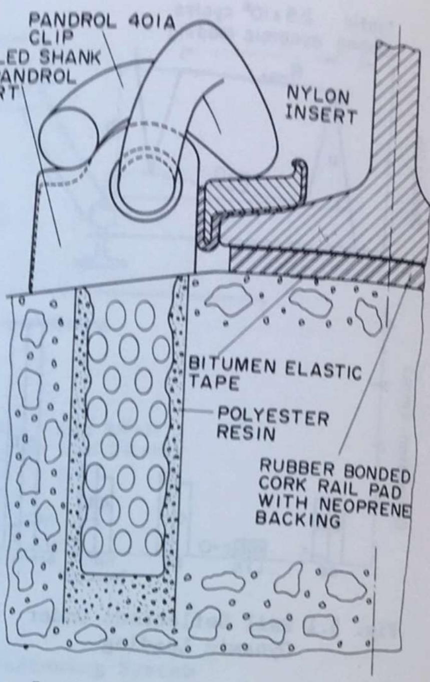

Fig. 5 Continuous Rail Support - Dutch Experiment

elastic reaction nearer the head might free them from head aches.

Meanwhile, using the NS type of rail fastener (Fig. 2.1), ORE sought to put slab track to quite severe proving on the test circuit of the Czechoslovakian Railways at Velim. Lengths were laid in a curve of 450 m radius (3.9° curve) without cant, and a train of loaded ore cars ran at a speed to give a radial acceleration of 0.65 m/sec^2 (61 km/h = 38 mph). After cumulative loading of 50 million metric tons, and no intermediate maintenance, the geometrical state of the track had varied very little from the original condition.

4. OTHER MODES OF ELASTICITY

Today, continuous welded rails are taken as normal for main lines. Where rail joints must be incorporated, the so-called breathing lengths must move in the fasteners. Either a frictional or elastic grip can in principle limit the movement. Before slip occurs there is usually a small elastic deformation and this is useful in distributing tractive or braking forces to the slab and without provoking rail creep. Eisses [7] has made a particular study of longitudinal elasticity in relation to precast slab track, and suggests that a fastener should permit 2.8 mm movement before slip occurs.

Two other modes of elastic restraint remain possible, both torsional and in longitudinal planes, namely horizontal and vertical. The first of

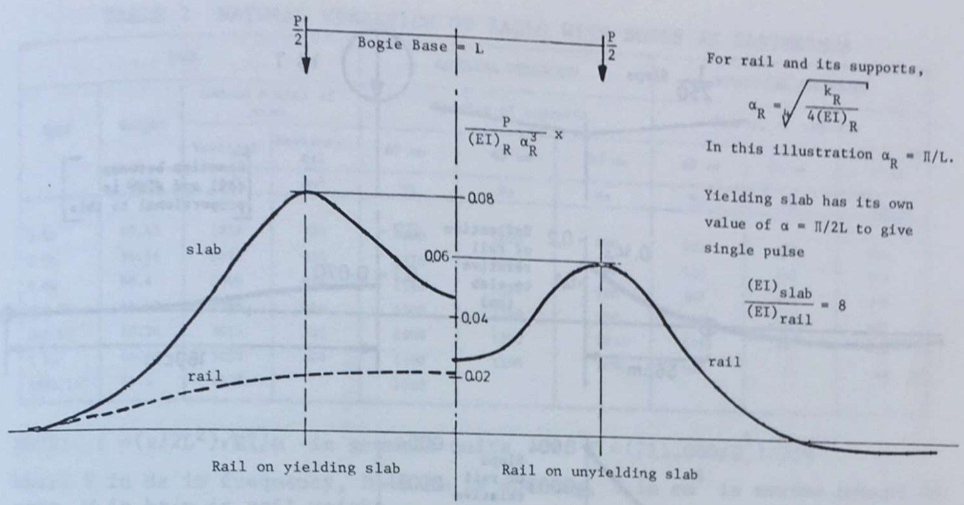

Fig. 6 Deflections Under Bogie

these has wielded importance by contributing to resistance against lateral buckling. A slab is eminently suitable for taking over that function. However, even in a slab track there remains a tenable role for these restraints by influencing vibration within the audio range.

However complicated may be deformations of the rails in the vicinity of the wheels, it is obvious that vibration can endure after the passage of a train, according to several modes of natural frequency. Indeed any railway man knows they can be excited ahead of a train, providing a strictly unauthorized listening post. Amongst these pervasive vibrations are some with nodes at every fastener. Table 2 shows primary frequencies of bending both vertically and horizontally, assuming the supports permit free angular movement. Any restraining of angular movement would raise the frequencies.

All these frequencies are well within the range of human hearing and the rail is a fairly large loud speaker. Moreover, those emitted waves which might have been absorbed by ballast are reflected by a hard concrete slab. Human sensitivity tends to be most acute at frequencies of about 1,000 Hz, diminishing gradually for frequencies below or above.

In this mode, the frequencies of horizontal bending are entirely below 1,000 Hz. Thus there is absolutely no merit in seeking to raise them by intentionally increasing angular restraint. The vertical vibrations are above 1,000 Hz for closely spaced fasteners, so rotational stiffness in a vertical plane has some slight acoustic merit.

Lower frequencies excited by the passage of trains inevitably have longer distances between nodal points, which may travel along the rail. Between nodes the rail must move within the fasteners. Thus the restraining stiffness of the fasteners exerts its effect on frequency. Travelling in a train over a series of different test lengths as at Radcliffe-on-Trent gives

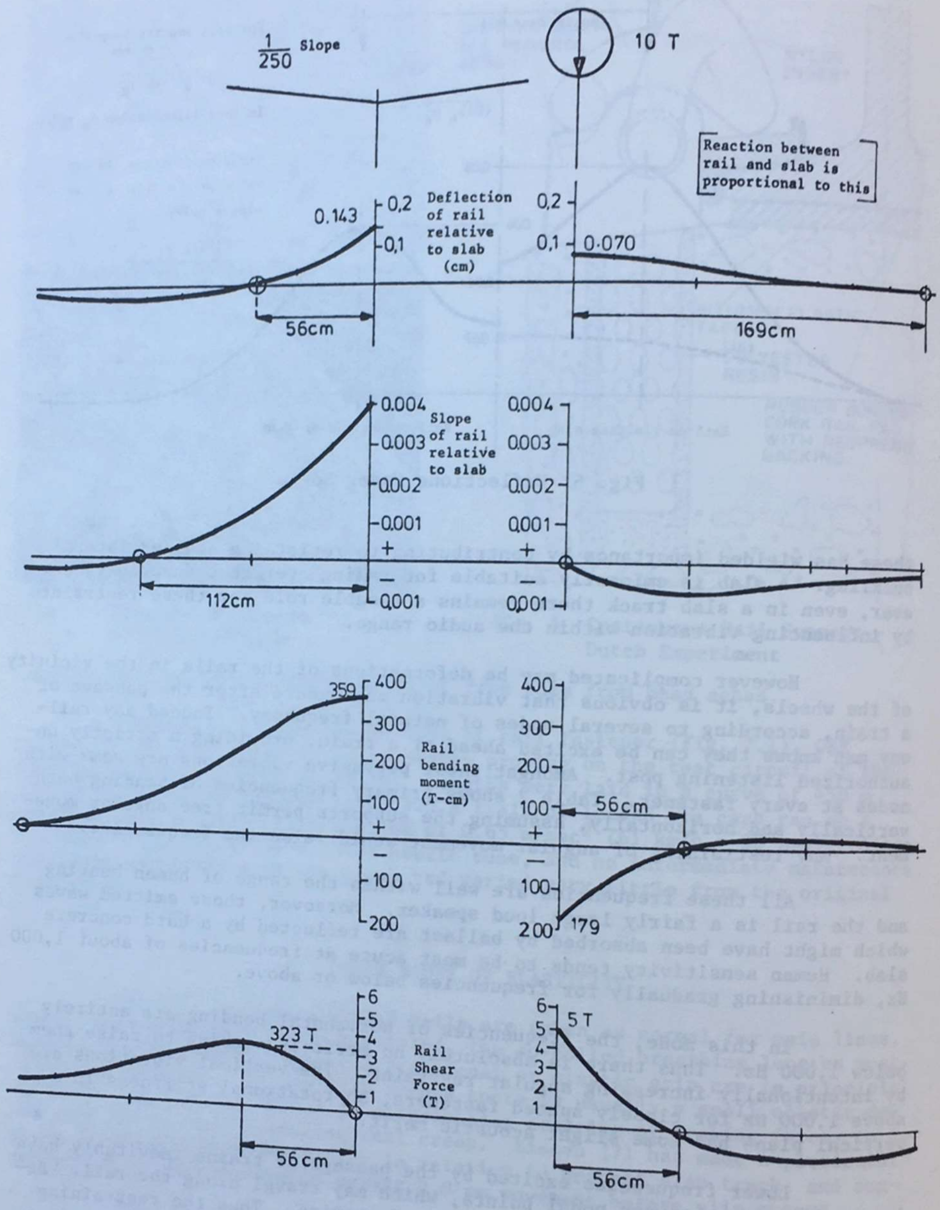

Fig. 7 Effects of Grade Change and Wheel Load

TABLE 2 NATURAL VIBRATION OF RAILS WITH NODES AT FASTENINGS

RAIL				VERTICAL FREQUENCY			HORIZONTAL FREQUENCY		
Type	Weight	Second Moment of Area		Spacing of supports			Spacing of supports		
		Vertical	Horizontal	60 cm	65 cm	70 cm	60 cm	65 cm	70 cm
	Kg/m	cm^4	cm^4	Hz	Hz	Hz	Hz	Hz	Hz
S 49	49.43	1819	320	1200	1020	880	500	428	370
S 54	54.54	2073	359	1220	1040	900	510	432	373
S 60	60.4	2760	454	1310	1120	960	542	461	398
UIC 54	54.43	2346	414	1300	1110	950	546	465	401
UIC 60	60.34	3055	513	1400	1200	1030	578	491	424
S 64	64.92	3252	604	1400	1190	1030	605	514	444
AREA 14C	69.4	4029		1510					

NOTE: $f = (\pi/2L^2)\sqrt{EI/m}$ in general units or $F = (713.000/D^2)\sqrt{J/W}$

where F in Hz is frequency, D in cm is spacing, J in cm^4 is second moment of area, W in kg/m is rail weight.

a surprisingly convincing subjective impression how the dominant tone tends to rise with stiffness. Evidence can be discerned from power spectral analysis, even if the total noise is unchanged. In turn that leads into the fascinating study of human preferences.

5. DAMPING

It may be well to clarify terminology, not just for linguistic pedantry, but to avoid some of the hazy notions imported by words like "cushioning".

Elasticity does not signify damping. The bending of a steel rail is elastic, but it can vibrate for a long time when freely suspended absorbing energy only by its own molecular structure. Likewise, external elastic support might absorb little vibrational energy.

There are materials which provide so called high hysteresis, which should be preferred. However, they do not necessarily conform with the mathematical notion termed "hysteretic damping".

Energy absorption can be provided in some materials better under shear deformation than direct strain, so the manner of deformation must be recognized for obtaining effective conversion of mechanical energy into heat. The form and thickness must be considered in relation to thermal dissipation, since temperature rise usually changes damping characteristics. A ribbed form of elastic pad deforms partly by shear, but reduces contact area with thermally conducting surfaces. Holes in an elastic pad have similar effect.

Surfaces rubbing in contact can provide frictional loss of mechanical energy, but usually at the cost of abrading the material, as, for example, the stones or particles constituting ballast. According to most theoretical approximations used for mathematical convenience, vibration is never completely killed, but continues in ever decreasing amplitude. Yet, when the force becomes too small to overcome the sliding friction between individual grains, a state of rest is consistent with general knowledge of static friction. It leads to explanation of random rest positions after passage of a train; the effect is quite apart from the uni-directional settlement or bedding of the granular material. The extent to which such "dry friction" is present may indeed explain the ability of ballast to limit vibration, and perhaps point the way to desirable damping characteristics of truly "cushioning" pads.

Imperfect elasticity, when Hooke's law of proportionality fails to apply, is usually, but not always accompanied by fairly enhanced hysteresis. However, for very small vibrational displacements superimposed on more fundamental deformations it seems sensible to base calculations on a stress-strain relationship more like the tangent modulus, assuming pre-loading more appropriate to the static reactions. Little absorption of energy can be expected where the loops of load times force are very small as for example in the test result of Fig. 8.

The principal need for damping is to destroy vibrations in the audio range. The continuous elastic layer has more chance of attenuating the bending of rails over very short wavelengths. In any case, covering a large part of the rail surface must provide useful acoustic screening.

6. PRACTICAL ASPECTS

Design for adequate strength poses no particular difficulty. Primitive methods of attaching rails to slabs or bridge decks without resilient pads belong to the past. Elastic support now allows reasoned calculation of major stressing in slabs, fasteners, and the bolts holding them together, even if dynamic effects remain empirical. Indeed, if the slab is adequate, the fasteners have a more constant support condition than with ballast. Hence their loading should be less erratic and ill effects of voids should be completely eliminated.

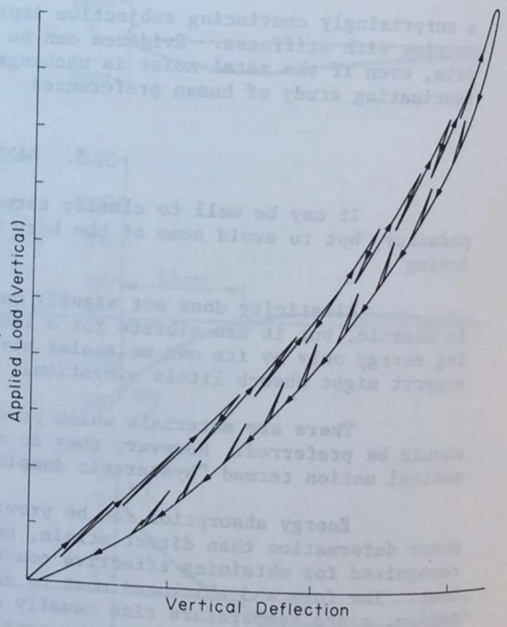

Fig. 8 Typical Load - Deflection Characteristic for a Fastening Assembly under Vertical Load (Showing effect of small amplitude load variations)

Laboratory tests [8] in which various fastenings were subjected to over two million cycles of combined vertical and transverse loading, promoted confidence for ORE and member railways to use them in slab track for service trains.

The sections laid at Radcliffe-on-Trent have carried about 3 million long ton per year of normal traffic since April 1969, including a fair proportion of 56.000 lb axles [9]. Since then, more ambitious lengths have been installed in lines carrying denser traffic. One in Germany at Oelde and Rheda is subject also to high speeds.

Together with the quite severe testing in Czechoslovakia, these tests have also demonstrated the ability of fasteners to retain geometrical condition. Initial play in the fasteners tends to close after the first few trains, and thereafter the component parts remain in tight contact. A simple form is good engineering. Too many elements mean too much play in total, because each part must have its own manufacturing tolerances. The rail itself is not the easiest section to roll, and any inaccuracy of the head relative to the base forms part of the system of tolerances.

In thinking of high speeds, very fine accuracy is relevant. Table 3 shows some requirements drawn from British railways. Ride comfort demands

TABLE 3 BR RECOMMENDED TOLERANCES

	160 km/h to 200 km/h 100 mph to 125 mph		Below 160 km/h to 120 km/h 99 mph to 75 mph		Below 120 km/h to 80 km/h 74 mph to 50 mph		Below 80 km/h 49 mph and below	
	A		B		C		D	
	I	M	I	M	I	M	I	M
GAUGE	-1 mm +3 mm	-1 mm +6 mm	-1 mm +4 mm	-1 mm +7 mm	-2 mm +5 mm	-2 mm +8 mm	-3 mm +6 mm	-3 mm +10 mm
Variation	3 mm in 2 metres		4 mm in 2 metres		5 mm in 2 metres		6 mm in 2 metres	
CANT	±2 mm	±5 mm	±2 mm	±6 mm	±3 mm	±8 mm	±3 mm	±10 mm
Variation	See twist							
TWIST 1 in: Measured Over 3 metres	750	600	600	400	600	400	600	400
ALIGNMENT on 20 metres overlapping chords	±3 mm	±4 mm	±4 mm	±5 mm	±5 mm	±6 mm	±6 mm	±8 mm
Variation	See gauge							
TOP-UNEVENNESS 20 metres	6 mm	10 mm	8 mm	12 mm	10 mm	14 mm	12 mm	16 mm
Variation	See Cant and Twist							
If datum below rail height	$10\ \text{mm} = {}^{+3}_{-7}$ or ± 5 etc.							

I denotes installation; M denotes maintenance very close limits in variation of line and level, cross level and gauge. For short wavelength irregularities it is even desirable to approach fractions of a millimeter. The possibility of avoiding abrupt changes in height or alinement, consequent upon joining new rails to old, must also be provided by means of fine adjustment.

Infinitely variable adjustment of alinement has been achieved in the fastenings of the Dutch Railways (Fig. 9) by means of eccentric bushes on the anchor bolts. The method relies on frictional grip, but has in practice given satisfaction. Other methods giving generally a millimeter step in adjustment, include alternative and sometimes reversible rail clamps.

Height adjustment is usually by shims under the rail foot or under the base plate. One millimeter is generally regarded as the coarsest step permissible. It must be remembered that the elastic element takes up about half a millimeter at the adjusted fastener, together with smaller changes at neighboring fasteners as the rail redistributes reactions.

Japanese developments are proceeding with a thermo setting resin placed in plastic bags under the rail. Heating elements are incorporated in rail pads located above and below the resin filled bag like a sandwich. Heating electrically and with pressure applied to the resin, the rail can be held by wedges at any desired height while the resin solidifies.

The total range of adjustment must envisage compensation for inaccuracies of the slab during installation and any subsequent changes thought likely to occur during its lifetime.

The method of installation and type of slab is quite important. Where the British type of continuous paving is practicable, there seems no difficulty in achieving a surface accuracy within about 3 mm, apart from an initial short length where the paver tends to be unstable. Surface grinding, at extra cost, can be tolerated

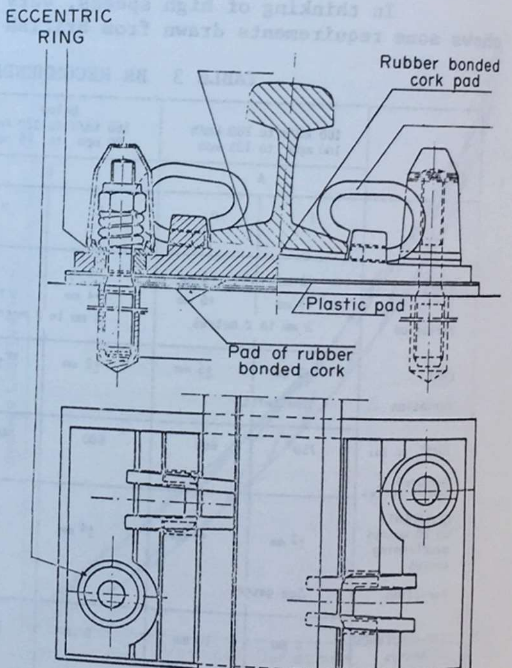

Fig. 9 Adjustment by Eccentric Ring

only over such short portions. Without entering discussions on further improving guidance techniques, there is no point in paying for more accuracy in the slab surface than can reasonably be compensated by initial setting of the fastener. Here a resin mortar seating can compensate while the rail is held independently in correct position.

To hold the rails in position with fastenings and anchor bolts suspended while a complete slab is cast, has some advocates. The method complicates pouring and compacting the concrete. Nevertheless at Rheda even concrete cross ties have been embedded in a slab cast around them. It effectively avoids all danger of shrinkage cracks passing through the fixing bolts, which has been a principal cause for loosening of bolts.

For pre-cast slabs laid directly on the earth formation or even on a prepared foundation of weak concrete or stabilized ballast, the surface accuracy cannot be expected to match that of a slab cast in place. Perhaps the tolerances would be doubled to about 6 mm.

It is suggested that fasteners should be designed to allow from 10 to 20 mm of vertical adjustment depending on the type of slab. Coarse steps of about 5 mm might be supplemented by provision for fine adjustment within ranges of 6 or 7 mm.

Transverse adjustment is needed to allow for the various tolerances and clearances. The Dutch Railways have found 6 mm either side of a central position sufficient for bridge installations, but it may not be adequate in other circumstances. The German Federal Railways prefer to allow 10 mm either way.

Where slab track is installed with intent to introduce higher speed, a rather special circumstance can occur. Often the full advantage cannot be exploited immediately. Neighboring sections may still have to be upgraded; rolling stock or signalling may still be awaited. On curves, the superelevation should really be changed at the time of introducing the higher speed, rather than when the slab is laid. A proposed solution [10] is to cant the slab surface to an intermediate angle and provide two levels of fastenings, which may be transposed at the appropriate time (Fig. 10). The method permits correct inclination of the rails relative to canted rolling plane.

A primary benefit sought from slab track is to reduce or avoid interruption of traffic for maintenance. Saving direct cost of maintenance cannot generally justify the high capital investment in slab track. The cost of stopping or diverting traffic can, however, be enormous. Economic justification therefore depends very much on details of construction enabling maintenance adjustments to be performed quickly between trains.

In several designs, the adjustment of height requires the rail to be lifted well clear of its seating for changing packing shims. Hence, to correct a fault in level, a considerable number of fasteners must be freed. The bow in the rail will be difficult to achieve if it is in tension, or perhaps even more difficult to eliminate if it is under compression.

One practical point is in relation to track circuits. A wet slab surface offers less electrical resistance than cross ties, so the fasteners should be well insulated and the form of their seating on the slab should avoid waterlogging.

Practical difficulties have sometimes been encountered by holes for anchor bolts fouling steel in the slab. A method of indexing positions of transverse reinforcing relative to fastenings must be evolved. Longitudinal reinforcing needs a designed clearance of about 5 cm (say 2") from bolt holes, bearing in mind the inaccuracies inherent in positioning the bars or mats, effects of handling, overlap of bars, vibrating the concrete and any track curvature.

A continuous elastic layer under the rail poses problems of drainage, and on some metropolitan lines, electric cables of fairly large size are required to pass under the rail at many places. A little forethought in fixing the minimum gap between rail foot and slab can spare awkward botching.

7. FUTURE TRENDS

In highly developed countries the picture emerging is that the increasing demand for movement of people and their goods allows an important share for guided, and automated land transport. It appears pre-eminent in many cases for heavy traffic of ores and minerals and a necessity for passenger transport where saturation of other modes is reached, as in the North East Corridor in the U.S.A. and cities throughout the world.

Heavy mineral traffic creates insistent demands for increasing axle loads, which damage conventional track and impose heavy burdens of maintenance. Very short intervals for maintenance are pre-supposed for any line justifying slab construction, hence the fasteners must facilitate rail changing and any adjustments.

The passenger aspects may be seen in two major categories. Over moderately long distances, high speeds are the necessary inducement to the customer, and with a tolerable comfort that requires extremely fine standards of track geometry. Concentration of traffic, the necessary inducement for capital investment, presupposes that normal maintenance must be practicable during very short intervals. Slab track is potentially attractive, provided that the fasteners facilitate fine adjustment between trains and rapidity in changing worn rails. The desirable features of adjustment devices are that they should be infinitely variable, if possible without bolts, which may rust and they should not require appreciably greater displacement of the rail than the desired correction for effecting the operation.

In urban transport, good acceleration and braking play a more important role than particularly high speed. Track maintenance must be limited to short occupations at night, often in a tunnel. Ride comfort at speeds on urban lines demands less finesse in adjustment, than in high speed lines.

In urban environments, however, noise and vibration must be minimised. In fact legislation appears imminent in many countries. ORE tests have shown that noise from slab track need be no greater than with ballast overlying

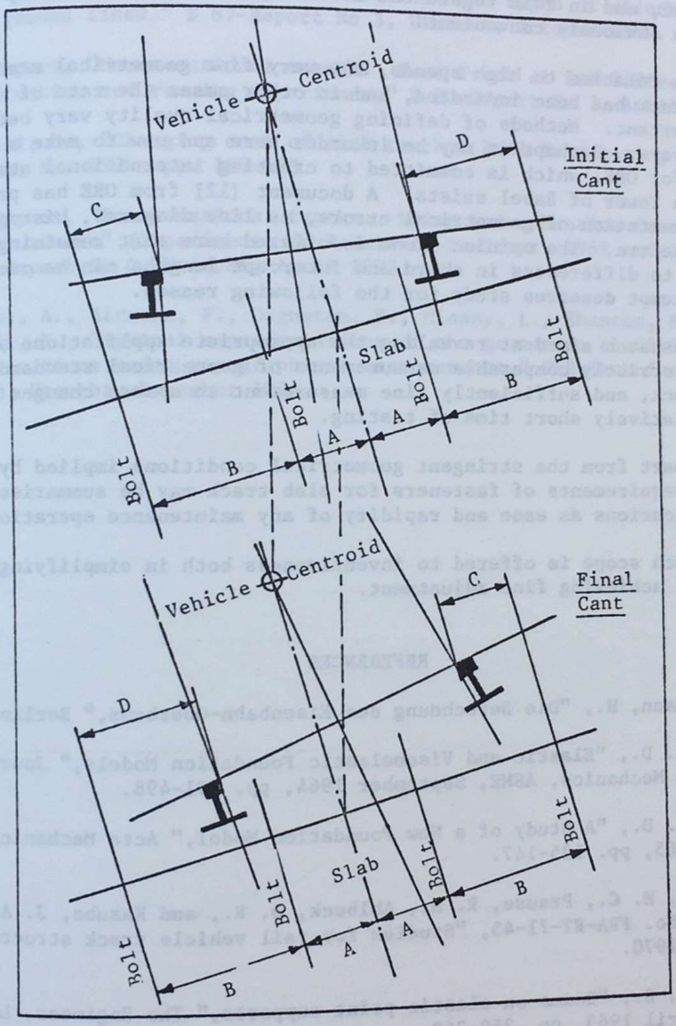

Fig. 10 Changing Cant on Slab by Transposing Fasteners

a concrete base [11].

Possible further improvement may be sought by enveloping the rail in elastomer with good damping properties. Restriction of air gaps is known to be beneficial, and in this regard the definite location of rails by direct fastening is obviously convenient.

In relation to high speeds, the very fine geometrical standard required of track has been indicated, and in other cases the rate of deterioration is important. Methods of defining geometrical quality vary between different railways. Perhaps it may be in order here and now to make a little propaganda for ORE, which is committed to creating international standards where only a Tower of Babel exists. A document [12] from ORE has proposed unified presentation of geometrical errors, as line diagrams, histograms and power spectra. The opinion alone is offered here that remaining difficulties due to differences in chord and intercept lengths can be overcome and that the attempt deserves study for the following reason.

Research aimed at revealing the appropriate applications of slab track needs strictly comparable measurements of geometrical standard for all kinds of track, and sufficiently fine measurement to assess changes occuring within a relatively short time of testing.

Apart from the stringent geometrical conditions implied by high speed, the requirements of fasteners for slab track may be summarised for all likely applications as ease and rapidity of any maintenance operations.

Much scope is offered to inventiveness both in simplifying maintenance and in achieving fine adjustment.

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