

Optimizing train stopping patterns and schedules for high-speed passenger rail corridors



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ABSTRACT

High-speed railway (HSR) systems have been developing rapidly in China and various other countries throughout the past decade; as a result, the question of how to efficiently operate such large-scale systems is posing a new challenge to the railway industry. A high-quality train timetable should take full advantage of the system's capacity to meet transportation demands. This paper presents a mathematical model for optimizing a train timetable for an HSR system. We propose an innovative methodology using a column-generation-based heuristic algorithm to simultaneously account for both passenger service demands and train scheduling. First, we transform a mathematical model into a simple linear programming problem using a Lagrangian relaxation method. Second, we search for the optimal solution by updating the restricted master problem (RMP) and the sub-problems in an iterative process using the column-generation-based algorithm. Finally, we consider the Beijing-Shanghai HSR line as a real-world application of the methodology; the results show that the optimization model and algorithm can improve the defined profit function by approximately 30% and increase the line capacity by approximately 27%. This methodology has the potential to improve the service level and capacity of HSR lines with no additional high-cost capital investment (e.g., the addition of new tracks, bridges and tunnels on the mainline and/or at stations).

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1. Introduction

The planning process for public transportation consists of several consecutive planning phases. The process begins with network design, which is typically then followed by line planning, timetabling, and vehicle and crew scheduling (Schöbel, 2012). Train timetables specify the arrival and departure times of trains between yards, terminals, sidings and every given point along a rail line or network. Train timetable scheduling plays an important role in the management and operation of complex railway systems.

Before scheduling train timetables, a railway operator usually defines a train service plan – a framework for train service and timetables based on certain strategic decisions, including origin–destination pairs for travel demand, station settings, operating capacities and planning parameters (Chang et al., 2000). Different train service plans produce passenger service schemes with different characteristics, such as the frequency, trip time and stopping patterns between different stations along a rail line. The level of service is a key factor that affects travelers' decisions in choosing their preferred transportation

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modes (Tong et al., 2012). Indirectly, the train service plan affects the fare revenue and profitability of a train operator. This “profit” factor is important to consider when scheduling train timetables.

Many scholars have provided excellent surveys of the inherent connections among different optimization problems in the field of railway routing and scheduling (Caprara, 2010), such as backtracking search (Adenso-Díaz et al., 1999), look-ahead search (Sahin, 1999), and the continuous approximation approach (Freyss et al., 2013). Over the past four decades, researchers have quite extensively studied the Train Timetable Problem (TTP), leading to the development of various railway operation models and techniques (Assad, 1980; Hansen, 2009; Walker et al., 2005; Cacchiani and Toth, 2012). Previous TTP optimization models have typically focused exclusively on train scheduling or passenger service demands, although additional models have attempted to consider both factors at small scales.

Brännlund et al. (1998) introduced a Lagrangian relaxation method for searching in the timetabling problem of a railway operator, namely, the scheduling of a set of trains to obtain a profit-maximizing timetable while not violating track capacity constraints. D’Ariano et al. (2007) investigated a new concept of a flexible timetable as an effective policy for improving punctuality without decreasing the capacity usage of the lines. They used three greedy heuristics and a branch-and-bound algorithm for conflict resolution, but they did not test them on different networks. Lee and Chen (2009) used a four-step process to optimize both train paths and train timetables. By decomposing the original complex problem into four parts and solving each part alone, their heuristic method was able to produce solutions for realistic scenarios. Liu and Kozan (2011) addressed train-scheduling problems based on prioritized and non-prioritized trains. Their model was required to conform to blocking and no-wait constraints in specific environments. Corman et al. (2014) presented a thorough assessment of the possible applications of an optimization-based framework for the evaluation of different timetables over a large network. Sun et al. (2014) proposed a multi-objective optimization model to minimize the degree of deviation for train rerouting on a high-speed railway network, considering the average train travel time, energy consumption and user satisfaction. To minimize the total train travel time, Zhou and Zhong (2007) modeled limited track resources via headway constraints and reformulated them as additive constraints to chronologically eliminate train conflicts. Meng and Zhou (2014) developed an innovative integer programming model for the problem of train dispatching on an N-track network by simultaneously rerouting and rescheduling trains using a time-space network-modeling framework. Shafia et al. (2012) illustrated a novel and robust train-timetabling problem for a single-track railway line to compute buffer times. All these models focused on train scheduling under given capacity constraints.

Another subset of previous studies focused on passenger service demands or train service planning. Peeters and Kroon (2008) used a branch-and-bound method to solve the problem of railway rolling stock circulation with a given timetable to meet passengers’ demands. Considering passenger flow, Deng et al. (2009) analyzed the relation between stopping schedules and passenger transfer choices. They built a bi-level model considering travel cost and the number of train stops. To decrease passenger transfer waiting time in a network, Petersen et al. (2012) proposed a planning approach that attempted to achieve a favorable trade-off between the two contrasting objectives of passenger service and operating cost by modifying the timetable. Kunitatsu et al. (2012) developed a micro-simulation system to simulate both train operation and passengers’ train choice behavior. Niu and Zhou (2013) and Niu et al. (2015) used the overall passenger waiting time as the objective and applied a genetic algorithm that indicated a train departure or no departure at every possible time point to optimize train timetables. Lin and Ku (2013) used two genetic algorithms, namely, a binary-coded genetic algorithm and an integer-coded genetic algorithm, to optimize stopping patterns for passengers to solve real-world problems with excellent performance. Espinosa-Aranda and Angulo (2015) proposed a constrained logit-type choice model that took the behavior of users into account. No models have optimized the train schedule to reach the maximum capacity of the railway lines.

Several previous studies have attempted to consider both train scheduling (including railway line capacity constraints) and passenger service demands, but only at small scales. Caprara et al. (2002, 2006) proposed a graph-theoretic formulation of the problem using a directed multi-graph in which nodes corresponded to departures/arrivals at a certain station at a given instant. Cacchiani et al. (2008) proposed heuristic and exact algorithms for the TTP on a corridor, including periodic and non-periodic. In their integral linear programming (ILP) formulation, each variable corresponded to a full timetable for a train, yielding a problem that was much simpler to solve. Cacchiani et al. (2010) considered the customary formulation of non-cyclic train timetabling, in which they sought a maximum-profit collection of compatible paths in a suitable graph. Their methods offered increased efficiency of the column generation algorithm and improved the experimental results but could not be applied to a large-scale problem. Min et al. (2011) proposed a column-generation-based algorithm focusing on the train-conflict resolution problem. Yang et al. (in press) considered the minimization of the total dwelling time and total delay between the actual and expected departure times to optimize both train stop pattern and train timetabling problems at the tactic level. They also point out, when applying to large real-world instances, an efficient heuristic algorithm is needed to speed up the searching process.

In this paper, we propose a new methodology using a column-generation-based algorithm to simultaneously account for both passenger service demands and train scheduling to optimize train timetables. The framework of our proposed methodology is illustrated in Fig. 1. In our model, the “train service plan” provides input parameters related to passenger service demands. The percentage of the total trains and the total number of trains required for each origin-to-destination (OD) pair on a rail line or network to satisfy the passenger transport demand can be determined by analyzing the passenger flow OD matrix. The decision variables correspond to the “stopping pattern” (specifying at which stations each train stops) and the “stopping time” (specifying how long each train stops at intermediate stations). In the model and algorithm, the first step is to optimize the trains’ stopping pattern while guaranteeing the train service plan constraints, whereas the second step is to optimize the train departure times while guaranteeing the stopping duration constraints, headway constraints and station

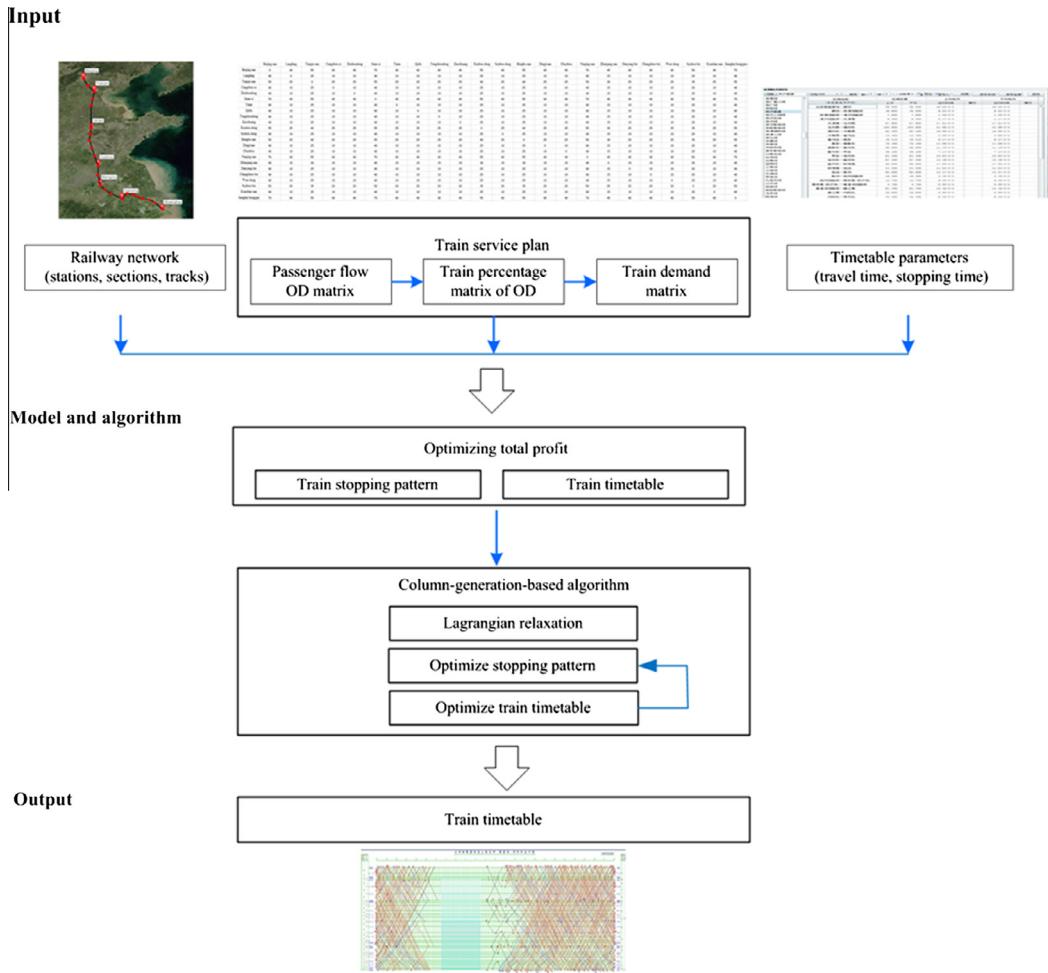


Fig. 1. Framework of the train timetable optimization methodology.

capacity constraints. The specific details regarding the column-generation-based algorithm are discussed in Section 3 of this paper. After several iterations, the output is a near-global-optimal train timetable.

We offer the following contributions to the growing body of research on the TTP:

- (1) We introduce a new railway timetable optimization model (Section 2) to simultaneously consider the train service plan and the train schedule.
- (2) We propose the use of an innovative approach (Section 3) to solve a very-large-scale train timetable problem using a column-generation-based heuristic.
- (3) We illustrate the use of the optimization model and the column-generation-based algorithm to improve the timetable of a real-world HSR system (Section 4).

The remainder of this paper is organized as follows. In Section 2, we present a conceptual illustration of the TTP based on a train service plan and propose an innovative train timetable optimization model. In Section 3, we use a Lagrangian relaxation method to transform the model into a large-scale linear programming problem and propose a column-generation-based algorithm to solve that problem. In Section 4, we present a real-world case study to demonstrate the applicability of the proposed methodology for obtaining improved solutions compared with the current timetables of HSR systems. Finally, conclusions and recommendations for further research are presented in Section 5.

2. Problem formulation

A train timetable defines train departure times, arrival times at each station and travel times in OD sections. It is an essential plan for the operation of a railway system. In this paper, we assume that the travel time in each OD section is constant for trains of the same class and that the total travel time includes the acceleration, retardation delay and stopping time at each station.

2.1. Problem description

In a physical rail network, a line plan specifies a train route from an origin to a destination with a determined stopping pattern. A train service plan consists of several line plans for different trains to satisfy a given travel demand. A train timetable must satisfy the travel demands for each of the OD pairs on a rail line or network as specified in the train service plan. Certain trains may have different stopping patterns, stopping at different sets of intermediate stations. To illustrate the concept of a “train service plan”, in Fig. 2, we illustrate four scenarios of different train service plans for three trains and four stations. At least one train must serve each OD pair, representing all possible combinations between any two stations. Scenarios 1 and 2 conform to this travel demand requirement because passengers can travel from any station to any other station; in Scenario 3, passengers traveling from Station 1 to Station 2 cannot be served; and in Scenario 4, passengers traveling from Station 2 to Station 3 cannot be served. Therefore, Scenarios 3 and 4 are infeasible. For rail operators, a key decision basis for defining a train service plan is the passenger flow assignment, that is, the estimated travel path choices of passengers traveling from their origins to their destinations via their desired trains (Fu and Benjamin, 2013; Fu et al., 2015). A rail operator will always seek the most efficient schedule and combination of line plans to achieve the optimal balance between stop locations and frequencies.

When there are n trains and m stations on a railway line, $2^{m \times n}$ different stopping patterns can be produced. Different stopping patterns can affect the train timetable. In Fig. 3, we present a different set of four scenarios, each with three trains with different stopping patterns. We assume that the stopping time is 2 min, the minimal time interval or headway between consecutive trains is 3 min, and at least one train stops at Stations 2 and 3. We observe that all scenarios satisfy the constraints except Scenario 4, in which passengers traveling from Station 2 to Station 3 are not served. Scenarios 1 and 2 require 20 min for all trains to complete their runs, whereas Scenario 3 requires only 18 min. Therefore, the timetable of Scenario 3 is the best among the four. From this example, it is evident that different stopping patterns can lead to different train timetables. Even with the same stopping pattern, different train departure times will also lead to different train timetables (Scenarios 2 and 3). To obtain an optimal timetable, we must determine the train stopping patterns and schedules (e.g., departure times) simultaneously – this is the underlying motivation of this paper.

To illustrate the train timetable problem, we use a space–time diagram, as shown in Fig. 4. The horizontal axis corresponds to time, and we discretize consecutive times into sufficiently small increments. In a typical railway system, the time unit is one minute. The vertical axis corresponds to space and represents stations and OD sections. Before a train departs from its origin, the time window (i.e., the feasible time span in which the train may depart and arrive at its destination) must be determined, and the best time to depart within that time window must be chosen. In the development of a train timetable, each train must be assigned to a specific feasible space–time window in the rail line or network.

When only one train is serving a rail line or network, with no potential schedule conflicts with other trains, it departs and arrives based on its own line plan. Its timetable is defined when its time of departure from the origin and its stopping pattern are determined. When more than one train is serving a rail line or network, conflicts, such as scenarios in which more than one train are simultaneously attempting to use certain station tracks or block sections, may occur. Certain trains may need to stop at a station to wait for other trains to pass through, which could mean that these trains cannot depart at their optimal

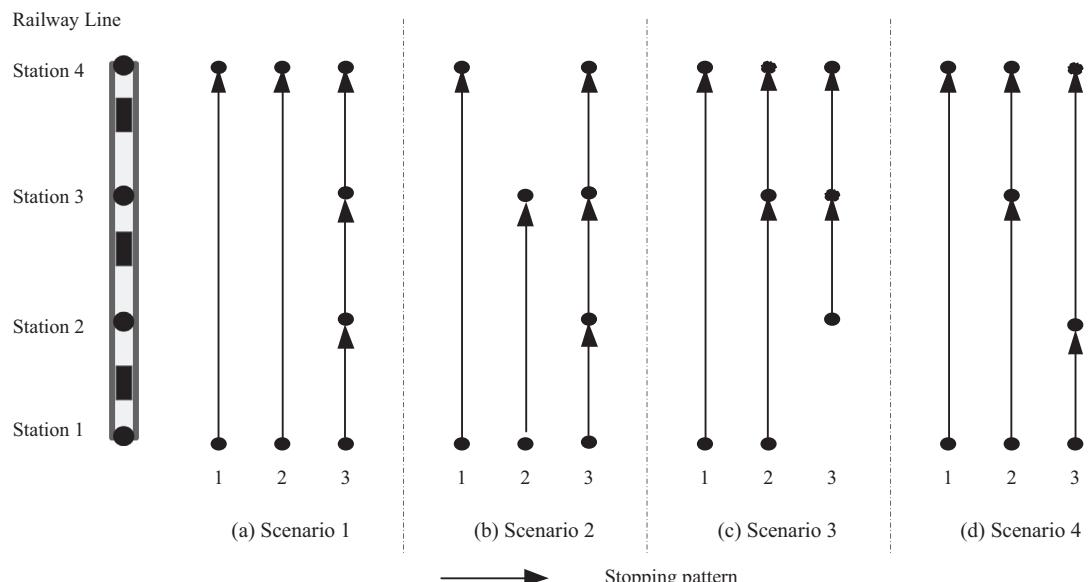


Fig. 2. Illustration of feasible scenarios (1,2) and infeasible scenarios (3,4) for serving a demand for travel from any station to any other station.

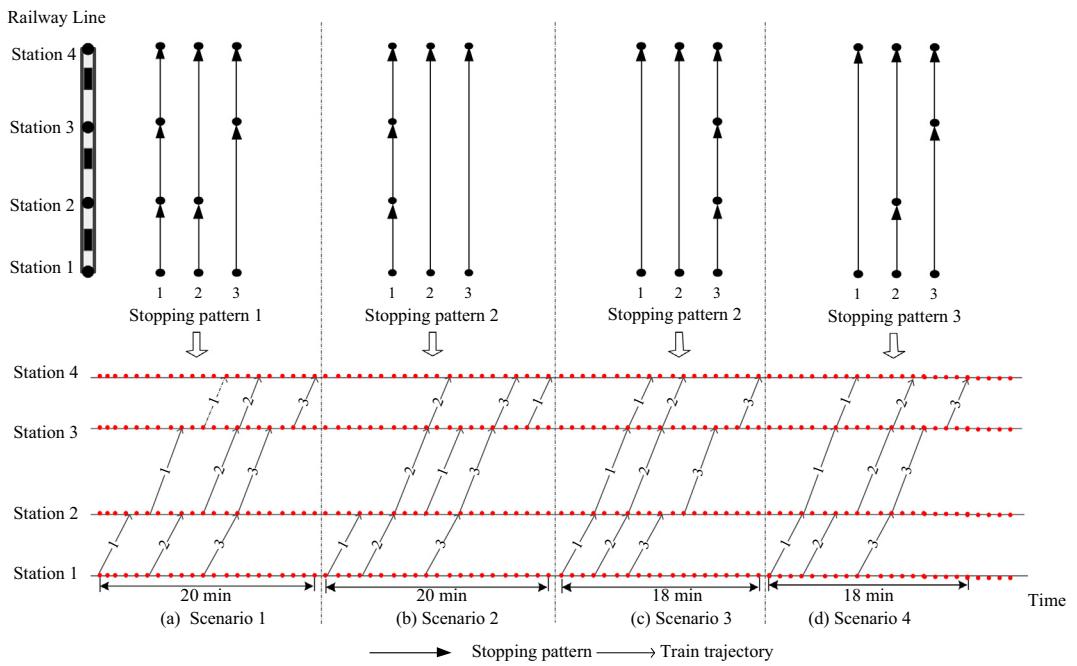


Fig. 3. Train timetables for different scenarios with different stopping patterns.

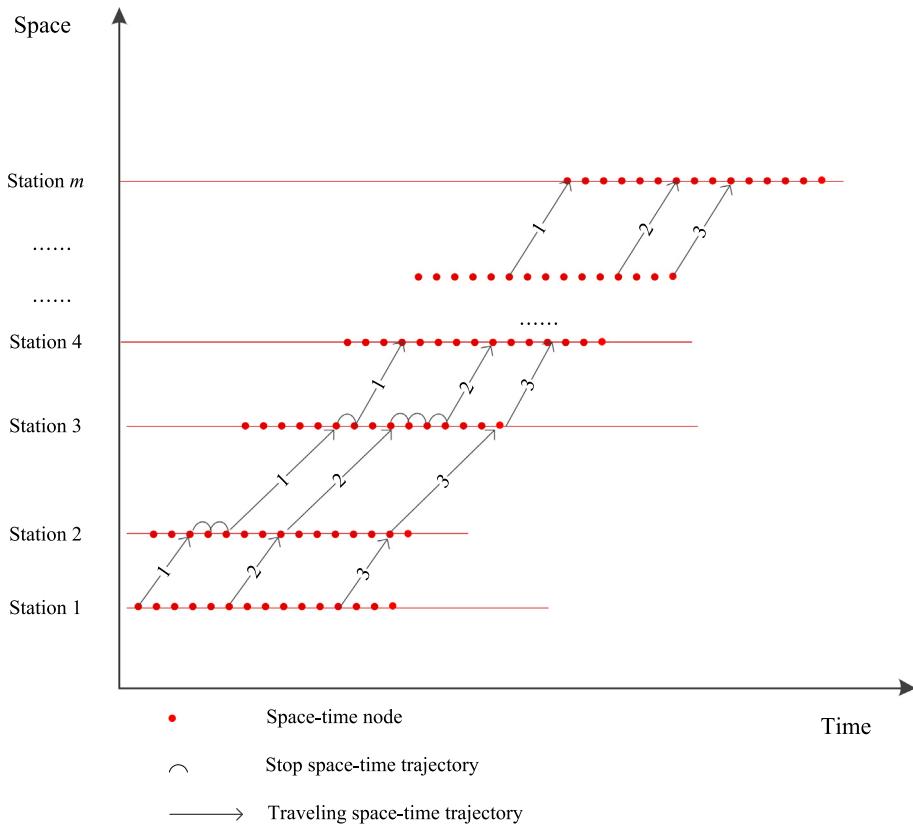


Fig. 4. A space–time diagram illustrating a train timetable.

times and that their stopping times will increase. Because of the constraints imposed by other trains and other potential limited resources, certain trains may not depart at their optimal times or may wait too long at some stations; thus, the comfort and satisfaction of passengers on those trains will decrease.

To evaluate the quality of a train timetable, we consider the “profit” of the rail line or network. In the TTP, the travel time, arrival time, departure time, stopping time and stopping pattern are important parameters. For passengers, when the arrival time and departure time are acceptable, the total travel time, which is influenced by the number of stops and the stopping time, is of great importance to their travel mode decisions and satisfaction. A longer stopping time or a greater number of stops increases the total travel time, which may decrease passengers’ satisfaction and transportation efficiency. This could lead to lower fare revenue and a reduction in the total profit to the operator of the railway line. Several studies have reported that the total loss of travel time for passengers is linearly related to the train service level (Chang et al., 2000; Wang and Ni (2008)). Hence, the profit of a rail line or network is expected to have an inverse relationship with the number of stops and the stopping time.

2.2. Notation

The general subscripts, input parameters and decision variables used in our mathematical formulations are listed in **Tables 1–3**.

The train schedule variable $x_{ij}^{t,s}(k)$ corresponds to two types of space–time trajectories: (1) a traveling space–time trajectory, where i and j are two adjacent stations and (2) a stopping space–time trajectory, where $j = i$ and $s = t + 1$ for each discrete time unit at a station (see Fig. 5).

The train service variable $y_{ij}(k)$ corresponds to the train stopping pattern. $y_{j,i}$ ($j = o(k)$) indicates that train k can serve passengers traveling from the origin station to station i , and $y_{o(k),i}(k) = 1$ indicates that train k stops at station i . Fig. 6 illustrates the relationship between the stopping pattern and the train service variable. In this figure, train k travels from Station 1 to Station 4 and stops at Station 2.

2.3. Train profit definition

As previously mentioned in Section 2.1, a train trajectory’s profit is based on the number of stops it contains and the stopping time. The objective of the optimization model in this paper is to maximize the profit from all trains given a certain train service plan. The profit of train k ’s trajectory, $\pi(k)$, is defined by the following linear formula, where $\alpha(k)$ is the penalty factor for the stopping time of train k and $\beta(k)$ is the penalty factor for the number of stops of train k . When train k is traveling a long distance, its $\alpha(k)$ and $\beta(k)$ values are relatively lower.

$$\pi(k) = \pi^{\max}(k) - \left[\alpha(k) \times \sum_i \sum_s \sum_t x_{i,i}^{t,s}(k) \right] - \left[\beta(k) \times \sum_i y_{o(k),i}(k) \right] \quad \forall k \quad (1)$$

Under the assumption that $\pi^{\max}(k) = 1$, $\alpha(k) = 0.03$, and $\beta(k) = 0.02$, Fig. 7(a) and (b) show one-way sensitivity analyses for the stopping time and the number of stops, respectively. Note that in the stopping time one-way sensitivity analysis, the number of stops is held constant at zero, which is unrealistic in the real world but is necessary in this illustration to demonstrate how the stopping time affects the profit. Fig. 7(c) is the overall relationship between the stopping time, number of stops and profit.

2.4. Mathematical model

Here, we introduce the basic optimization model (**Model 1**) used to maximize the profit from all trains:

$$Z_1 = \max_k \sum \pi(k) \quad (2)$$

Model 1 is subject to the following constraints.

2.4.1. Flow conservation constraint

The flow conservation constraint states that at each space–time node (i, t) , the outflow minus the inflow should be equal to 1 or -1 if the node is a source or a sink node, respectively; otherwise, it should be equal to 0.

Table 1
General subscripts.

i, j	Station indices, $i, j \in M$, where M is the set of stations, $ M = m$
k, k', k''	Train indices, $k, k', k'' \in N$, where N is the set of trains, $ N = n$
t, t', s	Time indices, $t, t', s \in R$, where R is the set of timestamps, $ R = r$

Table 2
Parameters.

v_{ij}	Minimum number of trains serving passenger flow from station i to station j according to the train service plan, $i \neq j$
$o(k)$	Origin station of train k
$q(k)$	Destination station of train k
$t_o(k)$	Earliest departure time of train k from origin station $o(k)$
$t_q(k)$	Latest arrival time of train k at destination station $q(k)$
w_i^{\max}, w_i^{\min}	Maximum and minimum stopping times at station i , respectively
f	Minimum departure headway (time interval) between two consecutive trains at a station
h	Minimum arrival headway (time interval) between two consecutive trains at a station
u_i	Number of sidetracks at station i (excluding main tracks)
$l_{ij}(k)$	Travel time of train k from station i to station j
$\pi^{\max}(k)$	Maximum profit for train k 's trajectory
ϕ^t	Maximum intermediate stopping time for each train at all intermediate stations
ϕ^s	Maximum number of stops for each train at all intermediate stations
$\alpha(k)$	Penalty for extra stopping time of train k
$\beta(k)$	Penalty for extra intermediate stops of train k

Table 3
Decision variables.

$x_{ij}^{t,s}(k)$	Train schedule variable (=1 if train k departs from station i at time t and arrives at station j at time s ; =0 otherwise)
$y_{ij}(k)$	Train service variable (=1 if train k can serve passengers traveling from station i to station j , $i \neq j$; =0 otherwise)

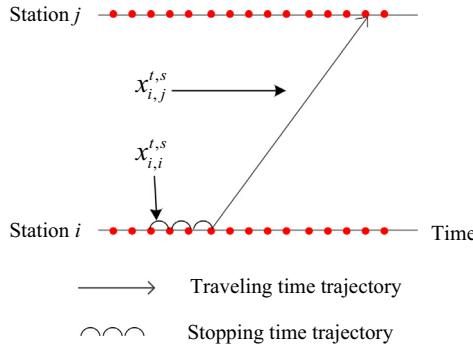


Fig. 5. Illustration of the train schedule variable (adapted from Tong et al. (2015)).

$$\sum_s \sum_j x_{ij}^{t,s}(k) - \sum_s \sum_j x_{j,i}^{s,t}(k) = \begin{cases} 1 & i = o(k), t = t_o(k) \\ -1 & i = q(k), t = t_q(k) \\ 0 & \text{otherwise} \end{cases} \quad \forall k, i, t \quad (3)$$

2.4.2. Train service plan constraints

Trains can stop only between the origin station and the destination station.

$$y_{o(k),i}(k) = 0 \quad i \leq o(k) \text{ or } i > q(k) \quad \forall k \quad (4)$$

There must be at least v_{ij} trains serving passengers traveling from station i to station j .

$$\sum_k y_{ij}(k) \geq v_{ij} \quad \forall i, j, i \neq j \quad (5)$$

2.4.3. Stopping duration constraints

A train is allowed to stop at each station for at most w_i^{\max} minutes and at least w_i^{\min} minutes.

$$\sum_s \sum_t x_{i,i}^{t,s}(k) \leq w_i^{\max} \times y_{o(k),i}(k) \quad \forall i, k \quad (6)$$

$$\sum_s \sum_t x_{i,i}^{t,s}(k) \geq w_i^{\min} \times y_{o(k),i}(k) \quad \forall i, k \quad (7)$$

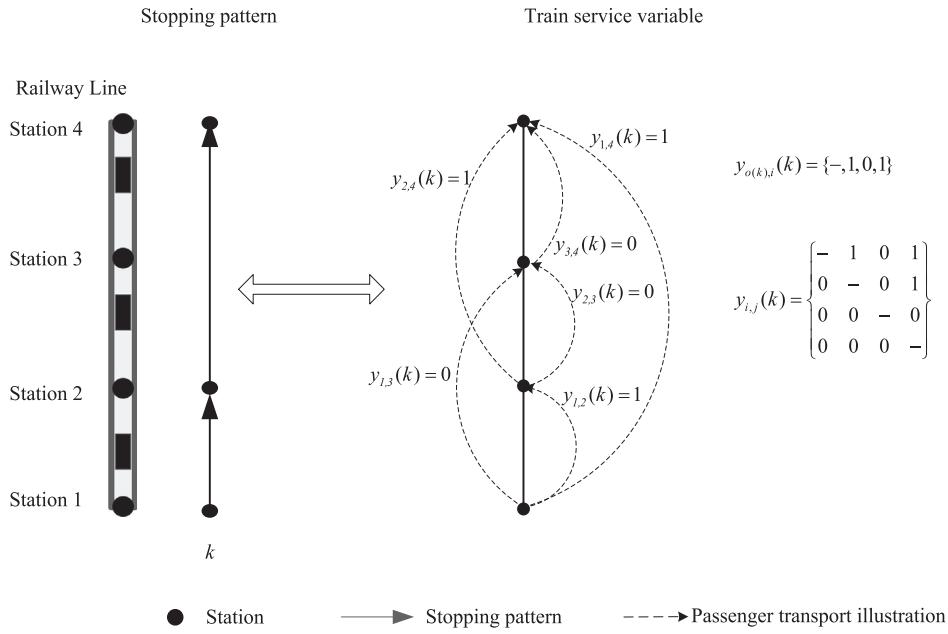


Fig. 6. Relationship between the stopping pattern and the train service variable.

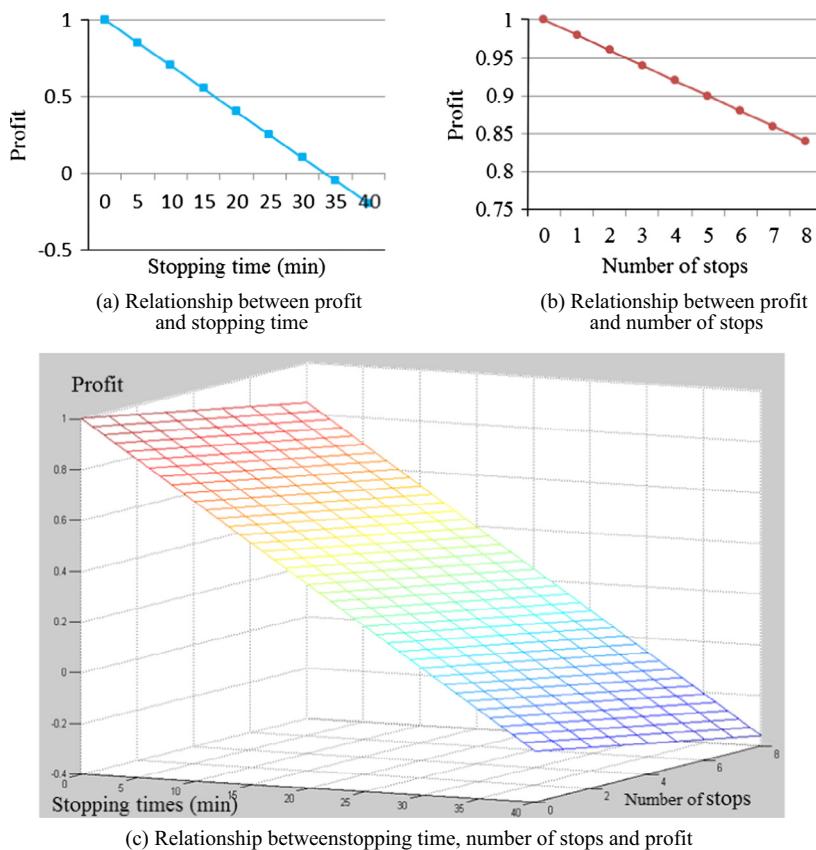


Fig. 7. Illustration of profits calculated using Eq. (1).

2.4.4. Headway constraints

Two trains traveling in the same direction cannot simultaneously depart from the same station, and a reasonable time interval between them is needed (typically, the minimum headway is based on the shortest braking distance between trains given the train types and signaling systems involved). The interval between two consecutive train departures from the same station i must be greater than or equal to the minimum departure headway f .

$$\sum_k \sum_j \sum_s \sum_{t \in (t', t'+f)} x_{ij}^{t,s}(k) \leq 1 \quad \forall i, t' \quad (8)$$

Similarly, the interval between two consecutive arrivals at the same station i must be greater than or equal to the minimum arrival headway h .

$$\sum_k \sum_i \sum_t \sum_{s \in (t', t'+h)} x_{ij}^{t,s}(k) \leq 1 \quad \forall j, t' \quad (9)$$

2.4.5. Station capacity constraint

Because of the finite capacity of a station, only a limited number of trains can stop at any given station at the same time. In other words, the number of waiting arcs (i, i, t, s) cannot exceed the number of sidetracks u_i at station i .

$$\sum_k \sum_s x_{ii}^{t,s}(k) \leq u_i \quad \forall i, t \quad (10)$$

2.4.6. Overtaking constraint

A train cannot overtake any other train on a single-track section, as shown in Fig. 8.

$$x_{ij}^{t', t'+l_{ij}(k)}(k) + \sum_{s \in [t, t'+l_{ij}(k)]} \sum_{t \in [t', t'+l_{ij}(k)]} x_{ij}^{t,s}(k') \leq 1 \quad \forall i, j, k, k', t' \quad (11)$$

Because of the headway constraints, the overtaking of trains in single-track sections may occur if and only if $l_{ij}(k) - l_{ij}(k') > f + h$.

Arguably, it is rare for two HSR trains traveling in the same direction to overtake each other. For example, in China, the high-speed railway network has an average section length of 25 km. Under the assumption that $f = 5$, $h = 5$, for an overtaking event to occur in a section, the speed difference between the faster train and the slower train must be greater than 200 km/h. However, there are only two types of HSR trains operating in China: 300 km/h trains and 250 km/h trains. For an overtaking event to occur in a section, the length of that section must be at least 250 km. Therefore, the overtaking constraint is not relevant in China. Therefore, this constraint is not considered in the following models for timetable optimization in HRS systems.

The total number of decision variables is $m \times m \times n \times (r \times r + 1)$. There are $m \times m \times n \times r \times r$ timetable continuity constraints, $m \times m$ train service plan constraints and $3 \times m \times r + 2 \times m \times n + r \times m \times n \times n$ constraints related to trains and stations. For the example of the Beijing-Shanghai HSR, which contains 23 stations and approximately 100 trains, the total number of variables is approximately 1.3×10^{11} . The sheer size of the timetable optimization problem for a typical HSR system necessitates an algorithm that can solve the problem effectively in a time-efficient manner.

3. Column-generation-based heuristic algorithm for solving the optimization problem

Column generation is an efficient algorithm that can decompose a large linear programming problem into several small parts to simplify the complexity of the calculation, especially for a linear programming problem with a total number of decision variables that is much greater than the total number of constraints. Therefore, it may be beneficial to solve the TTP presented in Section 2.4 using a column-generation-based algorithm.

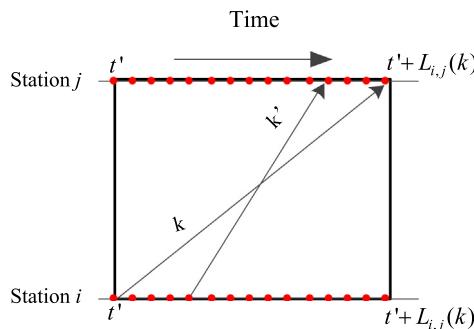


Fig. 8. Illustration of the overtaking restriction for a single-track section in one direction.

3.1. Outline of the algorithm

Initially, we relax constraints (6) and (7) to transform the model into a more straightforward linear programming problem by applying a Lagrangian relaxation method. Then, we implement a column-generation-based algorithm to obtain an optimal solution. Finally, we produce the solution and generate the train timetable to illustrate the computational results.

3.2. Lagrangian relaxation

Using Lagrangian relaxation, we first relax certain difficult-to-solve constraints in **Model 1**. By dualizing the stopping duration constraints, (6) and (7), into the original objective function in **Model 1** using the multipliers $\lambda_i^1(k)$ and $\lambda_i^2(k)$, we obtain the relaxed problem as **Model 2**, which has the following objective function:

$$Z_2 = \max \sum_k \left\{ \pi(k) + \sum_i \lambda_i^1(k) \times \left[w_i^{\max} \times y_{o(k),i}(k) - \sum_s \sum_t x_{i,i}^{t,s}(k) \right] + \sum_i \lambda_i^2(k) \times \left[\sum_s \sum_t x_{i,i}^{t,s}(k) - w_i^{\min} \times y_{o(k),i}(k) \right] \right\} \quad (12)$$

s.t. (3)–(5), (8), (9) and (10).

Model 2's objective function can be rewritten as follows:

$$Z_2 = \max \sum_k \left\{ \pi(k) - \sum_i \sum_s \sum_t [\lambda_i^1(k) - \lambda_i^2(k)] \times x_{i,i}^{t,s}(k) - \sum_i [\lambda_i^2(k) \times w_i^{\min} - \lambda_i^1(k) \times w_i^{\max}] \times y_{o(k),i}(k) \right\} \quad (13)$$

3.3. Implementation of the column-generation-based algorithm

Many linear programming problems are too large for all variables to be explicitly considered. Because most variables will be non-basic and will assume a value of zero in the optimal solution, in theory, only a subset of variables must be considered when solving the problem.

Column generation leverages this concept to generate only the variables that have the potential to improve the objective function – that is, to identify variables with reduced cost. The problem to be solved is split into two types of problems: the master problem and the sub-problems. The master problem is the original problem, except that only a subset of the variables is considered. A sub-problem is a new problem created to identify a new variable. The objective function of the sub-problem is the reduced cost of the new variable with respect to the current dual variables, and the constraints require that the variable obey the naturally occurring constraints. In our train timetable problem, the master problem represents the entire train timetable, and its objective function is the total profit of all trains. Each sub-problem is a train trajectory.

The process proceeds as follows. First, the master problem is solved; from the resulting solution, we are able to obtain dual prices for each of the constraints in the master problem. These dual prices can be used in the objective function of the sub-problems. Then, the sub-problems are solved. If the objective value of a sub-problem is positive, a variable with a positive reduced cost is identified. This variable is then added to the master problem, and the master problem is solved again. Re-solving the master problem will generate a new set of dual values, and the process is repeated until no more variables with a positive reduced cost are identified. All sub-problems return solutions with a non-positive reduced cost, and the corresponding solution to the master problem is optimal.

In the following sub-sections, we describe the implementation of the column-generation-based algorithm to solve the TTP model in greater detail.

3.3.1. Restricted master problem of column generation

First, we simplify Eq. (13) as follows:

$$Z_2 = \max \sum_k C_k \quad (14)$$

where

$$C_k = \pi(k) - \sum_i \sum_s \sum_t [\lambda_i^1(k) - \lambda_i^2(k)] \times x_{i,i}^{t,s}(k) - \sum_i [\lambda_i^2(k) \times w_i^{\min} - \lambda_i^1(k) \times w_i^{\max}] \times y_{o(k),i}(k) \quad \forall k \quad (15)$$

π can be decomposed into three terms, namely, a constant, a first-degree term of the number of stops, and a first-degree term of the stopping time.

$$\begin{aligned} C_k = & \pi^{\max}(k) - \left[\alpha(k) \times \sum_i \sum_s \sum_t x_{i,i}^{t,s}(k) \right] - \left[\beta(k) \times \sum_i y_{o(k),i}(k) \right] - \sum_i \sum_s \sum_t [\lambda_i^1(k) - \lambda_i^2(k)] \times x_{i,i}^{t,s}(k) \\ & - \sum_i [\lambda_i^2(k) \times w_i^{\min} - \lambda_i^1(k) \times w_i^{\max}] \times y_{o(k),i}(k) \quad \forall k \end{aligned} \quad (16)$$

Let $\lambda_i^\alpha(k) = \alpha(k) + \lambda_i^1(k) - \lambda_i^2(k)$ and $\lambda_i^\beta(k) = \beta(k) + \lambda_i^2(k) \times w_i^{\min} - \lambda_i^1(k) \times w_i^{\max}$; then, we can obtain a new formula as follows:

$$C_k = \pi^{\max}(k) - \sum_i \sum_s \sum_t [\lambda_i^\alpha(k) \times x_{i,i}^{t,s}(k)] - \sum_i [\lambda_i^\beta(k) \times y_{o(k),i}(k)] \quad \forall k \quad (17)$$

In Eq. (17), $\lambda_i^\alpha(k)$ is a penalty factor for the stopping time of train k at station i and $\lambda_i^\beta(k)$ is a penalty factor for the number of stops of train k at station i .

$$Z_2 = \max_k C_k \quad (18)$$

s.t. (3)–(5), (8), (9) and (10).

In our model, the train schedule and train service variables are two separate but related variables. Unlike in typical column generation algorithm implementations, we do not add columns for arcs. In our iteration procedure, we treat the variables associated with a given train trajectory as an entire column and optimize the train trajectories' profits. In other words, each column represents a set of arcs (variables) corresponding to a train trajectory (adapted from Cacchiani et al., 2008 and Li et al., 2014). Because of the limited search space for these new columns, we use a neighborhood search method to solve the sub-problems to obtain the optimal solution.

Initially, we obtain a feasible solution based on a train service plan. After the addition or removal of a train trajectory, $y_{i,j}(k)$ needs to be recalculated based on constraints (4) and (5) using CPLEX Optimization Studio. Then, the $x_{i,j}^{t,s}(k)$ are modified based on constraints (3), (8), (9) and (10). When $y_{i,j}(k)$ and $x_{i,j}^{t,s}(k)$ changes, more space may become available in the space–time diagram, and it may become possible to add train trajectories to improve the efficiency of the line, after which it is necessary to optimize $y_{i,j}(k)$ again, and the procedure is repeated.

Let us consider a simple example. Given $v_{i,j} = 1 \forall i, j \in M, i \neq j$ and a feasible train timetable, as shown in Fig. 9(a), we can first optimize $x_{i,j}^{t,s}(k)$, as shown in Fig. 9(b), resulting in more available space in the space–time diagram. Subsequently, we add a train trajectory, as shown in Fig. 9(c). Then, we optimize $y_{i,j}(k)$, as shown in Fig. 9(d).

In a complex network, we need to perform a large number of iterations to obtain an approximately optimal solution. In our column-generation-based algorithm, the RMP is intended to optimize the $y_{i,j}(k)$, and the sub-problems are intended to calculate the maximum reduced profits of the $x_{i,j}^{t,s}(k)$ and add columns to the RMP. After a new column is added, we must perform column management, check the profits of all train trajectories' and delete any train whose profit is lower than the threshold in each iteration.

3.3.2. Sub-problems of column generation

The sub-problems are solved to check whether a trajectory with a positive reduced profit exists before optimizing a train trajectory ($\forall k$) or adding an optimal train trajectory (k''). Constraints (3) and (4) are complementary constraints and are irrelevant for reducing cost and constraint (5) cannot be calculated in sub-problems. Thus, we consider only constraints (8)–(10) in the dual problems. Let the dual variables associated with constraints (8)–(10) be $\sigma_i(k)$ ($i \in (o(k), q(k))$, $\sigma_i(k) \in R$) and $\theta_i(k)$ ($i \in (o(k), q(k))$, $\theta_i(k) \in R$).

The reduced profit of train k 's trajectory is then given by

$$C_k - \sum_i \sigma_i(k) - \sum_j \theta_j(k) \quad k \in N \cup k'' \quad (19)$$

s.t. (8)–(10).

It is impossible for the reduced profit to be less than zero. Thus, we introduce a positive variable $\Gamma(k)$ to ensure that the reduced profit of a train trajectory is always positive. The variable $\Gamma(k)$ is computed as follows: $\Gamma(k) = \tau \times \beta(k) \times \sum_i W_i^{\min} / m$, where τ is the expected average number of stops for all trains. The reduced profit of a train trajectory is then given by

$$D_k = C_k - \sum_i \sigma_i(k) - \sum_j \theta_j(k) - \Gamma(k) \quad (20)$$

s.t. (8)–(10).

If $D_k \leq 0$, then all train trajectories have negative reduced profits and none need be added. Otherwise, the trajectory is found that is associated with the variable with the most positive reduced profit.

Before optimizing the train trajectory profits, we need to eliminate the redundant arcs based on: (1) the variable definitions and (2) the practical situation (including the time direction, the space direction, and the travel time in a section). As shown in Fig. 10, there may be a large number of arcs associated with a given point: traveling time arcs, such as 1, 2, 3, 4, and 5, and stopping time arcs, such as 6 and 7. Among these arcs, only arcs 1 and 6 are feasible. The others are infeasible. Arcs 2 and 7 are inconsistent with the variable definitions. Arcs 3, 4 and 5 do not respect the practical situation: the time direction of arc 3 is incorrect, the space direction of arc 4 is incorrect, and arc 5 does not conform to the travel time between

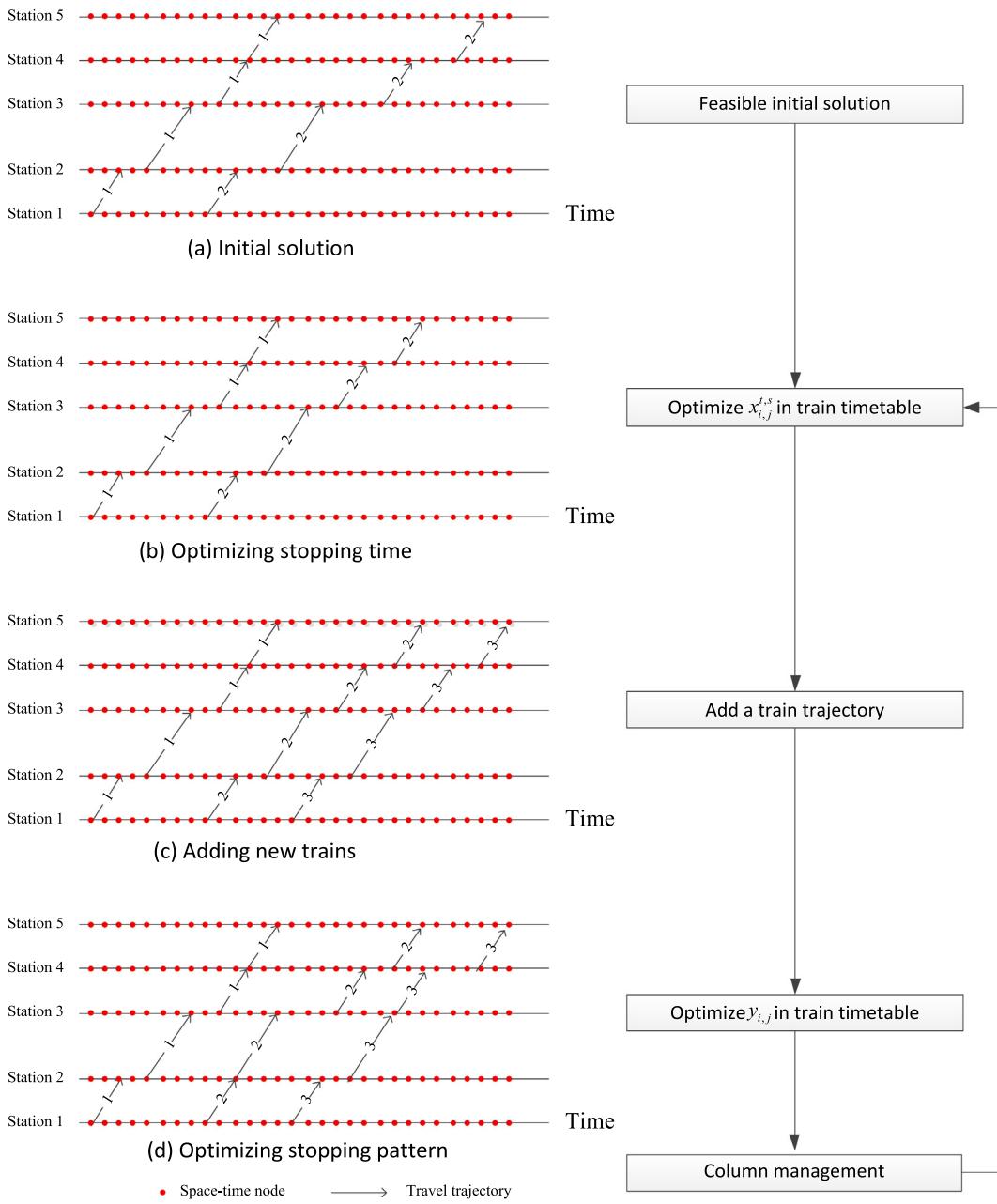


Fig. 9. Illustration of the column-generation-based approach.

Station 2 and Station 3. Although the number of variables is large, the search space for a feasible solution for a train trajectory is manageable.

Nevertheless, it is very difficult to find all feasible columns (as space-time trajectories), and we must use a neighborhood search method to solve the sub-problems to obtain a close-to-optimal solution and then calculate the maximum reduced cost.

3.3.3. Column generation solution procedure

Eqs. (18) and (20) are normative linear programming problems and thus are suitable to be solved using the column-generation-based algorithm. The following is the iterative procedure implemented to solve the problems, as summarized in Fig. 11.

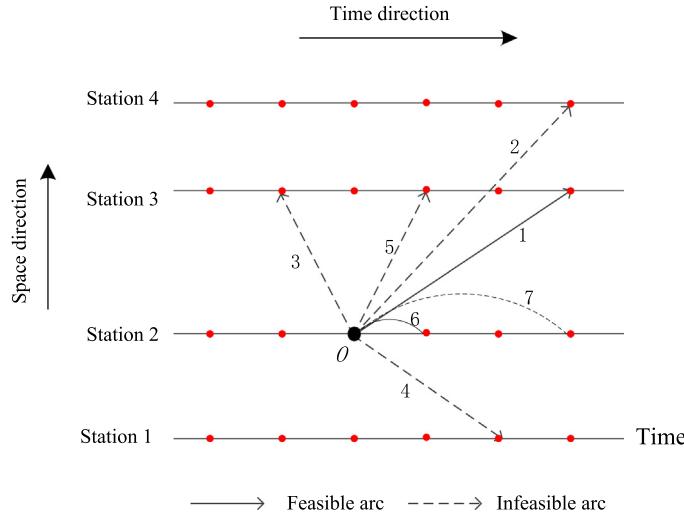


Fig. 10. Illustration of the elimination of redundant space-time arcs.

Step 1. Obtain an initial feasible solution.

Obtain an initial feasible solution of the TTP. First, in accordance with the train service plan, including the passenger flows and the locations of electric multiple unit depots, we design different types of train operating intervals and calculate the probabilities of trains stopping at stations. Based on this, we randomly generate a number of train trajectories.

Step 2. Initialize $y_{ij}(k)$, $x_{ij}^{ts}(k)$.

Step 3. Solve the restricted master problem.

The restricted master problem (**Model 2**) is a linear programming problem. We use CPLEX Optimization Studio as the LP solver. This procedure guarantees that the train timetable is optimal for the current total number of trains and the train service plan.

Step 4. Calculate the sub problems.

We calculate the sub-problems, D_k . If $D_k \leq 0$ ($\forall k \in N \cup k''$), then the profits of all train trajectories are approximately optimal. Otherwise, we need to add a column (optimize a train trajectory or add an optimal train trajectory) to the restricted

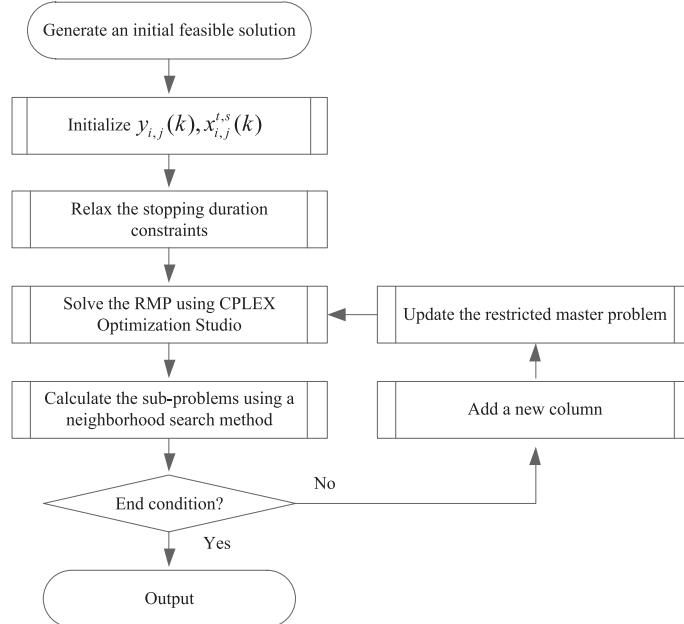


Fig. 11. Procedure of the solution methodology.

master problem. If $D_k \geq 0$ or the maximum number of iterations has been reached, we proceed to **Step 6**; otherwise, we proceed to **Step 5**.

Step 5. Update the restricted master problem.

We delete those trains whose profits are lower than the threshold. We update $x_{i,j}^{t,s}(k)$, $y_{i,j}(k)$, update the coefficients of the objection function, and return to **Step 3**.

Step 6. Present the output.

The final results of the model are $x_{i,j}^{t,s}(k)$, $y_{i,j}(k)$. We also convert the results into an explicit graph to visualize the train timetable

4. Case study: Beijing–Shanghai HSR line

In this section, we consider the Beijing–Shanghai high-speed railway in China to test our optimization model and algorithm. The scheduling algorithms were implemented in Microsoft Visual Studio 2010 on the Windows 7 OS, and we used CPLEX Optimization Studio as the LP solver. All experiments were performed on a PC with an Intel Core Duo 2.0 GHz CPU and 3 GB of RAM. The calculation time for the algorithm was within 3 min.

4.1. Introduction to the Beijing–Shanghai HSR

The Beijing–Shanghai high-speed railway connects Beijing to Shanghai. It has a total length of 1302 km (819 miles) and is the world's longest high-speed line ever constructed in a single phase (Fig. 12 shows the route). More than 100 million passengers were transported via this system in 2014. During peak hours, trains can run every five minutes. The trains are designed to reach a maximum speed of 380 km/h. There are two types of trains currently operating on this line: Grade G and Grade D. The highest speed of a Grade G train is 300 km/h, and that of a Grade D train is 250 km/h. The maintenance time window is from 0:30 A.M. to 4:30 A.M., which means that no ordinary train operations occur during this time span. From 4:30 A.M. to 7 A.M., only several test trains and short-distance trains are run on the line. Therefore, our time span of interest is defined as extending from 7 A.M. to 0 A.M.

4.2. Description of the experimental setting

The actual timetable of the HSR line uses 220 trains within the defined time horizon. Table 4 summarizes the possible line plans (only the major origin station and destination station are indicated) and their percentages. Table 5 shows the travel



Fig. 12. Diagram of the Beijing–Shanghai high-speed railway line: 23 stations and 1302 km.

times in each section for different types of trains. [Table 6](#) lists each station with its name, number of tracks and distance from Beijing Nan Station. Different types of trains run at different speeds and have different travel times in each section. The data in [Tables 4 and 5](#) are calculated based on the real train timetable of February 1st, 2015, and the data in [Table 6](#) were obtained from a publicly available domain ([Wikipedia, 2015](#)) reporting the total numbers of tracks in the stations, not the tracks available on the Beijing–Shanghai HSR main line.

4.3. Alternative mixed-integer optimization model

Because of the enormous passenger flow traveling on HSR lines in China, we consider an alternative model (**Model 3**) that first maximizes the total number of trains to meet the traffic demand. The profit of train k 's trajectory, $\pi(k)$, is a binary variable that is defined as follows:

$$Z_3 = \max \sum_k \pi(k)$$

$$\pi(k) = \begin{cases} 1 & \sum_i \sum_s \sum_t x_{i,i}^{t,s}(k) \leq \phi^t \cap \sum_i y_{o(k),i}(k) \leq \phi^s \\ 0 & \text{otherwise} \end{cases} \quad \forall k \quad (21)$$

s.t. (3)–(10).

Model 3 identifies the total number of trains whose trajectories are acceptable according to the thresholds for stopping time and number of stops. In Eq. (21), ϕ^t is the threshold for the stopping time at all intermediate stations and ϕ^s is the threshold for the number of stops. When train k 's stopping time is not greater than ϕ^t and its number of stops is not greater than ϕ^s , $\pi(k)$ is equal to "1"; otherwise, $\pi(k)$ is equal to "0".

We used **Model 3** to calculate the profits of the train trajectories. This model represents is a mixed-integer nonlinearly constrained optimization problem. We used CPLEX Optimization Studio to solve the model and calculate the profit for the entire train timetable based on the total number of trains. We defined several values for ϕ^t , and we let $\phi^s = 10$. For each value of ϕ^t , we solved the problem 20 times while holding the other parameters constant. The average CPU calculation time for each run was 30 s.

[Table 7](#) shows the results obtained for different values of the maximum stopping time (ϕ^t). When the maximum accepted stopping time (ϕ^t) is increased, the threshold of feasibility for the train trajectories will decrease, allowing more trains to be added to the timetable. However, this may cause the incidence of conflicts and the stopping times of individual trains to increase, resulting in an overall decrease in the profit of the entire timetable. [Fig. 13](#) shows the relationship between the timetable profit and the accepted stopping time. The horizontal axis represents the maximum stopping time (ϕ^t); the vertical axis represents the profit of the train timetable (the number of trains). When $\phi^t \leq 20$, the profit increases with increasing ϕ^t , and it reaches its highest value of 296 at $\phi^t = 20$; beyond that point, the train profit no longer increases and the average train speed decreases. A comparison of these solutions, especially their speeds, indicates that increasing the maximum stopping time can improve the efficiency of the line capacity usage but will decrease passengers' satisfaction. Ultimately, we do not believe that **Model 3** is a good strategy for optimizing train timetables – its results are compared with those of **Model 2** in Section 4.4.

4.4. Results and analysis

In this subsection, we illustrate the use of **Model 2** and the implementation of the column-generation-based algorithm to calculate the trains' profits, and we compare the results with the current actual timetable and the results of the alternative **Model 3**. The parameters related to the optimization process are specified in [Table 8](#). The parameter values were selected based on our experience when performing sensitivity analyses to assess the effects of the parameters on the objective function. The total number of iterations was set to 2500.

In the optimization process, we first obtained an initial feasible train timetable that conformed to constraints (3)–(5), (8), (9) and (10). However, this initial train timetable based on the train service plan had excessive stopping time and an excess number of stops. Based on the initial solution, we used the column-generation-based algorithm to add or remove trains and modify their stopping patterns to improve the train timetable. We performed 20 tests, and their average CPU calculation time was 150 s. [Figs. 14 and 15](#) illustrate the iterative solution process for the up direction.

In [Fig. 14](#), the horizontal axis represents the number of iterations and the vertical axis represents the total number of trains or the average train speed. The lower line (solid line) represents the total number of trains, and the upper line (dashed line) represents the average train speed in kilometers per hour. The total number of trains and the average train speed both increased with further iterations and stabilized before the maximum number of iterations were completed.

[Fig. 15](#) shows the two key parameters related to the train timetable. The horizontal axis represents the number of iterations, and the vertical axis represents the average stopping time or number of stops per train. The upper line (dashed line) represents the average train stopping time in minutes, and the lower line (solid line) represents the average number of stops.

Table 4

Train percentages for major OD pairs.

Origin	Destination						
	Beijing Nan (%)	Tianjin Nan (%)	Jinan Xi (%)	Xuzhou Dong (%)	Bengbu Nan (%)	Nanjing Nan (%)	Shanghai Hongqiao (%)
Beijing Nan	–	0	7	1	5	15	40
Tianjin Nan	0	–	7	1	1	3	3
Jinan Xi	7	7	–	1	3	5	3
Xuzhou Dong	1	1	1	–	1	1	1
Bengbu Nan	5	1	3	1	–	0	0
Nanjing Nan	15	3	5	1	0	–	2
Shanghai Hongqiao	40	3	3	1	0	2	–

Table 5

Travel times for each section (min).

Section		Train grade	
Origin	Destination	Grade G	Grade D
Beijing Nan	Langfang	19	22
Langfang	Tianjin Nan	13	16
Tianjin Nan	Cangzhou Xi	18	22
Cangzhou Xi	Dezhou Dong	22	26
Dezhou Dong	Jinan Xi	20	23
Jinan Xi	Taian	13	15
Taian	Qufu	15	18
Qufu	Tengzhou Dong	12	14
Tengzhou Dong	Zaozhuang	8	9
Zaozhuang	Xuzhou Dong	14	16
Xuzhou Dong	Suzhou Dong	14	17
Suzhou Dong	Bengbu Nan	18	22
Bengbu Nan	Dingyuan	12	14
Dingyuan	Chuzhou	13	16
Chuzhou	Nanjing Nan	16	19
Nanjing Nan	Zhenjiang Nan	17	20
Zhenjiang Nan	Danyang Bei	6	7
Danyang Bei	Changzhou Bei	7	8
Changzhou Bei	Wuxi Dong	12	14
Wuxi Dong	Suzhou Bei	6	7
Suzhou Bei	Kunshan Nan	7	8
Kunshan Nan	Shanghai Hongqiao	15	17

The train stopping time considerably decreased and the number of stops slightly decreased as the iterative process progressed.

Table 9 summarizes the comparisons of several parameters, such as the number of trains, the profit of the trains, the stopping time, the number of stops and the train speed, for the following five cases:

Case 1: The actual train timetable that was used by the Chinese railway on February 1st, 2015. In this timetable, there are 220 trains (from 7 A.M. to 0 A.M.) and the average stopping time is 17 min. In 2015, the average daily passenger flow volume is approximately 312,000; the peak daily passenger flow is approximately 489,000.

Case 2: A train timetable determined using a Lagrangian relaxation algorithm from Yue et al. (in preparation). The problem is formulated to make full use of the system capacity to schedule the train timetable effectively. We also use a Lagrangian relaxation method to transform the original complex problem into sub-problems, where the objective of each sub-problem is to optimize a single train's timetable. The Lagrangian multipliers are computed using bundle methods. However, please note that the profit calculation method for this case is different from that applied in this paper.

Case 3: A train timetable wherein the profits of the train trajectories are calculated using **Model 3** and $\phi^t = 5$.

Case 4: A train timetable wherein the profits of the train trajectories are calculated using **Model 3** and $\phi^t = 20$.

Case 5: A train timetable wherein the profits of the train trajectories are calculated using **Model 2**.

In Table 9, the “Gap” is calculated using the following definition.

$$\text{Gap} = (\text{Upper bound on profit} - \text{Lower bound on profit}) / \text{Upper bound on profit} \quad (22)$$

Note that there are not the exact upper bound and lower bound due to the heuristics used in the lower bound search. For each train trajectory, the lower bound is C_k and the upper bound is $\pi^{\max}(k) - \sum_i [\lambda_i^\beta(k) \times y_{o(k),i}(k)]$.

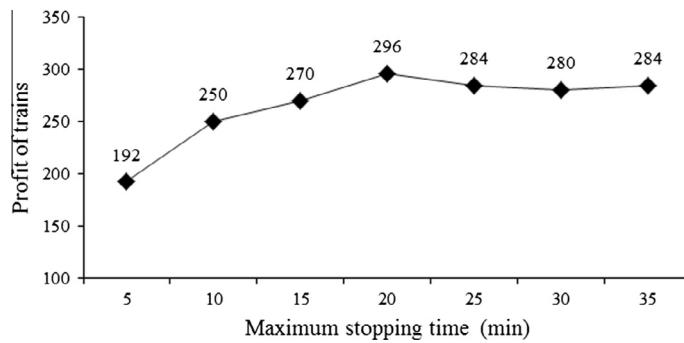
Table 6

Overview of Beijing–Shanghai HSR.

Number	Station name	Distance from Beijing (km)	Number of tracks		
			Up direction	Down direction	Shared
1	Beijing Nan	0	0	0	12
2	Langfang	59	1	1	0
3	Tianjin Nan	131	3	3	0
4	Cangzhou Xi	219	3	3	0
5	Dezhou Dong	327	3	3	1
6	Jinan Xi	419	9	8	0
7	Tai'an	462	3	3	1
8	Qufu	533	3	3	1
9	Tengzhou Dong	589	2	2	0
10	Zaozhuang	625	2	2	2
11	Xuzhou Dong	688	8	7	0
12	Suzhou Dong	767	3	3	0
13	Bengbu Nan	844	4	5	2
14	Dingyuan	897	2	2	0
15	Chuzhou	959	2	2	0
16	Nanjing Nan	1018	5	5	0
17	Zhenjiang Nan	1087	3	3	0
18	Danyang Bei	1112	2	2	0
19	Changzhou Bei	1144	3	3	0
20	Wuxi Dong	1201	3	3	0
21	Suzhou Bei	1227	3	3	0
22	Kunshan Nan	1259	3	3	6
23	Shanghai Hongqiao	1302	8	11	0

Table 7Comparisons for different values of the maximum stopping time (ϕ^t).

Maximum stopping time (ϕ^t)	5	10	15	20	25	30	35
Average stopping time (min)	4	9	11	14	17	19	22
Average trains' speed (km/h)	238	235	231	227	224	221	218
Profit of the entire train timetable using Model 3	192	250	270	296	284	280	284

**Fig. 13.** Comparison of the timetable profits obtained using **Model 3**.**Table 8**

Parameter definitions.

Symbol	Definition	Value
$\pi^{\max}(k)$	Profit of a train trajectory	1 (for trains of Grade G) 0.9 (for trains of Grade D)
λ_i^α	Penalty factors for stopping time	0.015–0.04
λ_i^β	Penalty factors for the number of stops	0.008–0.03

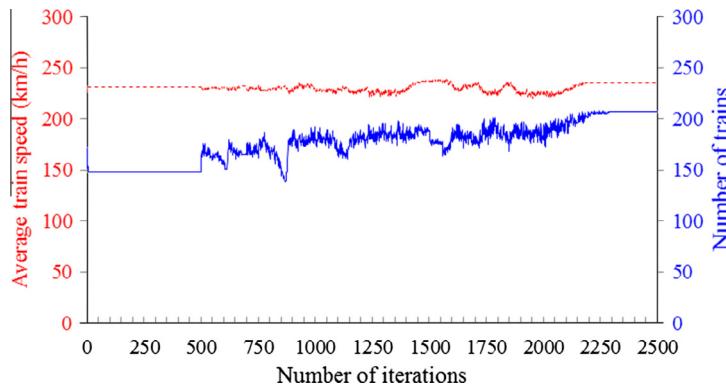


Fig. 14. Iterative solution process via column generation.

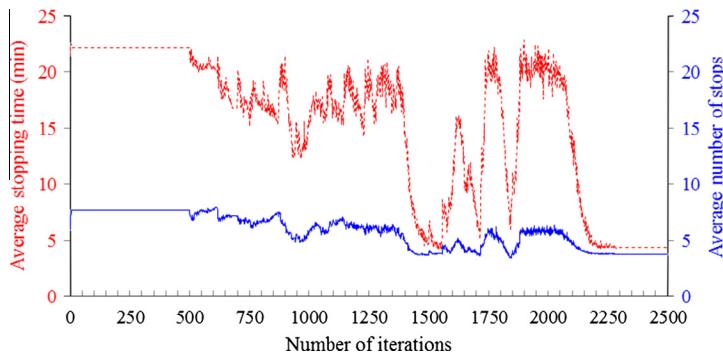


Fig. 15. Changes in the parameters during the iterative process.

Table 9

Comparisons of different train timetables.

Instance	Case 1	Case 2	Case 3	Case 4	Case 5
Number of trains	220	216	192	296	280
Ratio of number of trains (compared with Case 1)	100%	98%	87%	134%	127%
Number of trains in initial solution	–	216	160	160	160
Average stopping time	17	5	4	9	5.75
Average number of stops	7.1	2.3	8	6.0	2.81
Average speed (km/hour)	238.8	241.5	241.2	226.8	240.5
Average profit ($\bar{\pi}(k)$)	0.71	0.83	0.92	0.65	0.73
Profit of all trains ($\sum_{k \in N} \pi(k)$)	156	180	176	192	204
Ratio of profit (compared with Case 1)	100%	115%	112%	123%	130%
CPU calculation time (seconds)	–	95	30	35	150
Upper bound on profit	–	136570	192	256	224
Lower bound on profit	–	122066	176	192	204
Gap	–	10.6%	8.3%	25%	8.9%

First, we consider the gaps of the objective functions to evaluate their levels of optimality. **Case 1** is a real instance, for which the gap cannot be calculated, and **Case 2** is based on a different calculation method than the other three cases. The gaps for **Case 2**, **Case 3** and **Case 5** are all approximately 10%, and we believe that an approximately optimal solution was obtained in all of these cases. The solution of **Case 4** is the worst.

The following are several observations based on the results presented in Table 9:

- Compared with **Case 3** and **Case 4**, for which the alternative **Model 3** was used, the column-generation-based algorithm for optimizing the trains' profits using **Model 2** (**Case 5**) demonstrated better performance for optimizing train timetables. This algorithm can assign a sufficient number of trains and ensure that those trains' profits, based on speed, stopping time and number of stops, are relatively high. The results indicate a promising opportunity to increase the line capacity and the quality of service for passengers.

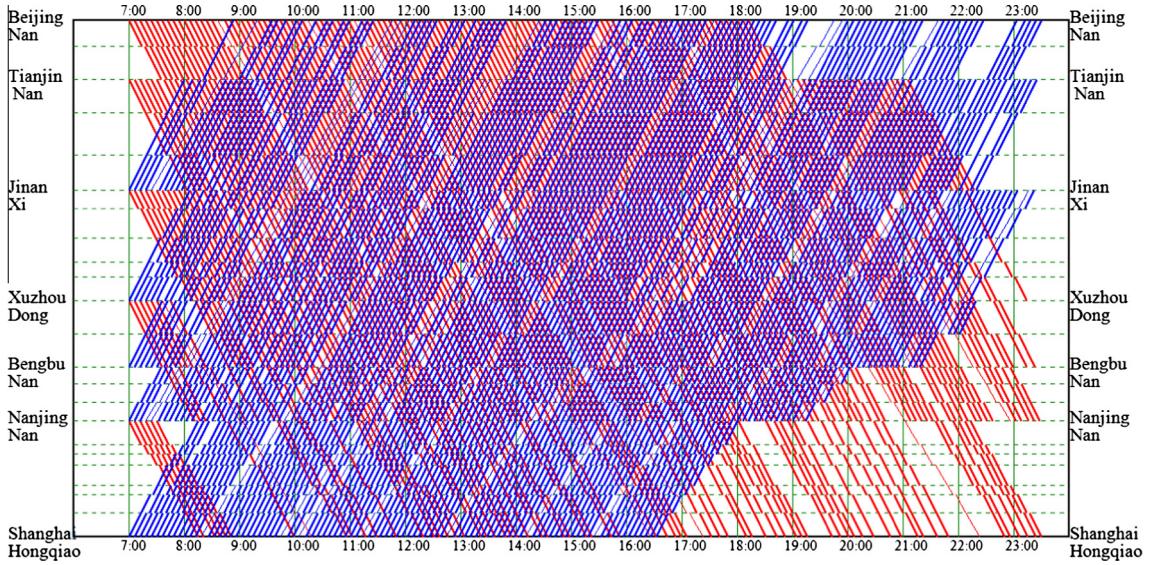


Fig. 16. An initial feasible solution for Case 5.

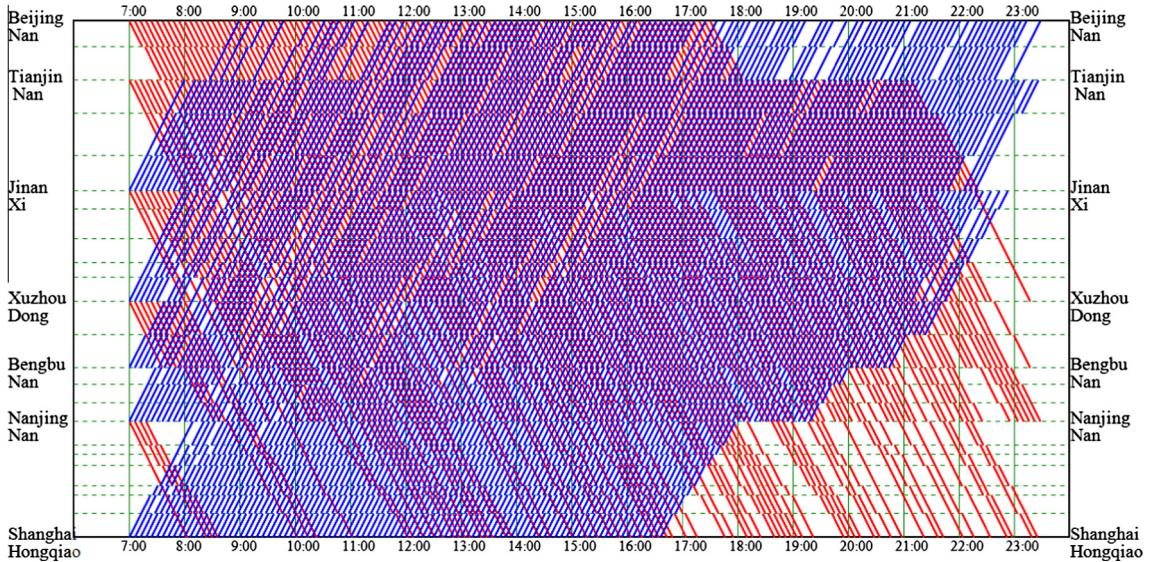


Fig. 17. The final solution for Case 5.

- From a comparison of **Case 2** and **Case 5**, we observe that the solution obtained using the Lagrangian relaxation algorithm (**Case 2**) has a high $\bar{\pi}(k)$, with a smaller stopping time and fewer stops. However, because of the lower number of trains, the profit in this solution is lower than that in the solution obtained via column generation, which means that it does not make full use of the existing line capacity to improve the service with additional trains.
- Compared with **Case 1**, the total number of trains and the total profit of the timetable obtained using the column-generation-based algorithm (**Case 5**) are much higher than those in the real timetable because of its lower stopping time and fewer stops. The number of trains increases by 27%, and the profit increases by 30%. This analysis indicates that the proposed model and algorithm can potentially increase the transport capacity and improve the degree of passenger satisfaction.

Fig. 16 shows the initial feasible solution and **Fig. 17** shows the optimized solution for **Case 5** for the time span of 7 A.M. to 0 A.M. In these figures, the red lines represent downward-going trains and the blue lines represent upward-going trains; the thicker lines represent trains of Grade G, and the thinner lines represent trains of Grade D. After the completion of the opti-

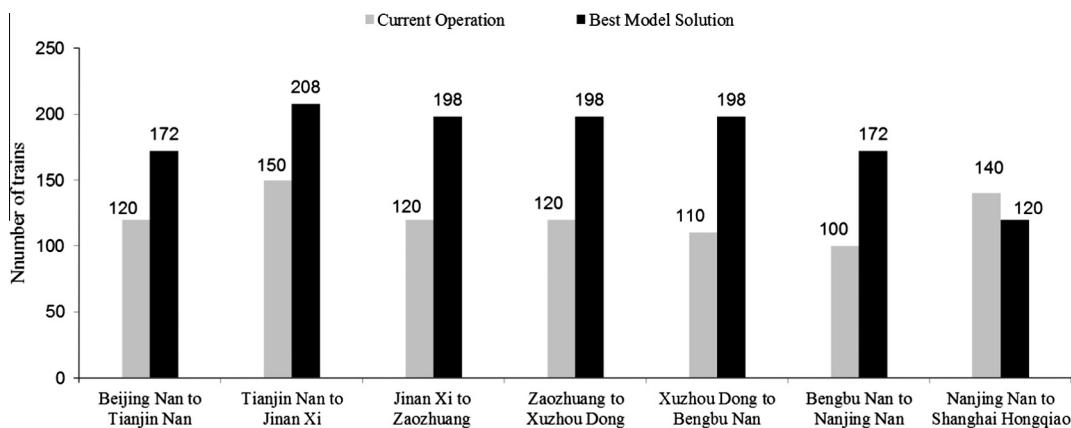


Fig. 18. Comparison of the numbers of trains operating in various sections.

mization process, more lines (train trajectories) had been added to the initial solution, as indicated by the greater density of lines in Fig. 17 compared with Fig. 16.

Fig. 18 compares **Case 1** with **Case 5** in terms of the total number of trains operating in each section. In each section except the Nanjing–Shanghai section, the numbers of trains in the timetable generated using the proposed optimization model (Case 5) are greater than those in the current actual timetable (**Case 1**). The result for the Nanjing–Shanghai section is irrelevant to the comparison because another HSR line, the Shanghai–Nanjing Intercity HSR line, also serves this section.

5. Conclusions

This paper proposes a new mathematical model for the optimization of train timetable scheduling based on a train service plan to optimize train timetables according to the trains' trajectories, including the stopping times and numbers of stops. This model can satisfy passenger transportation demands, enhance passenger satisfaction, improve the fare revenues and profits of rail operators, and improve the utilization of line capacity.

First, we obtain a feasible train timetable based on the specified train service plan; in this initial solution, however, the stopping times and numbers of stops for certain train trajectories are too high for passenger satisfaction. We apply a Lagrangian relaxation method to simplify the model and then use a column-generation-based algorithm to solve the resulting restricted master problem and sub-problems (the trains' stopping patterns are optimized in the restricted master problem, and the maximum reduced profits based on the stopping times are calculated in the sub-problems). In a case study of the Beijing–Shanghai high-speed railway, we obtained an improved solution that increases the number of trains by 27% compared with the current train timetable and increases the profit of the entire timetable by nearly one-third. In general, our innovative and integrated algorithm can quickly obtain a superior solution and take full advantage of the potential line capacity, thereby making it useful for application in the real world.

In addition, although the methodology and optimization model are presented in the context of HSR timetables in this paper, they can also be applied to any problems of passenger and transit rail train timetables and scheduling.

Our future research will focus on three major areas. First, we will establish an accurate mathematical model that will include the assignment of train units to obtain results that are more practical. Second, we will improve the computational performance of the algorithm by adjusting various parameters to speed up the algorithm's convergence and produce solutions that are more stable. Finally, we will consider other real-world high-speed railway networks to test the feasibility and effectiveness of the proposed methodology and optimization model.

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