Optimal Fueling Strategies for Locomotive Fleets in Railroad Networks

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Outline

• Background
• Model Formulation
• Optimality Properties and Solution Techniques
• Case Studies
• Conclusion
Fuel Price

- Fuel-related expenditure is one of the biggest cost items in the railroad industry
- Railroad fuel consumption remains steady
- Crude oil price sharply increases in recent years
Fuel Price

• Fuel (diesel) price influenced by:
  – Crude oil price
  – Refining
  – Distribution and marketing
  – Others

Source: Energy Information Administration.
Fuel Price

• Fuel price vary across different locations

• Each fuel station requires a long-term contractual partnership
  – Railroads pay a contractual fee to gain access to the station
  – Sometimes, a flat price is negotiated for a contract period

• US national fuel retail price, by county, 2009
Locomotive Routes in a Network

Candidate fuel station

Contracted fuel station
Motivation

• Usage of each fuel station requires a contractual partnership cost

• Hence, should contract stations and purchase fuel where fuel prices are relatively low (without significantly interrupting locomotive operations)
  – In case a locomotive runs out of fuel, emergency purchase is available anywhere in the network but at a much higher price
  – Each fueling operation delays the train
The Challenge
The Challenge

- Fuel cost vs. contract cost
  - Too few stations = high fueling cost (e.g., emergency purchase)
The Challenge

- Fuel cost vs. contract cost
  - Too many stations = high contracting costs
Problem Objective

• To determine:
  – Contracts for fueling stations
  – Fueling plan for all locomotives
    • Schedule
    • Location
    • Quantity

• To minimize:
  – Total fuel-related costs:
    • Fuel purchase cost
    • Delay cost
    • Fuel stations contract cost
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• Background
• **Model Formulation**
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Notation

• Set of candidate fuel stations, $N = \{1, 2, \ldots, |N|\}$
• Set of locomotives, $J = \{1, 2, \ldots, |J|\}$
• Sequence of stops for locomotive $j$, $S_j = \{1, 2, \ldots, n_j\}$, for all $j \in J$

For any location $i$
- $c_i = \text{Unit fuel cost}$
- $a_1 = \text{Delay cost per fueling stop}$
- $a_2 = \text{Contract cost per fuel station per year}$
- $M_i = \text{Maximum number of locomotives passing}$

$p = \text{Unit fuel cost for emergency purchase (} p > c_i \text{ for all } i)$
Notation

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For any locomotive $j$
- $b_j$ = Tank capacity
- $r_j$ = Fuel consumption rate
- $n_j$ = Number of stops
- $f_j$ = Travel frequency
- $g_j$ = Initial fuel
Notation

- Set of candidate fuel stations, $N = \{1, 2, \ldots, |N|\}$
- Set of locomotives, $J = \{1, 2, \ldots, |J|\}$
- Sequence of stops for locomotive $j$, $S_j = \{1, 2, \ldots, n_j\}$, for all $j \in J$

$l_{s}^{j} = \text{Distance between the } s^{th} \text{ and } (s+1)^{th} \text{ fuel stations that locomotive } j \text{ passes}$
Decision Variables

• For each station, contract or not?
  – \( z_i = 1 \) if candidate fuel station \( i \) is contracted and 0 otherwise

• For each locomotive, where to stop for fuel?
  – \( x_{s,j} = 1 \) if locomotive \( j \) purchases fuel at its \( s^{th} \) station and 0 otherwise
  – \( y_{s,j} = 1 \) if locomotive \( j \) purchases emergency fuel between its \( s^{th} \) and \((s+1)^{th}\) station and 0 otherwise

• How much to purchase?
  – \( w_{s,j} \) = Amount of fuel purchased at stop \( s \) of locomotive \( j \)
  – \( v_{s,j} \) = Amount of emergency fuel purchased between the \( s^{th} \) and \((s+1)^{th}\) stations of locomotive \( j \)
Formulation

\[
\begin{align*}
\min \quad & \sum_{j=1}^{[J]} \sum_{s=1}^{n_j} f_j \left[ \sum_{i=1}^{[N]} (c_i q_i s w_i^j + (p v_i s^j)) \right] + \alpha_1 \sum_{j=1}^{[J]} \sum_{i=1}^{n_j} f_j (x_i^j + y_s^j) + \alpha_2 \sum_{i=1}^{[N]} z_i \\
\text{s.t.} \quad & g_j + \sum_{s=1}^{k} (w_i^j + v_s^j - r_i^j l_s^j ) \geq 0, \quad \forall j \in J, \quad \forall k = 1, 2, \ldots, n_j - 1 \\
& \sum_{s=1}^{k} (w_i^j + v_s^j - r_i^j l_s^j ) \geq 0, \quad \forall j \in J, \quad k = n_j \\
& g_j + \sum_{s=1}^{k-1} (w_i^j + v_s^j - r_i^j l_s^j ) + w_k^j \leq b_j, \quad \forall j \in J, \quad \forall k = 1, 2, \ldots, n_j \\
& w_i^j \leq b_j x_i^j, \quad \forall j \in J, \quad \forall s = 1, 2, \ldots, n_j - 1 \\
& v_s^j \leq b_j y_s^j, \quad \forall j \in J, \quad \forall s = 1, 2, \ldots, n_j - 1 \\
& \sum_{j=1}^{[J]} \sum_{s=1}^{n_j} q_i^j s x_i^j \leq M_i z_i, \quad \forall i \in N \\
& x_i^j, y_s^j, z_i \in \{0, 1\} \quad \forall j, \forall s = 1, 2, \ldots, n_j - 1 \\
& w_i^j, v_s^j \geq 0, \quad \forall j \in J, \quad \forall s = 1, 2, \ldots, n_j - 1 \\
\end{align*}
\]

- Fueling cost + delay cost + contract cost
- Never run out of fuel
- Tank capacity never exceeded at fuel stations
- Must stop before purchasing
- Tank capacity never exceeded at emergency purchase
- Must contract fuel stations for usage
- Integrality constraints
- Non-negativity constraints
Problem Characteristics

- The MIP problem is NP hard…
  - Integration of facility location and production scheduling
- The problem scale is likely to be large
  - \( 2 \sum_{j=1}^{|J|} n_j + |N| \) of integer variables, \( 4 \sum_{j=1}^{|J|} n_j + |N| \) of constraints
  - For \(|J|=2500\) locomotives each having \( n_j=10 \) stops among \(|N|=50\) fuel stations, there are 50,050 integer variables and 100,050 constraints
- Commercial solver failed to solve the problem for real applications
- Hence, to solve this problem
  - Derive optimality properties to provide insights
  - Develop a customized Lagrangian relaxation algorithm
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Theoretical Findings

Optimality Condition 1

There exists an optimal solution in which a locomotive stops for emergency fuel only when the locomotive runs out of fuel.
Theoretical Findings

Optimality Condition 2

There exists an optimal solution in which a locomotive purchases emergency fuel only if the following conditions hold:

– Its previous fuel purchase (from either an emergency or fixed station) must have filled up the tank capacity
– If the next fuel purchase is at a fixed stations, then the purchased fuel should be minimum; i.e., the locomotive will arrive at the next station with an empty tank
Theoretical Findings

Optimality Condition 3

If a locomotive purchases fuel at two fixed fueling stations $s_1$ and $s_2$ (not necessarily adjacent along the route) but no emergency fuel in between, then there exists an optimal solution in which the locomotive either departs $s_1$ with a full tank, or arrives at $s_2$ with an empty tank.

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**Diagram:**
- Fuel level vs. Tank capacity
- Station $s$, $s+1$
- Not optimal
- Either of these two is optimal
Lagrangian Relaxation

- Relax hard constraints:
  \[
  \sum_{j=1}^{\left|J\right|} \sum_{s=1}^{n_j} q_{i,s}^j x_s^j \leq \sum_{j=1}^{\left|J\right|} M_i z_i, \quad \forall i \in N
  \]

- Then add them to the objective function with penalty:
  \[
  \sum_{i=1}^{\left|N\right|} u_i \left( \sum_{j=1}^{\left|J\right|} \sum_{s=1}^{n_j} q_{i,s}^j x_s^j - \sum_{j=1}^{\left|J\right|} M_i z_i \right)
  \]

Structure of the constraints:

- Fueling plan for locomotive 1
- Fueling plan for locomotive 2
- Fueling plan for locomotive \( j \)

Constraints

Variables

Facility location and fueling constraints
Formulation of Relaxed Problem

\[
\begin{align*}
\min & \quad \sum_{j=1}^{|J|} \sum_{s=1}^{n_j} f_j \left[ \sum_{i=1}^{|N|} (c_i q_i^j w_s^j) + (p v_s^j) \right] + \alpha_1 \sum_{j=1}^{|J|} \sum_{i=1}^{n_j} f_j (x_i^j + y_i^j) + \alpha_2 \sum_{i=1}^{|N|} z_i + \sum_{i=1}^{|N|} \left( \sum_{j=1}^{n_i} q_i^j x_s^j - \sum_{j=1}^{n_i} M_i z_i \right) \\
\text{s.t.} & \quad g_j + \sum_{s=1}^{k} (w_s^j + v_s^j - r_j l_s^j) \geq 0, \quad \forall j \in J, \ \forall k = 1,2,\ldots,n_j - 1 \\
& \quad \sum_{s=1}^{k} (w_s^j + v_s^j - r_j l_s^j) \geq 0, \quad \forall j \in J, \ k = n_j \\
& \quad g_j + \sum_{s=1}^{k} (w_s^j + v_s^j - r_j l_s^j) + w_k^j \leq b_j, \quad \forall j \in J, \ \forall k = 1,2,\ldots,n_j \\
& \quad w_s^j \leq b_j x_s^j, \quad \forall j \in J, \ \forall s = 1,2,\ldots,n_j - 1 \\
& \quad v_s^j \leq b_j y_s^j, \quad \forall j \in J, \ \forall s = 1,2,\ldots,n_j - 1 \\
& \quad x_s^j, y_s^j, z_i \in \{0,1\} \ \forall j, \forall s = 1,2,\ldots,n_j - 1 \\
& \quad w_s^j, v_s^j \geq 0, \quad \forall j \in J, \ \forall s = 1,2,\ldots,n_j - 1
\end{align*}
\]
Relaxed Problem

- After relaxing hard constraints the remaining problem could be decomposed into sub-problems
  - Each sub-problem solves the fueling planning for each locomotive

\[
\text{relaxed objective} = \sum_{j=1}^{\lfloor J \rfloor} z_j(u) + \sum_{i=1}^{\lfloor N \rfloor} z_i(\alpha_2 - u_i M_i)
\]

where \( z_j(u) \) is optimal objective function of \( j^{th} \) sub-problem
Sub-problem for the $j^{th}$ Locomotive

$$\min z_j(u) = \sum_{s=1}^{n_j} f_j \left( \sum_{i=1}^{[N]} (c_{i,s,j} w^j_s + p v^j_s) \right) + \alpha_1 \sum_{s=1}^{n_j} f_j (x^j_s + y^j_s) - \sum_{i=1}^{[N]} \sum_{j=1}^{[J]} \sum_{s=1}^{n_j} q_{i,s} x^j_s$$

s.t. \quad g_j + \sum_{s=1}^{k} (w^j_s + v^j_s - r^j l^j_s) \geq 0, \quad \forall k = 1,2,\ldots,n_j - 1

$$\sum_{s=1}^{k} (w^j_s + v^j_s - r^j l^j_s) \geq 0, \quad k = n_j$$

$$g_j + \sum_{s=1}^{k-1} (w^j_s + v^j_s - r^j l^j_s) + w_x^j \leq b_j, \quad \forall k = 1,2,\ldots,n_j$$

$$w^j_s \leq b_j x^j_s, \quad \forall s = 1,2,\ldots,n_j - 1$$

$$v^j_s \leq b_j y^j_s, \quad \forall s = 1,2,\ldots,n_j - 1$$

$$x^j_s, y^j_s, z_i \in \{0,1\}, \quad \forall s = 1,2,\ldots,n_j - 1$$

$$w^j_s, v^j_s \geq 0, \quad \forall s = 1,2,\ldots,n_j - 1$$
Sub-problem for Individual Locomotive

- Three types of possible “optimal” fuel trajectory
  - Type a: From one station to nonzero fuel at another station
  - Type b: From one station to zero fuel at another station, without emergency purchase
  - Type c: From one station to zero fuel at another station, after one or more emergency fuel purchases
Shortest Path Method

- We find a way to apply a simple shortest path method to solve the sub-problem
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Test Case

Network Information

Diagram showing network information with values along the axes and connections between points.
Test Case

Locomotive Route Information
Test Case

- 12 nodes, 8 locomotives
- $\alpha_1=100$, $\alpha_2=10,000$
- Tank capacity=2500
- Different fuel price for fixed stations between $2$ to $5$ and $7$ for emergency
- Frequency assumed 1 for all locomotives
- Consumption rate assumed 1 for all locomotives
Optimal Fuel Stations

$2 \quad $3 \quad $3.5 \quad $5$

$2 \quad $3 \quad $5 \quad $2$

$3 \quad $2 \quad $2.5 \quad $4$
Optimal Fueling Plan: Locomotive 1

- $2
- $3
- $3.5
- $5

$2 → $3 (1700) → $5 → $2 (2500)

$3 (1000) → $2 (2500) → $2.5 (750) → $4
Optimal Fueling Plan: Locomotive 2

(2100) $2 → (650) $3 → (1000) $3 → (2500) $2 → (2100) $3

$2, $3, $3, $2

$3.5, $5, $5, $2

$2.5, $4
Optimal Fueling Plan: Locomotive 8
Real World Case Study

- Full railroad network of a Class-I railroad company
- 50 potential fuel stations
- Thousands of predetermined locomotive trips (per week)
- Fuel price from $1.9 - $3.0 per gallon with average $2.5 per gallon
- Tank capacity 3,000 - 5,000 gallons
- Consumption rate 3 - 4 gallons per mile
- Contracting cost of fuel stations $1 - $2 billion per year
Real World Case Study

- Algorithm converges after 500 iterations in 1200 CPU seconds
- The optimality gap was less than 6%
- This model can efficiently reduce the total cost of the system
Real World Case Study

- Solution 1: Benchmark (current industry practice)
- Solution 2: Optimal fueling schedule using all current stations
- Solution 3: Global optimum (using an optimal subset of stations)
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Conclusion

• A Mixed Integer Programming (MIP) model
  – Integrates fuel schedule problem and station (location) selection problem
  – Considers fuel cost, delay, and fuel station contracting costs
• LR and other heuristic methods are developed for large-scale problem with good computational performance
• We developed a network representation and shortest path method for solving scheduling sub-problems
• This problem was later used as a competition problem at INFORMS Railway Applications Section (RAS)
Thank you.
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